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Exploring Effective Pedagogical Approaches for Teaching Linear Algebra to Engineering Students: A Literature Review

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Abstract

A few years ago, when the authors embarked on the task of redesigning a linear algebra course for engineering students, there was a lack of available review papers providing a comprehensive foundation on the topic and an explanation of the current state of knowledge for engineering students. Undeterred, the authors undertook an extensive examination of existing literature related to linear algebra and carefully examined their applicability for engineering students, to successfully redevelop the course. Subsequently, the objective of this paper is to provide a list of pedagogical methods the authors has reviewed for teaching linear algebra courses that applicable to engineering students. The ultimate intention is to assist future scholars who may find themselves in a similar position as the authors, enabling them to save a significant amount of time by benefiting from the insights presented in this paper.

In this paper, the examined methods were divided into two broad categories: (1) pedagogical methods focusing on specified linear algebra contents such as “span”, “linearly independence”, “linear transformation”, etc., and (2) general instructional pedagogical methods focusing on the course instead of specific topics, such as “flipped classroom”, “active learning”, “technology integration” etc. We read more than 70 literatures and only included those methods that are applicable for teaching engineering students in this paper. For instance, we excluded the literature that investigated different approaches to master proofs of some linear algebra concepts since there is not much emphasis on proofs as other topics like applications for engineering students. To ensure a comprehensive review, the author utilized multiple search engines and delved into various databases, including conference papers and mathematical education journals, to access a diverse range of valuable methodologies.

Introduction

The importance and usefulness of linear algebra have continued to grow, and a solid grounding in linear algebra becomes essential in many fields. Problems in a wide range of disciplines, such as engineering, computer science, operations research, economics, and statistics, are solved using linear algebra techniques. Many of the ABET program criteria specifically list the knowledge and application of linear algebra as a necessary skill.

Teaching linear algebra poses unique challenges due to the abstract nature of its core concepts such as vector spaces, linear transformations, and eigenvalues/eigenvectors. Research by Carlson et al. [1], Dorier [2], and Wawro et al. [3] has documented the difficulty students face in grasping these foundational principles.

One major issue is the struggle to visualize abstract concepts, which is crucial for understanding the geometric implications of vector spaces, linear independence, and transformations. This difficulty in visualization has been extensively discussed in studies by Dubinsky [4], Dorier and Sierpinska [5], Klasa [6], Dogan [7], and Harel [8]. Moreover, there is a noticeable trend among students to prioritize learning problem-solving procedures over developing a conceptual framework, which can hinder their ability to apply principles to novel situations [9], [10].

Linear algebra represents a shift from computational math, like calculus, to more theoretical concepts, which can be challenging for students, particularly those who are more comfortable with numerical computations [11]. Traditional teaching methods may not effectively address these conceptual challenges. To enhance understanding, pedagogical approaches should incorporate active learning and problem-solving strategies, as advocated by Stewart et al. [12] and Rensaa et al. [13]. Furthermore, the integration of technology in teaching should aim to support conceptual understanding rather than mere procedural knowledge [14].

The diverse mathematical backgrounds of students entering linear algebra courses further compound the complexity, requiring instructors to tailor their teaching to accommodate varying levels of prior knowledge [15]. Additionally, conventional methods of assessing student comprehension may not accurately gauge understanding of linear algebra concepts, emphasizing the need for improved assessment practices [15]. Considering these challenges, there is a pressing need to develop innovative teaching and assessment strategies to facilitate deeper understanding of linear algebra concepts.

In this paper, we examined more than 70 papers including textbooks, and extracted the methods and resources that can be applied to engineering students.

