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Facilitating Learning of Projectile Problems with a Unified Approach

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Solving Projectile Problems by using a Unified Approach

Abstract

What projectile problems share in common is a given object accelerates downward with the acceleration due to gravity, which should be the key to solving this type of problems. However, students are often challenged by variations in given conditions (e.g., initial velocity or launch angle). They may try to follow different procedures for different given conditions while they neglect the essential common characteristics of projectile problems. This paper will provide a unified calculus-based approach which simplifies problem solving by shifting students' attention from variations of surface features to the governing principle of projectile problems. First, we will justify the instructional benefit of this approach from the perspective of cognitive load theory. Then, we will demonstrate how this approach can solve projectile problems by using the same set of equations with the only different unknowns. Examples and assessment results will be provided to demonstrate the effectiveness as well as the challenges when teaching this approach.

Introduction

Analyzing projectile motion is a typical type of kinematics problems in Dynamics. Although students have learned how to solve this type of problems in Physics I, they often face challenges because of their deficiency in general problem solving skills and weak foundation in math and physics. Students' deficiency in problem solving has been well documented in the literature addressing the difference between novices and experts. When novices solve a problem, they will work backwards from the goal to the given using a means-ends strategy. In contrast, experts, using existing knowledge held in long-term memory, often solve the problem forward from the givens to the goal. The means-ends strategy often imposes a heavy cognitive load because learners are required to simultaneously consider the givens, the goal, the differences between them, and possible strategies which might reduce the differences. As a result, learners' working memory, with the limitations restricted to capacity and duration, may experience cognitive overload. According to Cognitive Load Theory, learning will be hindered if cognitive load exceeds working memory resources. Therefore, we need to design an instructional strategy to avoid cognitive overload.

The learning challenge might also be caused by students' bad habits in solving dynamics problems. For example, students may start to write equations without specifying a coordinate system and identifying the givens and finds carefully. Students usually take these habits as trivial carelessness, but it is these bad habits that prevent students from developing solid problem solving skills for learning dynamics. For this reason, we need to develop a training plan to help them build good habits not only for learning dynamics but also for general problem solving.

In this paper, a unified approach is proposed to serve as one stone aiming for two birds: avoid cognitive overload and help students develop certain good habits for problem solving. This approach is developed according to instructional guidelines offered by cognitive load theory. Although the effectiveness of the approach requires more rigorous experimental educational research to be demonstrated, we hope this unique work-in-progress could provide insights into projectile motion instruction. We will first provide a brief introduction to cognitive load theory which is the theoretic framework for the proposed approach. We will then explain why the traditional approach to solving projectile problems might not help students learn. Finally, we will present this approach to illustrate its benefits of avoiding cognitive overload and helping develop good habits for problem solving.

Theoretic Framework: Cognitive Load Theory

Cognitive load theory is a comprehensive set of instructional principles based on human cognitive learning processes [1-2]. Since our working memory can only process 7 ± 2 items at one time, learning will be hindered if information to be processed exceeds those limits [1-2]. Based on this rule, cognitive load theory provides specific instructional guidelines which minimize wasted mental resources and put limited mental resources to work in ways to maximize learning.

Cognitive Load imposed on working memory can be divided into two categories: intrinsic cognitive load and extraneous cognitive load [2]. As indicated by the names, intrinsic cognitive load refers to the mental work determined by the intrinsic nature of learning materials that the learner needs to acquire for achieving learning goals while the extraneous cognitive load refers to the mental work which is unnecessary and extraneous to learning goals. Intrinsic load is primarily determined by instructional goals or the intrinsic nature of the material; extraneous load is solely caused by the instructional procedures in use. Since intrinsic load is related to the complexity of learning materials, difficult learning materials will of course impose more cognitive load than easy ones. Generally, we cannot change intrinsic load during instruction due to its intrinsic nature. By contrast, extraneous load must be controlled through instructions because it wastes limited mental resources which should be committed to intrinsic load to maximize learning. Instruction guidelines provided by cognitive load theory are to balance mental work to achieve efficient learning.

