Finite Element Method - A Tool for Learning Runway Design

Abstract

A computer program was developed by idealizing flexible pavement into a finite element continuum. A layered pavement was idealized as an axisymmetric solid with finite boundaries in both radial and axial directions. The axisymmetric body was then divided into a set of ring elements, rectangular in section and connected along their nodal circles. Because of symmetry, the three-dimensional problem reduces to a two dimensional case. The program is capable of handling changes of material properties such as Resilient Modulus and Poisson's Ratio in both vertical and horizontal directions. Several other elastic multilayered computer programs are available for the structural analysis of a pavement such as ELSYM, BISAR, and ILLIPAVE. However this program is suitable for eliminating tensile stresses in granular layers by stress transfer method. Moreover this program is tailor made for analyzing runway pavements making it user friendly. Thus the demand for time and energy for learning initialization process of other advanced software is eliminated.

Students have successfully used this program for not only designing the runways but also optimizing their design by simulation. The power of simulation of the program enhanced the students' learning of runway design by providing them a feel for the large ranges of weather, load and material conditions that exist in the country. Statistical tests were conducted and results were documented on the power of simulation.

Development of Finite Element Analysis

A computer program was developed by idealizing the flexible pavement into a finite element continuum.

In this investigation a layered pavement system was idealized as an axisymmetric solid with finite boundaries in both radial and axial directions, as shown in Fig 1. The axisymmetric body was then divided into a set of ring elements, rectangular in section and connected along their nodal circles. The finite elements are actually complete rings in the circumferential direction, and the nodal points at which they are connected are circular lines in plan view. Because of axisymmetry, the three-dimensional problem reduces to a two-dimensional case similar to a plane strain problem. Tensile stresses and strains were taken to be positive, and compressive stresses and strains negative. For each element the four nodal points were numbered in the clockwise direction. Each node has two degrees of freedom.

Displacement Functions

The two displacement components in a solid continuum varied as complicated functions of position. A number of approaches, including power series and Fourier series expansions, have been proposed by several researchers to represent the behavior of displacement components inside each element. Because of the assumptions made about these functions, the accuracy of the answer increases as the element size decreases. For this investigation, the displacement functions inside each element were approximated by the following:

$$u(r,z) = b_1 + b_2 r' + b_3 z' + b_4 r' z'$$

$$v(r,z) = b_5 + b_6 r' + b_7 z' + b_8 r' z'$$

where for each element the local coordinate system r', z' was used, which has its origin at the center of each element.

Global Stiffness

Physical representation of three point Gaussian integration for calculating elemental stiffness is shown in Fig.1. The stiffnesses of all elements were assembled to obtain the total stiffness for the system. The required assembly was accomplished by using the element-node table and displacement-code number array. Because the system stiffness matrix comes out to be banded and symmetric, it is assembled in half-band form. A consistent load vector was formulated using the principle of virtual work. [(1)]

Boundary Conditions

Nodal points on the vertical boundary at a distance of 12 radii from the center and on the centerline were constrained from radial movement; those on the bottom boundary were not allowed to move vertically or horizontally. The bottom boundary was fixed at a depth of 50 radii in accordance with the findings of Duncan et al. [(2)]

Verification of Elastic Finite Element Analysis

The validity of the finite element computer program was established by comparing the results with those of Ahlvin and Ulery [(3)]. The vertical and radial stresses along the center of the load at a radial distance of 0.752 radii (from the centerline of the load) obtained by the program, were compared with those obtained by the elastic half-space analysis after Ahlvin and Ulery as shown in Figures 2 and 3. The deflections and stresses computed by this procedure agree closely with those obtained from the elastic half-space analysis.

Several other elastic multilayered computer programs (e.g., ELSYM, BISAR, ILLIPAVE and FEPAVE) are available for the structural analysis of a pavement, and any of them also can be used. However, the finite element method using the principle of stress transfer developed by Zienkiewicz et al. [(1)] is suitable for eliminating tensile stresses in unbound granular layers. Moreover this program is tailor made for analyzing runway pavements making it user friendly. Thus the demand for time and energy for learning the initialization process of other advanced software is eliminated.

Scope of the software

The software can analyze a multilayered pavement structure up to five layers. The material properties can be varied for each finite element both in horizontal and vertical directions. The material properties to be input are Resilient Modulus and Poisson’s Ratio. Up to 50 million standard axles of heavy duty traffic can be handled by the program. The pavement temperature range is -30 deg. F to 140 deg. F. Default values are suggested by the program whenever appropriate. The software can handle the values for all needed variables that exist in the continental United States. More details on the variables that exist in the continental United States can be found from Brooks (a/k/a Matthews) and Pandey [(4)] and Brooks (a/k/a Matthews) and Monismith [(5)]. The results obtained from the software were extensively documented[(6,7)]. The software can also be used for enhancing creative performance of the students [(8)]. In this paper the software was successfully used by the students as a tool for learning the subject [(9)] as documented in the results section.