Brief History

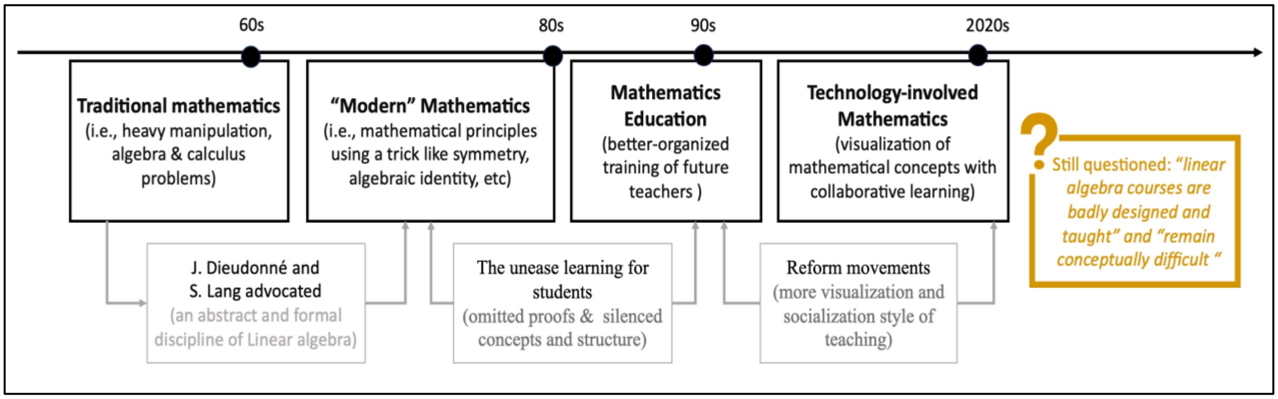
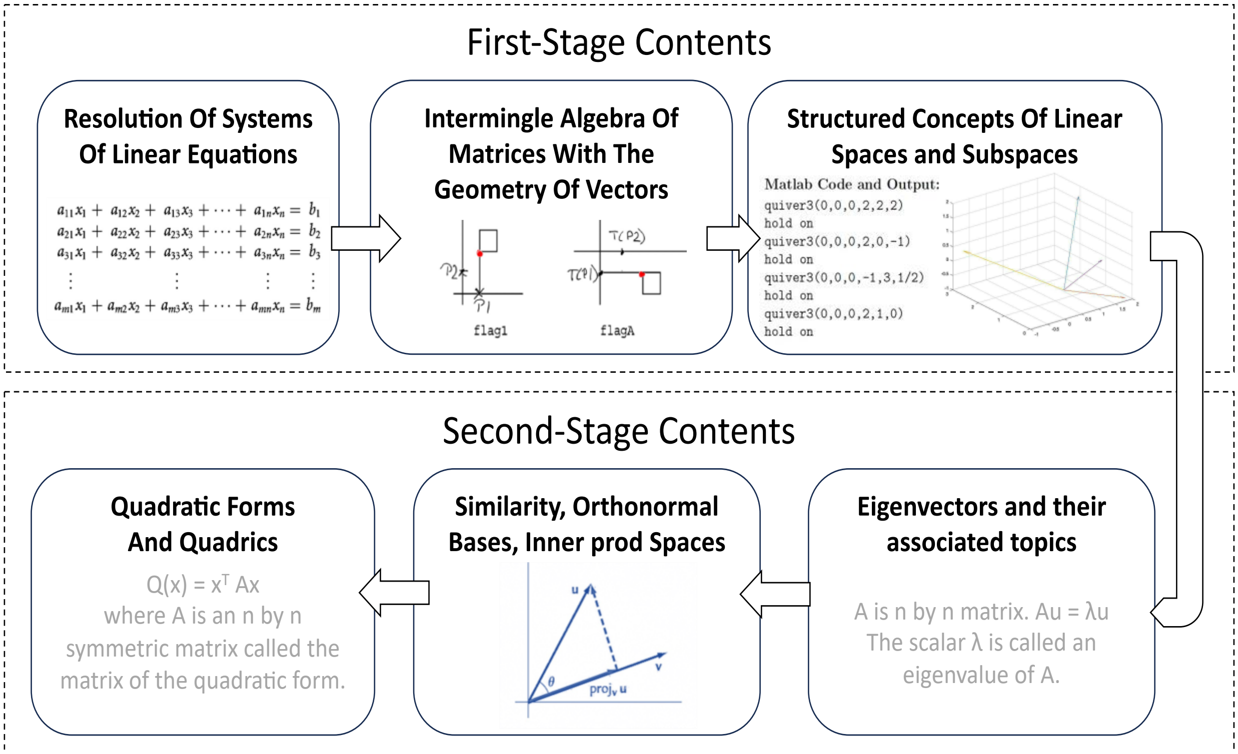


Figure 1. The History of Linear Algebra

In the 1960s (Figure 1), a group of mathematicians, notably associated with the Bourbaki School such as J. Dieudonné and S. Lang, advocated for a significant overhaul in mathematics education, labeling it "Modern Mathematics." They believed that what students learned should match how mathematicians were logically constructing math at the time. Consequently, Linear Algebra became a highly abstract and formalized subject. Aspects like Analytic Geometry and Matrix Calculus were largely overlooked in favor of emphasizing complex mathematical structures. The focus shifted away from intuition towards a more rigorous exploration of these structures, making Linear Algebra a challenging and abstract discipline for students during that period.

By the 1980s (Figure 1), educators started to notice a growing sense of frustration among students. They observed increasing failures, drop-outs, and widespread confusion. In response, there was a gradual shift towards simplifying mathematical subjects. Many educators started reducing undergraduate mathematics to sets of tasks to be completed, often omitting proofs and in-depth analyses of concepts and structures. This approach led to concerns about the quality of mathematical education, culminating in a crisis highlighted by the National Science Foundation (NSF) in 1986. In response to this crisis, a new field emerged: Mathematics Education. Universities began to establish departments or institutes dedicated to improving the training of future mathematics teachers at all levels.

Then by 1990s (Figure 1), advancements in technology have spurred numerous reform movements aimed at enhancing the teaching of mathematics. These movements emphasize the importance of visualizing mathematical concepts and promoting collaborative learning. Regard to linear algebra, the Linear Algebra Curriculum Study Group (LACSG) was formed in 1990 [2] in the United States, recommending that the curriculum of linear algebra should match the needs of client disciplines and could be taught with technologies. However, some researchers, like Dorier and Sierpinska [5], expressed skepticism, stating that linear algebra courses were often poorly designed and taught. They stated that regardless of the teaching approach, linear algebra remained a challenging subject both cognitively and conceptually. Conversely, Artigue [16] advocated strongly for the use of Computer Algebra Systems (CAS), while acknowledging the need to maintain awareness of how mathematics is learned within such software environments.



