

# EXPERIMENTAL VERIFICATION OF POWER SYSTEM CURRENT AND VOLTAGE SYMMETRICAL COMPONENTS

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**ABSTRACT:** This paper reports results of an experimental approach to determining symmetrical components of both currents and voltages in an unbalanced three-phase electric power system. An active filter circuit described in a previous publication has been enhanced with current shunts which convert secondary current of current transformers to a measurable voltage. This enhancement allows the measurement of symmetrical components of currents while the circuit previously presented was capable of measuring only symmetrical components of voltages. Combining the two capabilities allows experimental verification of calculations which determine symmetrical components of voltages and currents associated with unbalanced and ungrounded loads.

## 1. Introduction

This paper reports results of an experimental approach to determining symmetrical components of both currents and voltages in an unbalanced three-phase electric power system. An active filter circuit described in [1] has been enhanced with current shunts which convert secondary current of current transformers to a measurable voltage. This enhancement allows the measurement of symmetrical components of currents while the circuit presented in [1] was capable of measuring only symmetrical components of voltages. Combining the two capabilities allows experimental verification of calculations which determine symmetrical components of voltages and currents associated with an unbalanced and ungrounded wye-connected load. Reference [2] examines this configuration in an end-of-chapter problem which illustrates that line to floating neutral voltage of the load contains only positive- and zero-sequence components, while load currents have only positive- and negative-sequence components. This paper presents experimental data which verify calculated values.

The University of Alaska Fairbanks (UAF) electric power option students complete a four-credit energy conversion course in the fifth semester and four-credit electric power courses in the sixth and seventh semesters. The following material appears in the first electric power course in support of teaching symmetrical components. The author has used this laboratory exercise for several years and has found that it enhances student understanding and acceptance

of symmetrical components as a significant and useful tool for power system analysis. It also provides an opportunity to discuss benefits of isolation provided by two-winding voltage and current transformers.

## 2. The Laboratory Exercise

### 2.a. Overview

The original motivation for this laboratory exercise was a very interesting and useful paper by C.A. Gross and L. Thompson [1]. That paper describes an active filter consisting of a group of interconnected RC operational amplifier filters. The active filter has, as inputs, an arbitrary set of three voltage phasors which drive three inverting two-winding transformers that step the input voltages down from  $120 V_{\text{rms}}$  to  $7.5 V_{\text{rms}}$  (16:1 ratio). Six operational amplifiers have the non-inverting input grounded, and the inverting input connected to an input resistor and feedback components consisting of a parallel capacitor and resistor. The values of the two resistors and the capacitor are selected so that at 60 Hz the voltage transfer function of these circuits is

$V_o/V_i = -0.5 + j0.866 = 1 / \underline{120^\circ}$ . This is equal to the operator “a” of the following standard equations which relate an arbitrary set of three phasors  $V_a$ ,  $V_b$ , and  $V_c$  to the so-called symmetrical components of the  $V_a$  phasor  $V_{a1}$  (positive sequence),  $V_{a2}$  (negative sequence), and  $V_{a0}$  (zero sequence).

$$V_{a1} = (1/3)(V_a + aV_b + a^2V_c), V_{a2} = (1/3)(V_a + a^2V_b + aV_c), \text{ and } V_{a0} = (1/3)(V_a + V_b + V_c)$$

The six operational amplifier circuits are connected to the input transformer secondaries and to ordinary three-input summing operational amplifiers to replicate the three equations listed above. Thus, the output of the active filter consists of the symmetrical components of the  $V_a$  phasor.

Reference [1] shows the results of using as inputs (1) a balanced set of three phasors in abc phase sequence (positive sequence output only), (2) a balanced set of three phasors in acb phase sequence (negative sequence output only), and (3) three equal phasors (zero sequence output only). In addition, [1] simulates a balanced three-phase fault (all inputs equal zero), a single line to ground fault, a line to line fault, and a double line to ground fault.