Use Cognitive Load Theory to Improve Projectile Motion Instruction

Cognitive load theory could help explain why the traditional strategy may hinder learning by imposing extraneous cognitive load. The focus of cognitive load theory is to optimize cognitive load imposed on working memory. If learning materials are new to learners, they will result in cognitive load. So let's first examine how many new items may occur when use the traditional approach to solve projectile problems with the example below. The solution to this problem is taken from the solution manual provided by the publisher [3]. Without an explicit note, all examples used in this paper use a Cartesian coordinate system with the origin at the initial position and upward as the positive y-direction.

Example: The skier leaves the 20° surface at 10 m/s. Determine the distance d to the point where he lands [Example 13.7 in 3].

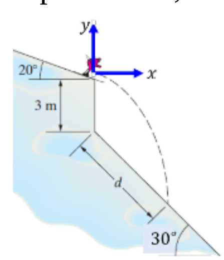


Table 1 Solution and Cognitive Load Analysis

Solution	Cognitive Load Analysis
$a_x = 0$, $a_y = -9.81$ m/s^2	Most students should be familiar with it so it will not be counted as a new item.
$v_x = 10 \cdot \cos 20°$ m/s,	No new item is introduced as most students should be able to figure out this step as well.
$v_y = -9.81 \cdot t - 10 \cdot \sin 20°$ m/s	Some students might take it as a new equation if they do not understand that this is simply the

	velocity representation of the motion with constant acceleration or they still need to list $v_f = v_i + a \times t$ in their formula sheet.
$s_x = 10 \cdot \cos 20° \cdot t$ m	Some students might also take it as a new equation they need to memorize if they do not truly understand what $a_x = 0$ means.
$s_y = -\frac{1}{2} \cdot 9.81 \cdot t^2 - 10 \cdot \sin 20° \cdot t$ m	Some students might take it as a new equation as well for the same reason.
When he hits the slope, we have $s_x = d \cdot \cos 30° = 10 \cdot \cos 20° \cdot t$ m,	Most students should be able to figure out the relationship.
$s_y = -3 - d \cdot \sin 30° = -\frac{1}{2} \cdot 9.81 \cdot t^2 - 10 \cdot \sin 20° \cdot t$ m	Most students should be able to figure out this step as well.
Solving these two equations together, we find $t = 1.01$ s, $d = 11.0$ m	If students still struggle with algebra, they may get stuck in this step. So solving the two equations may result in new items as well.

As explained above, this approach may result in more than five new items which might exceed the limit of working memory. Meanwhile, if students make a mistake in any intermediate step, they will not be able to solve the problem. Furthermore, if the given conditions are changed (e.g., d is given while v_0 is unknown), even the strategy is the same, students may take it as a new problem because the sequence of equations to be solved will change.

All these new items impose cognitive load on learners. If no instructional strategy can alter the load, the load should be generally treated as intrinsic load. You will find soon that the load can be reduced by the proposed unified approach. In other words, such load is extraneous load because it can be reduced by different instructional strategies.

Different from the traditional approach which is based on algebra, the proposed unified approach is calculus-based. Since students should be familiar with the general relationship between position, velocity, and acceleration (i.e., $\vec{a} = d\vec{v}/dt$ and $\vec{v} = d\vec{r}/dt$) and the characteristics of projectile motion (i.e., $a_x = 0$ and $a_y = -g$), they should easily set up equations of motion for each direction:
x direction: $v_x = dr_x/dt$, and
y direction: $a_y = dv_y/dt$ and $v_y = dr_y/dt$.

Because all these equations are separable, students should be able to set up the following equations:

$$\int_{t_0}^{t_1} v_x\, dt = \int_{r_{0x}}^{r_{1x}} dr_x,$$
$$\int_{t_0}^{t} a_y\, dt = \int_{v_{0y}}^{v_y(t)} dv_y, \text{ and}$$
$$\int_{t_0}^{t_1} v_y(t)\, dt = \int_{r_{0y}}^{r_{1y}} dr_y,$$

where t_0 is the initial instant and r_{0x} and r_{0y} are positions in each direction at t_0, t_1 is the time of flight, r_{1x} and r_{1y} are positions in the x- and y- direction at t_1, and $v_y(t)$ is the function of the velocity in the y direction. Since these equations are set up based on the principle, they can be applied to solve all projectile problems with the only difference in unknowns. At this point, nothing is new to students so no cognitive load is imposed.