$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\ \vdots \qquad\quad \vdots \qquad\quad & \vdots \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

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quiver3(0,0,0,2,2,2)
hold on
quiver3(0,0,0,2,0,-1)
hold on
quiver3(0,0,0,-1,3,1/2)
hold on
quiver3(0,0,0,2,1,0)
hold on
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A is n by n matrix. $Au = \lambda u$
The scalar λ is called an eigenvalue of A.

$Q(x) = x^T Ax$
where A is an n by n symmetric matrix called the matrix of the quadratic form.

Figure 2: Typical Approach to Teach Linear Algebra

So, what content should be covered in a linear algebra class and how should it be taught now? (see Figure 2). LACSG played a significant role for educators to design the contents and teaching methods for the course. While there are different approaches varying from institutions and instructors, the typical approach to teaching linear algebra begins with solving systems of linear equations, followed by a blending of matrix algebra with vector geometry. It's only later in the curriculum that structured concepts like linear spaces and subspaces are introduced, along with topics like bases, and linear transformations. After that, students delve into eigenvectors and eigenvalues, exploring relationships of similarity between matrices and linear transformations. Finally, they learn to construct orthonormal bases in inner-product spaces and study quadratic forms and quadrics (see Figure 2).

Suggested Textbook

There are numerous textbooks in the market and extensive papers investigating different methods to teach and learn linear algebra, we aim to pick up those resources that are applicable for engineering students in this paper.

After reviewing many linear algebra textbooks such as [17], [18], [19], [20], [21], [22], [23], we recommend “Linear Algebra and Its Application” [24] for engineering students. This book has garnered significant praise from instructors and is widely adopted in current teaching practices. In comparison to other reviewed textbooks, "Linear Algebra and Its Applications" offers several notable advantages:

- It features a wealth of practical applications spanning various fields, with seven comprehensive Case Studies and 20 Application Projects.
- The book prioritizes visualization of concepts throughout, providing geometric interpretations for key concepts. This approach enhances student comprehension, as many learners find visual aids beneficial for understanding abstract ideas.
- Each section includes exercises that require the use of matrix programs such as MATLAB™, Maple®, or Mathematica. This hands-on approach reinforces learning and facilitates practical application of concepts.
- The textbook is accompanied by a dedicated website, offering extensive resources to support both learning and instruction. The site includes data files for 900 numerical exercises, as well as materials for Case Studies and Application Projects. Additionally, files compatible with MATLAB, Mathematica, Maple, and TI calculators are available, enabling students to explore fundamental mathematical and numerical concepts in linear algebra using these software tools.

Literature on Specific Topics

Many papers focus on students' conceptual understanding in linear algebra, particularly in topics like systems of linear equations, linear independence, and eigenvectors/eigenvalues. These papers examine trends in studying student reasoning, theoretical frameworks, and the creation of educational resources, which could be applied to the education for engineering students.

Systems. Table 1 presents a summary of studies that examine the use of systems of equations in an educational context, specifically their application and understanding among students. These studies collectively highlight the importance of practical applications, the challenges in transitioning between different forms of representation (symbolic and geometric), and the significance of understanding the relationships between equations and their symbolization. It reveals the educational strategies for teaching systems of equations to engineering students, who require a solid grasp of these concepts for real-world problem-solving.

Table 1. Literature about teaching systems of equations.

Literature	Topic	Findings	Relevance for Engineering Students
[25]	Introduced a traffic flow problem that could be solved using systems of equations.	Students were able to use the modeling task to extend their understanding of variables, equations, and systems of equations.	Engineering students often encounter real-world problems that can be modeled using systems of equations, such as traffic flow analysis or electrical circuit design. This study's approach to teaching systems of equations through practical applications can be beneficial for engineering students.
[26]	Focused on students' difficulties in transitioning between symbolic and geometric representations for systems of equations.	Students struggled with these transitions but used hand-drawn visual representations of partial planes, which aided in understanding.	Engineering involves both abstract mathematical representations and real-world applications. Understanding how students grapple with the symbolic and geometric aspects of systems of equations can help engineering educators design more effective instruction tailored to their needs.
[11]	Explored relationships between equations in a system and asked in-service teachers to recreate ideas about when two systems have the same solution.	The paper provides insights into how equations describe systems and their solutions.	Engineering often involves solving complex systems of equations, and understanding the relationships between equations is crucial for effective problem-solving. This paper's insights can be useful for engineering students as they tackle practical engineering problems.
[12]	Extended previous work by examining various ways students symbolize systems of equations and their solutions.	The study delves into how students represent and understand systems of equations symbolically.	Engineering disciplines heavily rely on mathematical modeling, and effective symbolization of systems of equations is fundamental. This research can help engineering students develop better symbolization skills, which are essential for their coursework and future careers.

