AC 2008-2967: FINITE ELEMENT METHOD - A TOOL FOR LEARNING HIGHWAY DESIGN

Robert Brooks, Temple University

Dr. Brooks is an Associate Professor and the Undergraduate Director of the Department of Civil and Environmental Engineering at Temple University. He was voted the "Transportation engineer of the year" by the ASCE-Philadelphia Section. Dr.Brooks' expertise includes finite element methods, highway and runway design, innovative materials in transportation engineering. He won the Tempe University College of Engineering's Teaching Award for the year 2008.

Asher Madjar, Temple University

Prof.Madjar is a research professor in the Department of Electrical and Computer Engineering, Temple University. His research interests are Microwave Photonics, MMIC design, R&D of Microwave Components and Subsystems, GaAs Microwave Devices and Monolithic Circuits.

William Miller,

Prof.Miller is a Senior Lecturer in the Department of Civil and Environmental Engineering, Temple University. He earned his PhD in Engineering from the University of Pennsylvania. He teaches Introduction to Engineering Course to all majors in Engineering. His specialty is Air Pollution and Control Engineering.

Keerthi V. Takkalapelli, Temple University

Keerthi is a Graduate Student in the Department of Civil and Environmental Engineerin, Temple University.

Finite Element Method - A Tool for
Learning Highway Design

A computer program was developed by idealizing flexible pavement into a finite element continuum. The analysis was carried out for a single wheel load of 9,000 lb with a tire pressure of 100 psi. A layered pavement was idealized as an axisysmmetric solid with finite boundaries in both radial and axial directions. The axisymmetric body was then divided into a set of ring elements, rectangular in section and connected along their nodal circles. Because of symmetry, the three-dimensional problem reduces to a two dimensional case. The program is capable of handling changes of material properties such as Resilient Modulus and Poisson's Ratio in both vertical and horizontal directions. Several other elastic multilayered computer programs are available for the structural analysis of a pavement such as ELSYM, BISAR, and ILLIPAVE. However this program is suitable for eliminating tensile stresses in granular layers by stress transfer method. Moreover this program is optimized for analyzing highway pavements and is also user friendly. The time and effort needed for incorporating initial conditions necessary in other advanced software is eliminated.

Students have successfully used this program for not only designing the highways but also optimizing their design by simulation. The power of simulation of the program enhanced the students' learning of highway design by providing them with a feel for the large ranges of weather, load and material conditions that exist in the country.

Development of Finite Element Analysis

A computer program was developed by idealizing the flexible pavement into a finite element continuum. The analysis was carried out for a single wheel load of 40.131(9000lb) distributed over a circular area of 152-mm radius with a tire pressure of 0.55MPa (80 psi).

Idealization

In this investigation a layered pavement system was idealized as an axisymmetric solid with finite boundaries in both radial and axial directions. The axisymmetric body was then divided into a set of ring elements, rectangular in section and connected along their nodal circles. The finite elements are actually complete rings in the circumferential direction, and the nodal points at which they are connected are circular lines in plan view. Because of axisymmetry, the three-dimensional problem reduces to a two-dimensional case similar to a plane strain problem. Tensile stresses and strains were taken to be positive, and compressive stresses and strains negative. For each element the four nodal points were numbered in the clockwise direction. Each node has two degrees of freedom.

Displacement Functions

The two displacement components in a solid continuum varied as complicated functions of position. A number of approaches, including power series and Fourier series expansions, have been proposed by several researchers[1,2] to represent the behavior of displacement components inside each element. Because of the assumptions made about these functions, the accuracy of the answer increases as the element size decreases. For this investigation, the displacement functions inside each element were approximated by the following:

$$u(r,z) = b_1 + b_2r' + b_3z' + b_4r'z' \quad (1)$$

$$v(r,z) = b_5 + b_6r' + b_7z' + b_8r'z' \quad (2)$$

where for each element the local coordinate system r',z' was used, which has its origin at the center of each element.