Results

Twelve students of Transportation Systems and Management class of 2005 learned and used the computer program for not only analyzing the existing conditions of the runways but also to optimize the design. A confidential survey was conducted using the questionnaire shown in Table 1. Table 2 shows 5

Performance Indices (PI) of the computer program. Table 3 shows the improved scores on the five performance indices due to the usage of the computer program. PI1 in Table 3 is the difference of Q1 and Q2 of Table 1. Similarly PI2 in Table 3 is the difference of Q3 and Q4 of Table1 and so on and so forth. The improvements in all the performance indices are statistically significant as determined by the ANOVA (F-test) test and as shown in Tables 4 and 5.

The statistical significance of each PI is confirmed by conducting t-test. The results are shown in Tables 6-10.

Conclusion

The computer program saved statistically significant time on the analysis and optimization of the design and increased students' knowledge statistically significantly on the application of geometric, material and environmental conditions for the design of runways.

Table 1. Questionnaire on the utility of the Finite Element Method

1. How much time in minutes was taken for analyzing the existing conditions using manual method?
2. How much time in minutes was taken for analyzing the existing conditions using the computer program?
3. How much time in minutes was taken for optimization process using manual method?
4. How much time in minutes was taken for optimization process using computer program?
5. What was your score on a scale 1-100 on the knowledge you have on the application of geometric conditions for the runway design before you learned the application of the computer program?
6. What was your score on a scale 1-100 on the knowledge you have on the application of geometric conditions for the runway design after you learned the application of the computer program?
7. What was your score on a scale 1-100 on the knowledge you have on the application of material conditions for the runway design before you learned the application of the computer program?
8. What was your score on a scale 1-100 on the knowledge you have on the application of material conditions for the runway design after you learned the application of the computer program?
9. What was your score on a scale 1-100 on the knowledge you have on the application of environmental conditions for the runway design before you learned the application of the computer program?
10. What was your score on a scale 1-100 on the knowledge you have on the application of environmental conditions for the runway design after you learned the application of the computer program?

Table 2. Performance Indices (PI) of the FEM Computer program

1. Time saved on the analysis of the existing conditions (PI 1)

2. Time saved on the optimization of the design (PI 2)

3. Increased knowledge on the application of geometric conditions for the design of runways (PI 3)

4. Increased knowledge on the application of material conditions for the design of runways (PI 4)

5. Increased knowledge on the application of environmental conditions for the design of runways (PI 5)

Table 3. Improved scores in % due to the FEM computer program on various performance indices

Student No.	PI1	PI2	PI3	PI4	PI5
1	69	77	17	18	13
2	75	79	21	17	17
3	76	84	24	21	16
4	72	78	20	17	17
5	69	83	23	21	13
6	70	85	26	20	16
7	79	77	22	18	12
8	77	86	19	19	15
9	68	86	20	20	14
10	73	80	21	19	15
11	67	76	25	18	16
12	77	87	16	20	17
13	71	79	26	17	13

Table 4. Detailed statistical results of ANOVA test on the Performance Indices of the FEM Computer Program

	PI 1	PI 2	PI 3	PI 4	PI 5
Mean	72.5	81.3	21.5	18.8	14.9
95% confidence interval for Mean	70.84 thru 74.23	79.61 thru 83.00	19.84 thru 23.23	17.15 thru 20.54	13.23 thru 16.62
Standard Deviation	3.93	3.97	3.20	1.46	1.75
Hi	79.0	87.0	26.0	21.0	17.0
Low	67.0	76.0	16.0	17.0	12.0
Median	72.0	80.0	21.0	19.0	15.0
Average Absolute Deviation from Median	3.31	3.46	2.54	1.23	1.46

Table 5. Summary statistical results of ANOVA test on the Performance Indices of the FEM Computer Program

Source of Variation	Sum of Squares	d.f.	Mean Squares	F
Between	54151	4	13538	1451.
Error	559.8	60	9.331	
total	54711	64		

The probability of this result, assuming the null hypothesis, is less than 0.01

Table 6. t- test results for the Analysis of existing conditions

	Avg. Time (minutes)	SD
Manual Process	182	33
Computer program	35	3.6
Number of students	13	13
Average time saved	147 minutes (81%)/student/run	
t = 18.2		

With t-score so high, the p-value is 0.001, a score that formed the basis to reject the null hypothesis and conclude that the computer program made a statistically significant difference on the time saved for the analysis of the existing conditions.