Action, Process, Object, Schema (APOS). In linear algebra education, "Action, Process, Object, Schema" (APOS) theory has been used. Kazunga and Bansilal [27] applied APOS to assess teachers' matrix operation understanding. Trigueros [28] explored schema development during modeling activities. Trigueros [29] examined LAS development in an introductory course, while Oktac [30] investigated mental constructions. These studies provide valuable insights into linear transformation and other related concepts such as range, image, domain, dimension, and systems of linear equations.

Table 2 summarizes the learning processes of engineering students regarding vector spaces and linear algebra. It discusses the action of decomposing vector spaces [31], the process students use to reason about span and basis [32], [33], [34], and the object of study, linear independence [35], [36]. It also touches on the schema students develop for understanding these mathematical concepts and their applications in engineering.

Table 2. Literature about teaching span, linear independence, and basis.

Literature	Topic	Findings	Relevance for Engineering Students

[31]	Genetic decomposition of abstract vector spaces.	Explored components in binary operations and related aspects.	Limited direct relevance but understanding vector spaces is foundational for engineering fields like computer graphics and robotics.
[32], [33], [34]	Examining student reasoning for span, linear independence, and basis.	Highlighted students' varied reasoning modes and preferences.	Relevant for engineering mathematics courses, especially when dealing with vector spaces, linear transformations, and optimization.
[35]	Correlation between students' formal definitions and geometric interpretation of linear independence.	Found a lack of correlation between them.	Important for engineering students when working with vector spaces and geometric interpretations in fields like computer graphics and mechanical design.
[36]	Example generation and students' focus on row reduction for linear independence.	Students primarily used row reduction over understanding linear combination relations.	Good for engineering students in linear algebra courses when solving systems of equations and working with matrices.

For the concept of linear independence, in addition to the literature mentioned in Table 2, Ertekin et al. [35] found that teacher trainees were better at algebraic rather than geometric aspects of the concept. In contrast, Dogan-Dunlap [37] noted that geometric representations could enhance understanding when combined with algebraic elements. Wawro et al. [3] designed teaching sequences to help students integrate both algebraic and geometric interpretations of span and linear independence, aiming to support diverse learning styles.

Modes of thinking. Table 3 discusses the impact of different modes of thinking on engineering students' understanding of linear algebra concepts. Bagley and Rabin [38] explored the use of computational thinking in creating linearly independent sets of vectors in R^4, which is especially beneficial for engineering disciplines like graphics and design. Dogan-Dunlap [37] analyzed students' homework on vector independence in R^3, identifying 17 thinking modes, with a focus on geometric and algebraic reasoning. Dogan [39] assessed the impact of interactive web modules on students' thinking modes in linear algebra, finding that such tools enhance geometric reasoning skills for engineering problem-solving.

Table 3. Literature about modes of thinking.

Literature	**Topic**	**Findings**	**Relevance for Engineering Students**
[38]	Students creating linearly independent sets of vectors in R^4 using "computational thinking".	students' creative approaches to computational thinking and their ability to generate examples and assess linear independence.	Relevant for engineering students dealing with vector spaces and mathematical reasoning in fields like computer graphics and mechanical design.
[37]	Students' homework solutions involving linear independence and dependence of vectors in R^3, using Sierpinska's modes of thinking.	Identified 17 student thinking modes, with a focus on geometric and algebraic thinking modes in explaining linear independence.	Relevant for engineering students working with vector spaces and geometric interpretations in various engineering applications.
[39]	Interactive web-modules for linear algebra students with different levels of access, exploring various modes of thinking.	Refined categories into 9 student thinking modes, with some allowing for Sierpinska's modes of thinking. Students who used interactive modules outside of class and during class discussions were more likely to reason geometrically.	Helpful for engineering students in linear algebra courses, helping them develop geometric and algebraic reasoning skills applicable in engineering problem-solving.

Emergent models. Table 4 reviews research on emergent models in linear algebra education. Carcamo et al. [40], [41] showed their use in teaching span for cybersecurity applications. Wawro et al. [42] demonstrated how emergent models aid understanding of span and linear independence through imaginative scenarios. Plaxco and Wawro [43] and Rasmussen et al. [44] highlighted flexible student understanding and the importance of conjecturing in mathematical reasoning. Dogan [45] focused on problem-solving flexibility, while Zandieh et al. [46] and Payton [47] explored student reasoning, beneficial for both mathematics and, in certain contexts, engineering education.

Table 4. Literature about emergent models in teaching linear algebra.