Global Stiffness

The stiffnesses of all elements were assembled to obtain the total stiffness for the system. The required assembly was accomplished by using the element-node table and displacement-code number array. Because the system stiffness matrix comes out to be banded and symmetric, it is assembled in half-band form. A consistent load vector was formulated using the principle of virtual work. [3]

Boundary Conditions

Nodal points on the vertical boundary at a distance of 12 radii from the center and on the centerline were constrained from radial movement; those on the bottom boundary were not allowed to move vertically or horizontally. The bottom boundary was fixed at a depth of 50 radii in accordance with the findings of Duncan et al. [4]

Verification of Elastic Finite Element Analysis

The validity of the finite element computer program was established by comparing the results with those of Ahlvin and Ulery [5]. The vertical and radial stresses along the center of the load at a radial distance of 0.752 radii (from the centerline of the load) obtained by the program, were compared with those obtained by the elastic half-space analysis after Ahlvin and Ulery. The deflections and stresses computed by this procedure agree closely with those obtained from the elastic half-space analysis.

Several other elastic multilayered computer programs (e.g., ELSYM, BISAR, ILLIPAVE and FEPAVE) are available for the structural analysis of a pavement, and any of them also can be used. However, the finite element method using the principle of stress transfer developed by Zienkiewicz et al. [2] is suitable for eliminating tensile stresses in unbound granular layers. Moreover this program is tailor made for analyzing highway pavements making it user friendly. Thus the demand for time and energy for learning initialization process of other advanced software is eliminated.

Scope of the software

The software can analyze a multilayered pavement structure of up to five layers. The material properties can be varied for each finite element both in horizontal and vertical directions. The material properties to be input are Resilient Modulus and Poisson's Ratio. Up to 50 million standard axles of heavy duty traffic can be handled by the program. The pavement temperature range is -30 deg. F to 140 deg. F. Default values are suggested by the program whenever appropriate. The software can handle the values for all needed variables that exist in the continental United States. More details on the variables that exist in the continental United States can be found from Brooks (a/k/a Mattthews) and Pandey (6) and Brooks (a/k/a Matthews) and Monismith (7). The results obtained from the software were extensively documented (8,9). The software can also be used for enhancing creative performance of the students (10). In this paper the software was used by the students as a tool for learning the subject (11) as documented in the results section.

Results

Fifteen students of Transportation Engineering classes of 2003 learned and used the computer program for not only analyzing the existing conditions of the highways but also for optimizing the design. A confidential survey was conducted using the questionnaire shown in Table 1. Questions 5 through 10 were evaluated by students. These evaluations were checked against their combined scores in the mid and final examinations with and without the use of the computer program. The differences were not statistically significant as shown in Table 2. Table 3 shows 5 Performance Indices (PI) of the computer program. Table 4 shows the improved scores on the five performance indices due to the usage of the computer program. PI1 in Table 4 is the difference of Q1 and Q2 of Table 1. Similarly PI2 in Table 4 is the difference of Q3 and Q4 of Table 1. The improvements in all the performance indices are statistically significant as determined by the ANOVA (F-test) test results shown in tables 5 and 6.

The statistical significance of each PI is confirmed by conducting t-test. The results are shown in Tables 7-11.

Table 1. Questionnaire on the utility of the Finite Element Method

1. How much time in minutes was taken for analyzing the existing conditions using manual method?
2. How much time in minutes was taken for analyzing the existing conditions using the computer program?
3. How much time in minutes was taken for optimization process using manual method?
4. How much time in minutes was taken for optimization process using computer program?
5. What was your score on a scale of 1-100 on the knowledge you have on the application of geometric conditions for the highway design before you learned the application of the computer program?
6. What was your score on a scale of 1-100 on the knowledge you have on the application of geometric conditions for the highway design after you learned the application of the computer program?

7. What was your score on a scale of 1-100 on the knowledge you have on the application of material conditions for the highway design before you learned the application of the computer program?
8. What was your score on a scale of 1-100 on the knowledge you have on the application of material conditions for the highway design after you learned the application of the computer program?
9. What was your score on a scale of 1-100 on the knowledge you have on the application of environmental conditions for the highway design before you learned the application of the computer program?
10. What was your score on a scale of 1-100 on the knowledge you have on the application of environmental conditions for the highway design after you learned the application of the computer program?