The filter design described above has been modified at UAF with the addition of three 5-ohm, 50 W, 1% current shunt resistors and a switch to disconnect the secondaries of the original input transformers and replace them with the three current shunts so that the voltages developed across the current shunts become the filter input voltages instead of the transformer secondary voltages. This limits input current to approximately 3 A and also requires that  $180^\circ$  be added to the phase angles of the filter outputs, since the inverting effect of the original input transformers is no longer part of the filter transfer function. Also, since the current shunts are connected to a common neutral (ground), it is not possible to measure currents directly without the circuit being grounded through the shunt resistors. As will be shown later, this can be overcome with a set of three current transformers which provides isolation and thus the capability of measuring currents in ungrounded Wye or Delta load configurations.

## 2.b. Laboratory equipment requirements

Other than the symmetrical component active filter and a three-phase set of balanced voltages typically available from a 120V/208V four-wire laboratory power source, the only other equipment needed are reasonably accurate multimeters for measuring voltage and current, a phasemeter, three current transformers, and three resistive loads. Clamp-on current probes make measuring current easier if a limited number of multimeters is available.

## 2.c. Laboratory procedure

First, students are asked to test the symmetrical component filter by supplying (a) positive sequence input voltages only, (b) negative sequence input voltages only, and (c) zero sequence input voltages only. This tests all aspects of the filter and emphasizes the sequence component definitions.

Second, a grounded-Wye resistive load is connected to a balanced 120 V line-to-neutral three-phase source. Resistor values of approximately 45 ohms, 60 ohms, and 100 ohms are used for phase A, phase B, and phase C, respectively. Students are asked to measure the symmetrical components of the load phase currents and load voltages with the active filter. They are also asked to measure rms voltage and current magnitudes and respective phase angles in all three phases of the circuit. Finally, phase (line) currents are calculated using the measured current symmetrical components. Since the shunt resistor and load resistor values are temperature dependent, better accuracy is obtained by calculating resistor values using voltage and current measurements rather than ohmmeter measurements taken when the resistors are cold. It is also necessary to convert the voltages measured at the symmetrical component active filter outputs to equivalent current values by dividing by the corrected value of the 5-ohm current shunts.

Finally, part two is repeated with the neutral return current path removed so that the load is an ungrounded-Wye configuration. In this case, a 10:5 current transformer (CT) primary winding is inserted in each phase and the secondary windings supply current to the active filter 5-ohm shunts. Again, all circuit voltage and current magnitudes and phase angles are measured, including the symmetrical component active filter outputs. In this case, it is necessary to measure primary and secondary currents to accurately determine the CT ratio. With this information and knowledge of the shunt resistance values, it is straightforward to convert active filter output voltages to equivalent current magnitudes. Laboratory participants are asked to calculate sequence components of load phase current  $I_a$  using measured values of the three phase currents  $I_a$ ,  $I_b$ , and  $I_c$ , and compare with measured sequence currents.

Students find it instructive that sequence currents in unbalanced loads do not cause voltages across the load resistors having the same sequences. For example, The unbalanced grounded-Wye load currents have positive-, negative-, and zero-sequence current components, while the voltages across that load are positive sequence only. Also, the unbalanced ungrounded-Wye load currents consist only of positive- and negative-sequence components, while the voltages across the load resistors contain only positive- and zero-sequence components.

## 2.d. Comparison of laboratory data and calculations

This section shows data and calculations for the symmetrical component active filter test, the unbalanced grounded-Wye load case, and the unbalanced ungrounded-Wye load case. All angles are with respect to three-phase voltage source phase “a” line to neutral voltage  $V_{aN}$ .