Literature	Topic	Findings	Relevance for Engineering Students
[40], [41]	Using emergent models and mathematical modeling to teach span, focusing on creating secure passwords using vectors.	Successful transition from informal to formal understandings of span and spanning sets.	Cybersecurity and mathematical modeling.
[42]	Using emergent models in the context of a person traveling on two modes of transportation (magic carpet and hoverboard) to teach span and linear independence.	students engaged in transitioning from informal to formal ways of reasoning about span and linear independence.	More suitable for general mathematics education.
[43]	Span and linear independence among linear algebra students, including Travel, Geometric, Vector algebraic, and Matrix algebraic categories.	Students often had flexible understandings categorized into various concept image categories.	More beneficial for educators in mathematics courses.
[44]	Students generating examples and making conjectures about sets of linearly independent and dependent vectors in R^2 and R^3.	Emphasized ways of conceptualizing progressive mathematization and student thinking through conjecturing, generalizing, and justifying.	More focused on mathematical reasoning in linear algebra.
[45]	Student understanding of linear independence, focusing on connections students make and the need for flexibility in problem-solving.	students being able to activate non-routinized plans for problem-solving.	Can be applicable to broader mathematical education.
[46]	Student-generated everyday examples for basis and their use of metaphors like travel and building in understanding vector spaces.	Identified codes related to vector relations and characteristics in everyday examples.	Limited direct relevance to engineering students; more suited for mathematics education.
[47]	Student reasoning about linear independence in an active learning approach for large lectures.	Identified hub concepts like pivot positions and basic/free variables and highlighted common misconceptions.	More applicable to mathematics education, but understanding linear independence can benefit engineering students in certain contexts.

For the topic of vectors, planes, and their intersections in R^3, in addition to Table 4, Sandoval and Possani [26] analyzed the difficulties faced by students when working with different representations of vectors, planes, and their intersections in three-dimensional Euclidean vector space.

Eigen theory. Table 5 reviews studies on teaching eigen theory, emphasizing geometric understanding and modeling in engineering education. It includes using APOS theory [48] and software like GeoGebra [49] to help students conceptualize eigenvalues and eigenvectors, demonstrating benefits in subjects like structural analysis. It emphasizes the using of embodied thinking and dynamic geometry software for understanding eigen theory, while others focus on mathematicians' use of gestures for conceptualizing eigenvectors. These studies also cover the effective use of problem-based learning to deepen conceptual understanding in linear algebra.

Table 5. Literature about teaching eigen theory using geometric interpretations.

Literature	**Topic**	**Findings**	**Relevance for Engineering Students**
[48]	Genetic decomposition of eigenvalue, eigenvector, and eigenspace concepts using APOS framing in an economics modeling situation.	Effective for student motivation and learning, but students struggled with recognizing the nature of eigenspaces and their dimensions.	More suited for mathematics or economics education.
[49]	Using modeling activities to teach eigenvectors and eigenvalues to architectural students, including moments of inertia problems with GeoGebra software.	Tasks helped students solve problems in new ways and make connections between mathematical thinking worlds.	Potentially relevant for engineering students when dealing with structural analysis and moments of inertia in engineering design.
[50]	Exploring student thinking about eigenvector and eigenvalue concepts using Tall's three worlds, focusing on the embodied approach.	Encouragement to think in an embodied manner aided symbolic manipulations and understanding of the eigenequation.	Relevant for engineering students when studying eigenvalues and eigenvectors in structural analysis and control systems.
[51]	Initial learning about the eigenequation using Geometer's Sketchpad with a focus on student interactions with dynamic geometry software.	Students intentionally manipulating sketches to build their understanding of eigen theory relationships.	more applicable to mathematics education.
[52]	Exploring student approaches to advanced eigenvalue and eigenvector problems using Geometer's Sketchpad, involving synthetic-geometric and analytic-arithmetic approaches.	Student approaches evolved towards increased generalization.	More suited for mathematics education.
[53]	Analysis of mathematicians' gestures when discussing eigenvectors to explore the role of motion and gesture in conceptualizing eigenvectors.	Mathematicians use language and gesture to convey motion and time aspects in their conceptualization of eigenvectors.	Offers insights into abstract mathematical concept visualization.
[54]	Study on how students reinvented the diagonalization equation $A = PDP^{-1}$ using stretch factors and stretch directions, emphasizing geometric and motion aspects.	Highlighted ways of symbolizing and students' graphic representations of mathematical relationships.	More suitable for mathematics education.
[55]	Investigation of physics students' transfer of learning of eigenvalues and eigenvectors from prerequisite experiences to quantum mechanics.	Potential for well-designed tasks to help students transition from arithmetic to geometric and structural modes of thinking.	Relevant for engineering students in physics-related courses with quantum mechanics components.
[56]	Study on the teaching and learning of eigen theory in a Problem-Based Learning (PBL) context, emphasizing object and schema concept constructions.	High number of object and schema concept constructions among students, including those from lower tracks.	Relevant for engineering students, particularly in PBL environments, showcasing the potential for conceptual understanding in linear algebra.
[57]	Examination of students' understanding of eigenspaces, focusing on themes related to linear combinations of eigenvectors.	Identified seven themes in students' responses, including reasoning about linear combinations and the linear independence of eigenvectors.	Relevant for engineering students studying linear algebra, aiding in understanding eigenspaces and linear combinations.

For the topics of eigenvalues and eigenvectors, in addition to the above table, Caglayan [52] used dynamic geometry software to enhance math majors' understanding, while Beltrán-Meneu et al. [49] applied visual methods for architecture students. Gol Tabaghi [51] and Sinclair & Gol Tabaghi [53] found that geometry helps in understanding these concepts, with diagrams acting as a bridge between physical gesture and mathematical language. Andrews-Larson et al. [58] and Zandieh et al. [46] focused on geometric representations and reinvention of equations.