Table 2 Change of Students' self evaluations scores in Table 1 from their exam scores

Question #	Change in Score
Question 5	-4
Question 6	+5
Question 7	-5
Question 8	-3
Question 9	+4
Question 10	+4

Table 3. Performance Indices (PI) of the FEM Computer program

1. Time saved on the analysis of the existing conditions (PI 1)

2. Time saved on the optimization of the design (PI 2)

3. Increased knowledge on the application of geometric conditions for the design of highways (PI 3)

4. Increased knowledge on the application of material conditions for the design of highways (PI 4)

5. Increased knowledge on the application of environmental conditions for the design of highways (PI 5)

Table 4. Improved scores in % due to the FEM computer program on various performance indices

No. of students	PI 1	PI 2	PI 3	PI 4	PI 5

1	81	90	20	18	14
2	79	89	17	16	12
3	84	93	23	20	15
4	77	86	16	17	12
5	86	97	15	19	16
6	78	94	25	19	15
7	83	88	20	16	16
8	85	96	18	18	14
9	76	92	22	20	12
10	80	87	19	16	16
11	74	84	24	17	15
12	75	95	19	19	13
13	88	85	21	17	13
14	87	83	24	20	7
15	82	91	21	18	14

Table 5. Detailed statistical results of ANOVA test on the Performance Indices of the FEM Computer Program

	PI 1	PI 2	PI 3	PI 4	PI 5
Mean	81.0	90.0	20.3	18.0	13.6
95% confidence interval for Mean	79.27 thru 82.73	88.27 thru 91.73	18.53 thru 22.00	16.27 thru 19.73	11.87 thru 15.33
Standard Deviation	4.47	4.47	3.01	1.46	2.32
Hi	88.0	97.0	25.0	18.0	16.0
Low	74.0	83.0	15.0	16.0	7.00
Median	81.0	90.0	20.0	18.0	14.0
Average Absolute Deviation from Median	3.73	3.73	2.40	1.20	1.60

Table 6. Summary statistical results of ANOVA test on the Performance Indices of the FEM Computer Program

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F
Between	84702	4	21175	1870.
Error	792.5	70	11.32	

total	85494	74		

The probability of this result, assuming the null hypothesis, is less than 0.001

Table 7. t- test results for the Analysis of existing conditions

	Avg. Time (minutes)	SD
Manual Process	202	37
Computer program	39	4
Number of students	15	15
Average time saved	163 minutes (81%)/student/run	
t = 17.18		
p = 0.001		
Statistically significant?	Yes	

Table 8. t- test results for the Optimization process

	Avg. Time (minutes)	SD
Manual Process	1094	182
Computer program	109	11
Number of students	15	15
Average time saved	985 minutes (90%)/student/run	
t = 20.3		
p = 0.001		
Statistically significant?	Yes	

Table 9. t-test results for the Increased knowledge on the application of geometric conditions for the Highway Design

	Knowledge	SD
Manual Process	64%	8.3
Computer program	77%	7.9
Number of students	15	15
Increased Knowledge	20.3% (from 64 to 77)	
t = 4.4		
p = 0.001		
Statistically significant?	Yes	

Table 10. t-test results for the Increased knowledge on the application of wide variety of material conditions for the Highway Design

	Knowledge	SD

Manual Process	61%	6.9
Computer program	72%	7.1
Number of students	15	15
Increased Knowledge	18% (from 61 to 72)	
t = 4.3		
p = 0.005		
Statistically significant?	Yes	

Table 11. t-test results for the Increased knowledge on the application of environmental conditions for the Highway Design

	Knowledge	SD
Manual Process	60%	7.3
Computer program	67%	9.1
Number of students	15	15
Increased Knowledge	12% (from 60 to 67)	
t = 2.66		
p = 0.01		
Statistically significant?	Yes	

Conclusions

1. A computer program based on Finite Element Method for the design of highways was developed. This program was used for the analysis of the existing conditions and for optimizing the design of highways. This software has an advantage over other available software where time and effort would be required to set up initial conditions.

2. Students saved 163 minutes (81%) for each analysis of existing conditions of highways.

3. Students saved 985 minutes (90%) for each optimization process of the design.

4. Students increased 20.3% of their knowledge on the application of geometric conditions for the highway design.

5. Students increased 18% of their knowledge on the application of material conditions.

6. Students increased 12% of their knowledge on the application of environmental conditions for the design.

7. The above improvements were statistically significant as supported by the results of t- and F-tests.

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