### 2.d.1. Symmetrical component active filter test

A. Positive sequence input:

$$V_{aN} = 120.4 \text{ V} / \underline{0^\circ}, V_{bN} = 120.4 \text{ V} / \underline{240^\circ}, V_{cN} = 120.4 \text{ V} / \underline{120^\circ}$$

Active filter output:

$$V_{a1} = 7.50 \text{ V}, V_{a2} = 0.05 \text{ V}, V_{a0} = 0.05 \text{ V}$$

B. Negative sequence input:

$$V_{aN} = 120.4 \text{ V} / \underline{0^\circ}, V_{bN} = 120.4 \text{ V} / \underline{120^\circ}, V_{cN} = 120.4 \text{ V} / \underline{240^\circ}$$

Active filter output:

$$V_{a1} = 0.07 \text{ V}, V_{a2} = 7.51 \text{ V}, V_{a0} = 0.05 \text{ V}$$

C. Zero sequence input:

$$V_{aN} = 120.4 \text{ V} / \underline{0^\circ}, V_{bN} = 120.4 \text{ V} / \underline{0^\circ}, V_{cN} = 120.4 \text{ V} / \underline{0^\circ}$$

Active filter output:

$$V_{a1} = 0.05 \text{ V}, V_{a2} = 0.03 \text{ V}, V_{a0} = 7.51 \text{ V}$$

### 2.d.2. Unbalanced grounded-Wye load case

$$V_{aN} = 120.0 \text{ V} / \underline{0^\circ}, V_{bN} = 120.0 \text{ V} / \underline{240^\circ}, V_{cN} = 120.0 \text{ V} / \underline{120^\circ}$$

$$R_a = 51.1 \text{ ohms}, R_b = 71.4 \text{ ohms}, R_c = 97.6 \text{ ohms}$$

$$I_a = 2.35 \text{ A} / \underline{0.2^\circ}, I_b = 1.68 \text{ A} / \underline{240.2^\circ}, I_c = 1.23 \text{ A} / \underline{120.3^\circ}$$

Using average shunt resistance = 5.063 ohms (at operating temperature), the symmetrical components of  $I_a$  measured at the output of the active filter are:

$$I_{a1} = 1.72 \text{ A} / \underline{-1.1^\circ}, I_{a2} = 0.330 / \underline{23.2^\circ}, I_{a0} = 0.326 / \underline{-23.7^\circ}$$

Calculating symmetrical components of  $I_a$  from measured values of  $I_a$ ,  $I_b$ , and  $I_c$  gives:

$$I_{a1} = 1.75 \text{ A} / \underline{-0.2^\circ}, I_{a2} = 0.326 / \underline{23.6^\circ}, I_{a0} = 0.325 / \underline{-23.4^\circ}$$

The largest magnitude error is 0.03 A (for  $I_{a1}$ ). The largest angle error is 0.9° (for  $I_{a1}$ ).

Point “n” is the common point for the load resistors and point “N” is the voltage source neutral. Current  $I_{nN}$  = neutral return current =  $3I_{a0} = I_a + I_b + I_c$ .  $I_{nN}$  determined by  $3I_{a0}$  using the filter output is  $0.978 \text{ A} / \underline{-23.7^\circ}$  while measured  $I_a + I_b + I_c = 0.975 \text{ A} / \underline{-23.4^\circ}$ . The magnitude error is 0.003 A and the angle error is 0.3°.

### 2.d.3. Unbalanced ungrounded-Wye load case

$$\begin{aligned} V_{aN} &= 119.9 \text{ V} / \underline{0.1^\circ}, \quad V_{hN} = 120.2 \text{ V} / \underline{240.0^\circ}, \quad V_{cN} = 120.1 \text{ V} / \underline{120.2^\circ} \\ R_a &= 51.1 \text{ ohms}, \quad R_b = 71.4 \text{ ohms}, \quad R_c = 97.6 \text{ ohms} \\ I_a &= 2.09 \text{ A} / \underline{5.3^\circ}, \quad I_b = 1.83 \text{ A} / \underline{229.6^\circ}, \quad I_c = 1.50 \text{ A} / \underline{126.1^\circ} \end{aligned}$$

Again, point “n” is the common point (floating neutral) for the load resistors and point “N” is the voltage source neutral. The measured value of neutral to neutral voltage between the end points of the now disconnected neutral current return path  $V_{nN} = 22.9 \text{ V} / \underline{157.5^\circ}$ .