Application-based instructional approaches. Recent studies have employed real-world contexts to teach linear algebra, enhancing its relevance for engineering students. For example, Cárcamo et al. [40] used password generators to explain span and linear independence, while Possani et al. [25] modeled traffic flow to teach systems of linear equations. Andrews-Larson et al. [58] and Martin et al. [59] applied image manipulation to illustrate linear transformations. Zandieh et al. [54] used stretching space for eigenvalues and eigenvectors, demonstrating linear algebra's applicability in solving practical engineering problems. These context-driven approaches, anchored in real-life scenarios, bridge theoretical concepts with practical applications, making abstract ideas more tangible and directly relevant to a variety of engineering fields.

Literature on General Instructional Innovations

For literatures the authors have reviewed, different instructional innovations of teaching linear algebra were covered. We include a list of approaches that are applicable to engineering students below and describe several of them in details at the beginning of this section.

Pedagogical Designs with Maple and Cabri. Klasa [6] developed several pedagogical approaches for teaching Linear Algebra concepts, including linear transformations, eigenvectors, quadratic forms, conics with changes of bases, and singular values. These approaches were enhanced by the utilization of both the geometric micro-world Cabri and the computer algebra system Maple. The impact was not studied in the paper, but the method could be tried for engineering linear algebra education.

When utilizing the CAS Maple, communication primarily occurs in a symbolic and computational manner. However, to introduce a geometric perspective, instructors can employ animations created by the teacher. While these animations aim to enhance students' geometric comprehension, Maple often assumes the role of a director, limiting student interaction to passive observation. In collaborative Maple exercises, students typically adapt provided examples to new scenarios. In contrast, the geometric micro-world of Cabri offers students the freedom to generate their own visualizations, manipulate transformations and bases, adjust quadratic forms, manipulate vectors, and conduct independent explorations. This environment fosters a deeper understanding of both geometric and conceptual aspects of Linear Algebra. Maple primarily serves as a computational tool in this context. Moreover, in team-based computer lab settings, students engage actively with their peers and occasionally with the instructor, creating a dynamic learning atmosphere that enhances comprehension and collaboration.

Lab, Online Assessments, Application in Interactive Jupyter Notebooks. Silva et al. [60] redesigned the linear algebra course with multiple innovations and students reflected positively about this approach in the paper. Firstly, there was a reorganization of the course structure. The traditional linear algebra curriculum, typically consisting of three lecture hours per week, was redesigned. The theoretical components were condensed into two lectures per week, held on Mondays and Wednesdays, while the examples and applications were allocated to a single lecture per week, taking place on Fridays. Instead of the conventional expository teaching method for presenting examples and applications, the third lecture hour was transformed into a computational lab session, adopting a flipped classroom approach. During these sessions, students engaged in computer-based activities that applied linear algebra concepts to real-world scenarios. These examples were initially introduced during the Monday/Wednesday lectures to emphasize the strong connection between theoretical concepts and their practical applications, fostering a synergistic relationship between lectures and labs. The real-world examples served to motivate students by showcasing the relevance of linear algebra and helping them apply these concepts in practical contexts. Additionally, homework assignments and exams were transitioned to an online assessment platform called PrairieLearn [61]. Furthermore, collaborative learning was facilitated by carefully designing tasks, assigning team roles, utilizing Jupyter notebooks for interactive work, and involving undergraduate course staff in the lab sessions.

Active Learning, Code Concepts, Application Projects, MATLAB Auto Grader. Li [62] redeveloped the course for engineering students by integrating MATLAB into it with several innovations and the study showed positive feedback from students about this approach. The course redesign incorporates several key enhancements aimed at fostering a more dynamic, computational, and applied learning experience. Firstly, each 50-minute class session held on Mondays, Wednesdays, and Fridays is divided into two segments. The first segment involves instructors teaching main topics, while the second segment sees students actively engaging in group problem-solving sessions to apply and reinforce their linear algebra skills. Secondly, instructors utilize pre-written MATLAB live scripts to visualize abstract concepts during lectures, followed by students tackling more complex problems using MATLAB. Thirdly, the course integrates "coding core linear algebra concepts" as a fundamental component of course tasks, enhancing students' computational proficiency and understanding of the core concepts. Fourthly, five application projects have been introduced to the curriculum, providing opportunities for students to apply linear algebra concepts in real-world contexts. Lastly, the adoption of MATLAB Auto Grader for projects and certain tasks streamlines assessment processes and provides valuable feedback to students.

Flipped Classroom. Love et al. [63] implemented a flipped classroom approach, where students engaged with course materials outside of class, allowing in-class time for active problem-solving and collaborative learning. This study showed that the flipped classroom section demonstrated greater improvement between sequential exams compared to their peers in the traditional lecture section, despite similar performance on the final exam. Additionally, students in the flipped classroom section reported positive perceptions of their learning experience, particularly highlighting the benefits of collaboration and instructional videos in the course.