Table 7. t- test results for the Optimization process

	Avg. Time (minutes)	SD
Manual Process	985	164
Computer program	98	10
Number of students	13	13
Average time saved	887 minutes (90%)/student/run	
t = 21.2		

With t-score so high, the p-value is 0.001, a score that formed the basis to reject the null hypothesis and conclude that the computer program made a statistically significant difference on the time saved for the analysis of the existing conditions.

Table 8. t-test results for the Increased knowledge on the application of geometric conditions for the Runway Design

	Knowledge	SD
Manual Process	67 %	8.7
Computer program	81 %	8.3
Number of students	13	13
Increased knowledge	20.8% (from 67 to 81)	

t = 4.1

With t-score so high, the p-value is 0.001, a score that formed the basis to reject the null hypothesis and conclude that the computer program made a statistically significant difference on the time saved for the analysis of the existing conditions.

Table 9. t-test results for the Increased knowledge on the application of wide variety of material conditions for the Runway Design

	Knowledge	SD
Manual Process	64 %	7.2
Computer program	76 %	7.5
Number of students	13	13
Increased knowledge	18.8% (from 64 to 76)	

t = 3.9

With t-score so high, the p-value is 0.005, a score that formed the basis to reject the null hypothesis and conclude that the computer program made a statistically significant difference on the time saved for the analysis of the existing conditions.

Table 10. t-test results for the Increased knowledge on the application of environmental conditions for the Runway Design

	Knowledge	SD
Manual Process	62 %	7.7
Computer program	70 %	9.6
Number of students	13	13
Increased knowledge	12.9% (from 62 to 70)	

t = 2.51

With t-score so high, the p-value is 0.01, a score that formed the basis to reject the null hypothesis and conclude that the computer program made a statistically significant difference on the time saved for the analysis of the existing conditions.

Conclusions

1. A computer program based on Finite Element Method for the design of runways was used. This program was tailor made for the analysis of the existing conditions and optimizing the design of runways. Thus the demand for time and energy for learning initialization process of other advanced software is eliminated.

2. Students saved 147 minutes (81%) for each analysis of existing conditions of runways.
3. Students saved 887 minutes (90%) for each optimization process of the design.
4. Students increased their knowledge by 20.3% on the application of geometric conditions for the runway design.
5. Students increased their knowledge by 18% on the application of material conditions.
6. Students increased their knowledge by 12% on the application of environmental conditions for the design.
7. All the above improvements were statistically significant as supported by the results of t- and F-tests.

References

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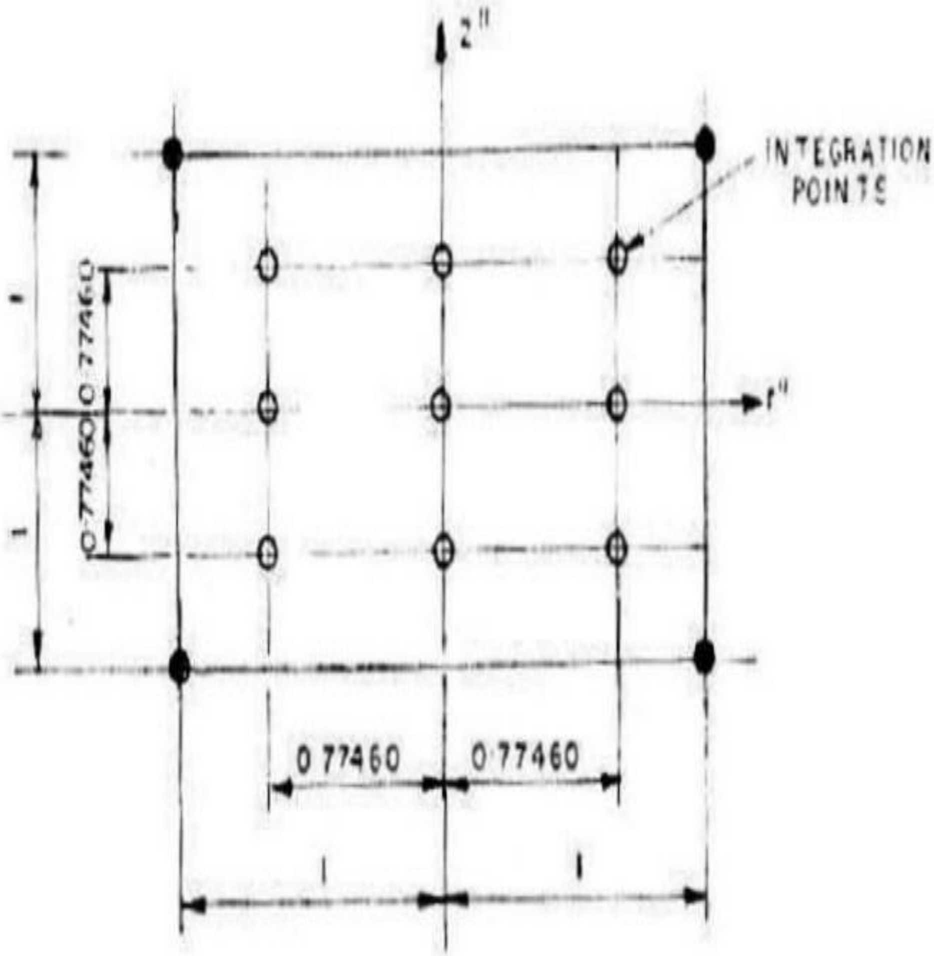
(9) S.Jahanian and Robert Brooks (a/k/a James Matthews), Multidisciplinary Project-A Tool for Learning the Subject, Journal of American Society of Engineering Education, April 1999, pp 153-162.