It takes three steps to apply the set of equations to solve projectile problems without finding numerical solutions:

Step 1: Specify a Cartesian coordinate system by choosing the origin and directions for each axis and represent the givens and finds in the coordinate system.

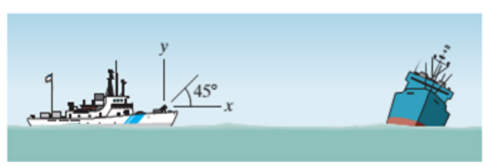
Step 2: Describe the motion in each direction with the set of equations given above.

Step 3: Identify unknowns for each equation and check whether the number of unknowns is equal to that of equations.

The beauty of this strategy lies in the fact that each step can be separated from the rest. Using specific assignments can achieve learning goals with less time and effort due to limited cognitive load imposed by each step. For example, some students may have trouble in representing Givens and Finds correctly if they have not developed good habits such as specifying a coordinate system before solving a problem. If that is the case, instructors can assign five projectile problems with only one requirement: explicitly draw a coordinate system on the figure of each problem and represent all givens with the chosen coordinate system. Students' attention will be focused on choosing an appropriate coordinate system and representing the sign and value for each given with the coordinate system. This assignment, with limited cognitive load, can be easily completed within 15 minutes and most students should be able to achieve the learning goal with this assignment. If this assignment is given prior to learning projectile motion, this step will not impose any cognitive load. For Step 2, as explained above, students should have no trouble understanding how the set of equations is set up. Then worked examples can be used to demonstrate how the same set of equations is used to solve projectile problems seemingly different to students. As a result, students will be convinced that this approach works. The only challenge encountered during instruction occurs at Step 3 which will be explained in detail later, but it is still manageable compared to the traditional approach.

We will use the following three examples to demonstrate how this approach is applied to solve projectile problems.

Example 1 [Problem 13.72 in 3]: Suppose that you are designing a mortar to launch a rescue line from coast guard vessels to ships in distress. The light line is attached to a weight fired by the mortar. Neglect aerodynamic drag and the weight of the line for your preliminary analysis. If you want the line to be able to reach a ship 300 ft. away when the mortar is fired at 45° above the horizontal, what muzzle velocity is required?



Given: $t_0 = 0$, $r_{0x} = 0$, $r_{0y} = 0$, $\theta = 45^{\circ}$, $v_{0x} = v_0 \cos\theta$, $v_{0y} = v_0 \sin\theta$, $r_{1x} = 300$ ft, $r_{1y} = 0$, $a_y = -32.2$ ft/s^2
Find: v_0 ft/s.

The motion in the x-axis:	$\int_{t_0}^{t_1} v_{0x}\, dt = \int_{r_{0x}}^{r_{1x}} dr_x$, (unknowns: t_1, v_0)
The motion in the y-axis:	$\int_{t_0}^{t} a_y\, dt = \int_{v_{0y}}^{v_y(t)} dv_y$, (unknown: $v_y(t)$) $\int_{t_0}^{t_1} v_y(t)\, dt = \int_{r_{0y}}^{r_{1y}} dr_y$, (unknown: none)

With three equations and three unknowns, the problem can be solved.

Example 2 [Problem 13.74 in 3]: When the athlete releases the shot, it is 1.82 m above the ground and its initial velocity is $v_0 = 13.6$ m/s. Determine the horizontal distance the shot travels from the point of release to the point where it hits the ground.

Given: $t_0 = 0$, $r_{0x} = 0$, $r_{0y} = 0$, $v_0 = 13.6$ m/s, $\theta = 30°$, $v_{0x} = v_0 \cos\theta$, $v_{0y} = v_0 \sin\theta$, $r_{1y} = -1.82$ m, $a_y = -9.81$ m/s^2
Find: r_{1x} m.

The motion in the x-axis:	$\int_{t_0}^{t_1} v_{0x}\, dt = \int_{r_{0x}}^{r_{1x}} dr_x$, (unknowns: t_1, r_{1x})
The motion in the y-axis:	$\int_{t_0}^{t} a_y\, dt = \int_{v_{0y}}^{v_y(t)} dv_y$, (unknown: $v_y(t)$) $\int_{t_0}^{t_1} v_y(t)\, dt = \int_{r_{0y}}^{r_{1y}} dr_y$, (unknown: none)

With three equations and three unknowns, the problem can be solved.