Running a flipped classroom involves several key steps:

- Preparation of Materials: Prepare pre-recorded lectures, instructional videos, readings, or other materials that cover the course content. Ensure these resources are accessible to students before each class session.
- Communication: Clearly communicate expectations and instructions to students regarding their pre-class preparation. Provide guidelines on how to engage with the materials effectively.
- Classroom Activities: Plan interactive and collaborative activities for in-class sessions. These activities should focus on applying and practicing the concepts covered in the pre-class materials.
- Facilitation: Act as a facilitator during class time, guiding students through discussions, group work, problem-solving activities, and answering questions that arise from their pre-class learning.
- Feedback: Provide timely feedback on students' pre-class work and performance during in-class activities. This helps reinforce learning and addresses any misconceptions.
- Assessment: Design assessments that align with the learning objectives of the course and reflect students' understanding of the material covered both in and out of class.
- Reflection and Adaptation: Continuously evaluate the effectiveness of the flipped classroom approach through student feedback, assessment results, and observation. Adjust as needed to improve student learning outcomes.
- Support: Offer support to students who may require additional assistance with understanding the pre-class materials or navigating the flipped classroom format. This could include holding office hours, providing extra resources, or offering one-on-one tutoring sessions.

However, Hardebolle et al. [64] and Se et al. [65] demonstrated that the flipped format did not have any significant impact on students' achievement compared to traditional lecturing but showed a reduced attainment gap for women and students with less prior knowledge in mathematics.

Inquiry-Oriented Linear Algebra (IOLA). Wawro [66] developed linear algebra course materials based on inquiry-oriented approach. Mathematical Association of America (MAA) has a special interest group on inquiry-based learning (IBL), with the purpose bringing practitioners and others interested in IBL together to share teaching resources and experiences, encourage, and publicize research related to IBL, and to promote the proliferation of IBL in Mathematics through conversation and professional development.

Inquiry-based learning (IBL) in Mathematics is a pedagogical framework in which students develop deep mathematical insights through collaborative, communicative, and comprehension-building activities. From Wawro’s chapter [66], the specific methodology used to develop IOLA is the Design Research Spiral, consisting of five primary phases: Design, Paired Teaching Experiment (PTE), Classroom Teaching Experiment (CTE), Online Work Group (OWG), and Web. In summary, these phases involve:

- Design Phase: Creating initial drafts of unit tasks and learning objectives.
- PTE Phase: Testing unit tasks with student pairs in a modified instructional environment.

- CTE Phase: Implementing unit tasks in a classroom setting, with a team member serving as the teacher-researcher.
- OWG Phase: Testing unit tasks in classroom settings with experienced IOLA users.
- Web Phase: Finalizing unit tasks and instructor support materials, then disseminating them on the IOLA website.

Concept-Rich Instruction Approach. Tashtoush et al. [67] introduced a teaching method based on the components of Concept-Rich Instruction Approach in Linear Algebra Course. This study concluded that significant improvements in performance were observed among those who utilized this approach compared to those who did not.

Adopting a teaching approach rooted in Concept-Rich Instruction entails guiding students towards practical applications and real-world scenarios to deepen their understanding of concepts. This method typically progresses through five key stages [68].

- Practice (Application): Emphasizing the significance of comprehensive learning, this stage involves enriching the teaching process with ample examples, exercises, and activities. Adequate repetition, reflective thinking, and concept visualization are essential to solidify understanding [69].
- Contextualization: By fostering discussions that consider students' diverse approaches and analyzing their mistakes, learners engage in a variety of applications to grasp the concept fully.
- Conceptual Understanding: This stage focuses on verbal expression, translation, and interpretation of concepts using words and symbols. Teachers act as mediators, guiding students in developing a deep understanding through verbal expression and ensuring correctness in meaning attribution [70].
- Re-contextualization (Concept Reframing): Linking new experiences with prior ones and exploring novel applications of concepts are key here. Learners must be trained to connect new experiences with previous knowledge, overcoming misconceptions from past experiences [71].
- Verification (Concept Mastery): Encouraging students to apply learned concepts in various contexts and problems is vital. This stage focuses on transferring understanding to practical situations, ensuring mastery of the concept.

Several factors influence student achievement, particularly related to teachers and instruction. Teachers play a crucial role in student success by employing diverse teaching strategies and considering individual differences, thereby fostering an inclusive learning environment.

Other General Instructional Approaches. The use of classroom response systems in teaching linear algebra, as explored by Cline et al. [72], represents a notable technological advancement in education. These systems enable real-time electronic feedback from students, facilitating the identification of conceptual misunderstandings. Through analyzing 781 responses across various institutions, the study highlights the effectiveness of challenging and theoretically focused questions in stimulating classroom discussion. Those questions, which prioritize interpretation over calculation, prove instrumental in enhancing student engagement and understanding, offering a valuable approach to teaching linear algebra to engineering students. A similar question platform called Question Press has been implemented in two of the sections taught by one of the co-authors, with the intention to compare student performance against other sections by the end of the semester and then improving future teaching and learning of linear algebra.