Load resistors are in series with CT primary windings. Undotted terminals of the CT primary windings are connected to the common floating neutral point “n”. The phase voltages across load resistors  $R_a$ ,  $R_b$ , and  $R_c$  and the CT primary windings are, respectively,  $V_{an} = 99.1 \text{ V} / \underline{5.2^\circ}$ ,  $V_{bn} = 125.3 \text{ V} / \underline{229.5^\circ}$ , and  $V_{cn} = 139.1 \text{ V} / \underline{126.0^\circ}$ .

Calculated symmetrical components of  $V_{an}$ , using measured values of  $V_{an}$ ,  $V_{bn}$ , and  $V_{cn}$ , are:  $V_{an1} = 120.1 \text{ V} / \underline{0.11^\circ}$ ,  $V_{an2} = 0.097 \text{ V}$  (should be zero), and  $V_{an0} = 23.2 \text{ V} / \underline{157.8^\circ}$ .

$V_{an0}$  should equal  $V_{nN}$ . They differ by 0.3 V and  $0.3^\circ$ .

Using shunt resistance determined by measured shunt voltage and current, CT ratio = 2.15 (determined by measured primary and secondary currents), and a CT phase angle error of  $1.1^\circ$ , the symmetrical components of  $I_a$  measured at the output of the active filter are:

$$I_{a1} = 1.79 \text{ A} / \underline{-0.3^\circ}, \quad I_{a2} = 0.349 \text{ A} / \underline{33.7^\circ}, \quad I_{a0} = 0.008 \text{ A} \text{ (should be zero)}$$

Calculating symmetrical components of  $I_a$  from measured values of  $I_a$ ,  $I_b$ , and  $I_c$ , gives:

$$I_{a1} = 1.79 \text{ A} / \underline{0.24^\circ}, \quad I_{a2} = 0.339 \text{ A} / \underline{32.4^\circ}, \quad I_{a0} = 0.005 \text{ A} \text{ (should be zero)}$$

The largest magnitude error is 0.01 A (for  $I_{a2}$ ). The largest angle error is  $1.3^\circ$  (for  $I_{a2}$ ).

## 3. Conclusions

The material presented in this paper has shown that the symmetrical component active filter described in [1] enhanced by the addition of current shunts provides a way to accurately measure symmetrical components of both currents and voltages. Excellent agreement exists between experimental data obtained from the active filter outputs and calculations based on measured currents and voltages.

Specific cases that were described include three tests of the active filter, unbalanced grounded-Wye and unbalanced ungrounded-Wye resistive load configurations. It is particularly

interesting to observe that currents which cause voltage drops across load resistors do not necessarily have the same symmetrical components as the voltage drops. Students have found this laboratory exercise very helpful in increasing their understanding and acceptance of symmetrical components as a computational tool for power system analysis.

#### **4. Bibliography**

- [1] C.A. Gross and L. Thompson, "Properties of Symmetrical Components by Use of Active Filters," *IEEE Transactions on Education*, Vol. E-25, No. 4, pp. 136-141, November 1982.
- [2] W.D. Stevenson, *Elements of Power System Analysis, Fourth Edition*, New York, McGraw-Hill, 1982, pp. 304.

#### **5. Biographical Information**

JOHN ASPNES earned B.S.E.E. and M.S.E.E. degrees at the University of Wisconsin, Madison in 1963 and 1965, respectively, and the Ph.D. degree in EE at Montana State University in 1976. Dr. Aspnes has been professor of electrical engineering at the University of Alaska Fairbanks since 1981, and was department head from 1983 to 1996. He is a member of ASEE, a senior member of IEEE, and a registered professional engineer in Alaska.