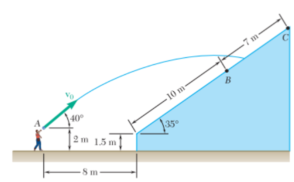
Example 3 [Problem 13.77 in 3]: A batter strikes a baseball 3 ft. above home plate and pops it up. The second baseman catches it 6 ft. above second base 3.68 s after it was hit. What was the ball's initial velocity, and what was the angle between the ball's initial velocity vector and the horizontal?
Given: $t_0 = 0$, $r_{0x} = 0$, $r_{0y} = 0$, $v_{0x} = v_0 \cos\theta$, $v_{0y} = v_0 \sin\theta$, $t_1 = 3.68$ s, $r_{1y} = 6 - 3$ ft, $r_{1x} = \sqrt{90^2 + 90^2}$ ft, $a_y = -32.2$ ft/s^2
Find: v_0 ft/s and θ.

The motion in the x-axis:	$\int_{t_0}^{t_1} v_{0x}\, dt = \int_{r_{0x}}^{r_{1x}} dr_x$, (unknowns: v_0, θ)
The motion in the y-axis:	$\int_{t_0}^{t} a_y\, dt = \int_{v_{0y}}^{v_y(t)} dv_y$, (unknown: $v_y(t)$) $\int_{t_0}^{t_1} v_y(t)\, dt = \int_{r_{0y}}^{r_{1y}} dr_y$, (unknown: none)

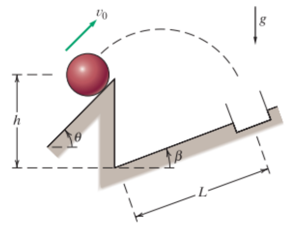
With three equations and three unknowns, the problem can be solved.

Results and Discussions

Two projectile problems (see below) were used to assess learning in a class with 35 students at a small private university during the spring 2016 semester. The Dynamics Concept Inventory 1.0 [4] was administered at the beginning of the semester to assess students' prior knowledge and the average grade was 34.5%. The first problem was given in a test after students just completed the projectile motion part and the second problem was assigned as homework three weeks later. For both problems students were required to provide Given and Find, set up equations, and show unknowns without finding the numerical solutions.



Problem 1 [5]: At halftime of a football game souvenir balls are thrown to the spectators with a velocity v_0. Determine the value of v_0 if the ball is to land at point B.



Problem 2 [6]: Greg has been playing skee ball all afternoon at the local arcade, but much to his dismay, he has not been able to get a single ball into the 100-point hole. The 100-point hole is located $L = 3$ ft from the base of the skee ball machine's backboard, which is angled at $\beta = 20°$ with respect to the horizontal. If the end of the launch ramp is $h = 2$ ft above the backboard's base and oriented at $\theta = 45°$ to ground, how fast should Greg project a ball up the launch ramp so that it lands in the 100-point hole? How long does it take for the ball to reach the hole?

Table 2 shows the assessment result with the percentage referred to correctness.

Table 2 Assessment Result

Problem	Date	Given/Find	Equations	Unknowns
1	Feb. 5, 2016	67%	26%	26%
2	Feb. 24, 2016	82%	56%	50%

We have to admit that the data itself cannot demonstrate the effectiveness of this approach because the two assessments were administered in two different situations. Our goal is to introduce this approach as work-in-progress to invite students and instructors to evaluate it and further refine it if it does help address learning challenges. So we will go beyond the data and take a deep look at common mistakes in each step and what can be easily changed through instruction/practice and what is hard to be corrected. This is also another benefit of this approach: provide detailed view of students' mistake to facilitate developing further instruction strategies.

Step	Common Mistakes
Given/Find	• Did not specify the coordinate system clearly. • Signs and values of Given/Find did not match the coordinate system. • Givens and/or finds were incomplete.
Equations	• Missed the subscripts (e.g., used r instead of r_x and r_y). • Did not use correct upper and/or lower limits in equations (e.g., used $\int_{t_0}^{t_1} a_y dt = \int_{v_{0y}}^{v_y(t)} dv_y$).
Unknowns	• Counted the same unknown twice (e.g., t_1 was counted unknown in equation $\int_{t_0}^{t_1} v_x dt = \int_{r_{0x}}^{r_{1x}} dr_x$ and it was counted unknown again in equation $\int_{t_0}^{t_1} v_y(t) dt = \int_{r_{0y}}^{r_{1y}} dr_y$). • Counted t in the equation $\int_{t_0}^{t} a_y dt = \int_{v_{0y}}^{v_y(t)} dv_y$ as an unknown.