Technology in education enhances engagement and allows for responsive teaching to student reasoning. Quinlan and Tennenhouse [73] found that while learning LaTeX improved students' mathematical precision, it did not enhance conceptual understanding. For engineering students, mastering LaTeX is not essential, though it could benefit some in their professional practices.

Nanes [74] advocates for a shift from traditional lectures to team-based learning (TBL) in medium-sized linear algebra classes, which has been linked to enhanced student attitudes, grades, and exam performance. This TBL method employs cycles of individual and team preparation tests, application activities that dominate class time, and consistent team-based peer reviews. The approach aims to improve historically struggling students and cultivate self-directed learning. While we recognize the benefits of TBL and encourage its use, incorporating such group work in classes is suggested rather than required.

In a study by Teixeira [75] an innovative approach to enhancing the teaching of linear algebra was examined. The method involves integrating peer instruction with a seminar strategy and leveraging didactic engineering to facilitate the comprehension of abstract concepts, foster innovative problem-solving research, and promote practical application of educational principles. To gauge the effectiveness of this approach, the Students' Evaluation of Educational Quality (SEEQ) questionnaire was administered and analyzed. The findings revealed that the blended method fosters a more dynamic learning atmosphere compared to traditional approaches, fostering the development of social interaction skills, and enhancing the learning process for students. Overall, the study demonstrated that the proposed method enhances student motivation and encourages reflection, crucial elements for cultivating a collaborative learning environment that positively influences academic retention and performance.

Other Aspects

Instructors’ Reasoning. Grenier-Boley [76] delved into the pedagogical approaches for presenting linear algebra's unifying concepts, which could be crucial for engineering students. The research highlighted that conventional instructional tactics can inadvertently simplify these concepts, which may prevent students from comprehensive understanding. For engineering students, who rely on a robust grasp of these principles, Grenier-Boley suggests a need for teaching strategies that promote an in-depth engagement with these foundational concepts to enhance their academic and professional competence.

Student learning experience. Rensaa [77] and Martínez-Sierra & García-González [78] explore the student experience in linear algebra, noting a tendency towards seeking correct answers over deep understanding, and a range of emotions from satisfaction to distress linked to classroom experiences. Andrews-Larson et al. [58] and Zandieh et al. [54] explore the effectiveness of inquiry-oriented approaches in linear algebra, focusing on student engagement with complex concepts like matrix multiplication and diagonalization through eigenvalues and eigenvectors. They emphasize the instructor's pivotal role in guiding students' conceptual development and facilitating a deeper understanding through strategic interventions and symbolizing techniques.

Fostering active learning and critical thinking, potentially enhances students' ability to apply mathematical concepts in practical contexts.

Role of technology. Several studies have focused on the synergy between different mathematical representations and visualization tools to aid student learning in linear algebra. Interactive Mathematica worksheets and dynamic geometry software like Geometer's Sketchpad and GeoGebra have been instrumental in making concepts like eigenvectors and eigenvalues more tangible. For instance, Gol Tabaghi [51] showed how vector manipulation can deepen the understanding of these concepts, while others have used GeoGebra to demonstrate polygon transformations and software like Photoshop for visualizing linear transformations. MATLAB's role in teaching engineering students through mathematical modeling was noted by Dominiquez-Garcia et al. [79]. However, Dogan [39] and Dogan-Dunlap [37] addressed the accessibility challenges posed by software reliance with the development of an interactive web-based module for visualizing vectors in R3. This shift to web-based resources, while improving accessibility, raises issues of scalability and browser compatibility. To address such challenges, one of the authors is developing GUIs to facilitate learning in applied mechanics. These GUIs aim to enhance the learning of math-based problems in engineering education through visual tools and interactive simulations, making complex concepts more accessible and applicable to real-life engineering scenarios.

Conclusion

This paper offers a comprehensive review of teaching methodologies for foundational linear algebra tailored to engineering students. Through the examination of over 70 literature sources, including textbooks, the authors categorized their findings into two main areas: research focusing on specific linear algebra contents and research focusing on general instructional pedagogical innovations. Notably, the authors delve into the implementations of general instructional pedagogical innovations such as Inquiry-Based Learning, Flipped Classroom, and Concept-Rich Instruction Approach, with the aim of facilitating quick adaptation by instructors. Additionally, the paper evaluates various textbooks and highlights "Linear Algebra and Its Application" [24] as a recommended resource for engineering students.

The significance of this review paper lies in its potential to revolutionize pedagogical practices and drive advancements in teaching linear algebra to engineering students. By providing a comprehensive foundation on pedagogical methods, the paper fills a critical gap in available resources and offers valuable insights for educators at all levels. Through its in-depth examination of methodologies and research techniques, this paper equips scholars and instructors with practical guidance, serving as an invaluable resource to navigate the complexities of teaching linear algebra with confidence and effectiveness. With its potential to shape the future of linear algebra education, this review paper lays the groundwork for enhanced learning experiences and fosters a deeper understanding of this fundamental mathematical subject.

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