	• When v_0 was not given, counted both v_{0x} and v_{0y} instead of v_0 as unknown.

For common mistakes in Steps 1 and 2, we designed corresponding assignments to address them.

Assignments for common mistakes at Step 1: Instead of letting them list givens and find on their own, we provided a template (see below) for them to fill. The template helped students organize seemingly scattered information in a meaningful way so students would less likely miss the givens. Assignments using this templated are suggested to be used prior to teaching projectile motion so students won't struggle to get this part correct while have trouble with Steps 2 and 3. We did not use this template until we found students did not do well in the test. With only two assignments, most common mistakes were corrected. Most students could list all givens without any mistake in signs, values, and/or units.

Given:

Information Relevant to the **Process**	$a_x =$ __________ft/s^2	$a_y =$ ______________ft/s^2
	Position	Velocity
Information Relevant to Instants $t_0 = 0$	$r_{0x} =$ ________ft $r_{0y} =$ ________ft	$\theta = 45°$ $v_{0x} =$ __________ft/s $v_{0y} =$ __________ft/s
Information Relevant to Instants t_1 (if t_1 is given, represent $t_1 =$ _______)	$r_{1x} =$ ________ft $r_{1y} =$ ________ft	

Find: (Do NOT miss the units.)

Figure 1 Template for Writing Given/Find.

Assignments for common mistakes at Step 2: For the subscript mistake, we realized it was caused by students' habit of neglecting details. So we required students to use MATLAB to find numerical values to force them to pay attention to every single detail they put on paper. Otherwise they will rush through equations and come up with some answer they do not understand or simply copy values from the solution manual or online solutions. We did not require them to find numerical values for every problem in each assignment. They usually only had one MATLAB assignment to find the numerical values as the process is the same. This assignment will help student develop the skill of using MATLAB for computing and focus students' attention to details.

For the upper/lower limit mistake, we assigned online multiple choice problems to let them choose appropriate upper limits and also asked them to explain why they chose the answer. The purpose of this assignment is to encourage them to understand why the equation is set up this way rather than memorize the equation. This assignment could also be done in class as a clicker question. Students could immediately catch their mistake through the discussion and feedback.

The common mistakes are caused by students' deficiency in algebraic thinking [7]. Students are supposed to achieve algebraic thinking but many students still stay in arithmetic thinking which can only deal with specific numbers rather than variables. When students with arithmetic thinking encounter an equation with all variables, they do not take the equation as an expression to represent the relationship between the variables. Instead, they will substitute all numerical values into the equation and isolate the unknown to calculate the value. They do not realize that the equation is solvable if there is only one unknown in the equation. They often experience difficulty if they encounter an equation with two unknowns. They do not realize that they merely need to find more equations to make the number of equations equal that of unknowns. Thus, teaching them to focus on finding unknowns rather than substituting in numbers was a big challenge.

Since my whole course is taught in this way, i.e., setting up all equations and specifying unknowns before solving equations, students start to learn this approach when learning straight-line motion. We also take the assignment design strategy similar to that used in developing assignments for Step 1: assign exercises with one single goal to reduce cognitive load so students can achieve learning easier and faster. For example, we will assign five problems and provide givens, find, and equations. Students only need to recognize unknowns. Then we may provide incomplete equations so students need to set up new equations while finding unknowns. It usually takes one month to help most students make this transition.

Conclusion

In this paper, we have shared a unique approach to teach projectile motion to avoid cognitive overload and help students develop good problem solving habits. In addition to presenting the initial assessment results, we also provided common mistakes students often make when use this approach and corresponding practice strategies to address these deficiencies. Future research should collect more data to explore how scaffolding through question prompts affects students' learning. Future work should also investigate the design of question prompts and the effect on learners with different background. As the ultimate goal of scaffolding is to achieve independent learning, research should be conducted to find out when scaffolding can be removed.

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