WEIGHTING FUNCTIONS : $H_1 = 0·555556$
$H_2 = 0·888889$
$H_3 = 0·555556$

FIG.1 PHYSICAL REPRESENTATION OF THREE POINT GAUSSIAN INTEGRATION

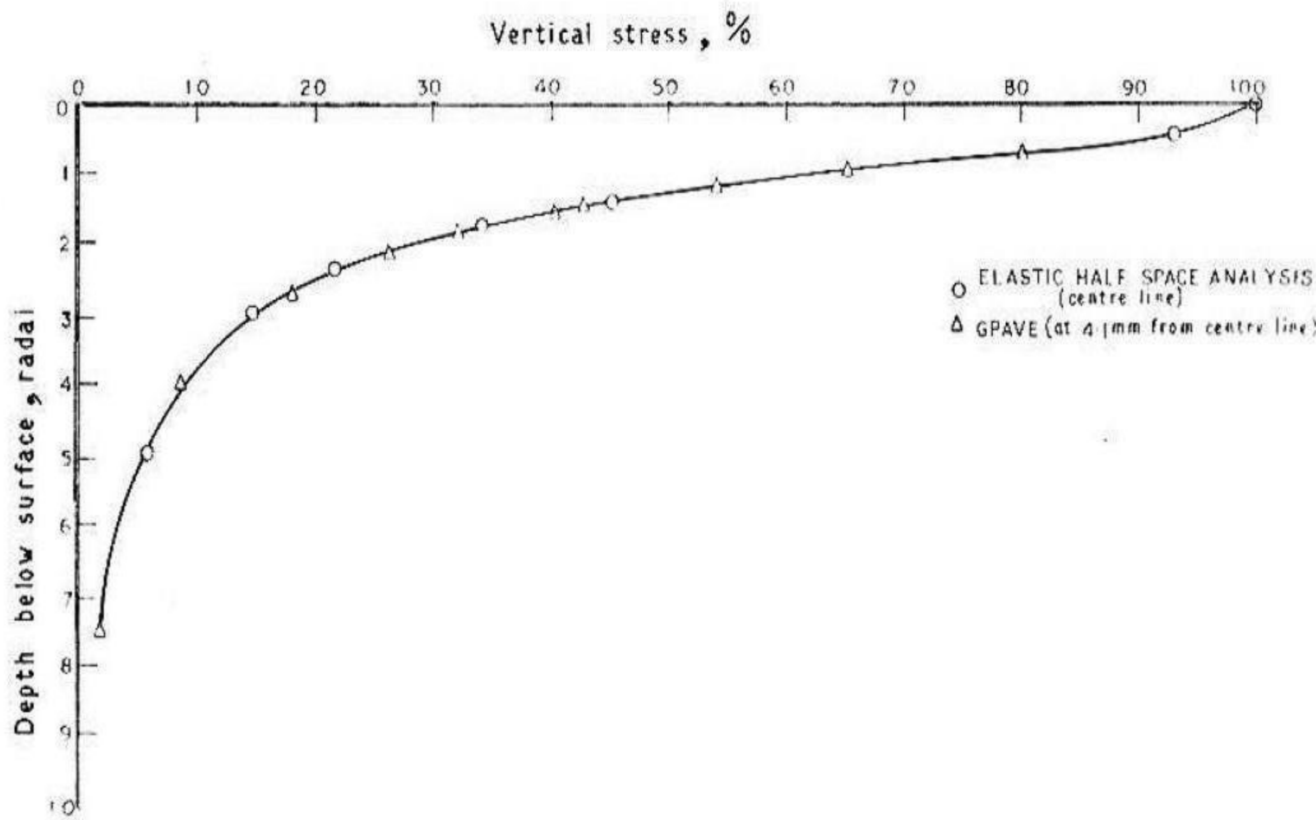


FIG 2 COMPARISON OF VERTICAL STRESS AT THE CENTRE LINE OF LOAD (GPAVE AND ELASTIC HALF SPACE ANALYSIS)

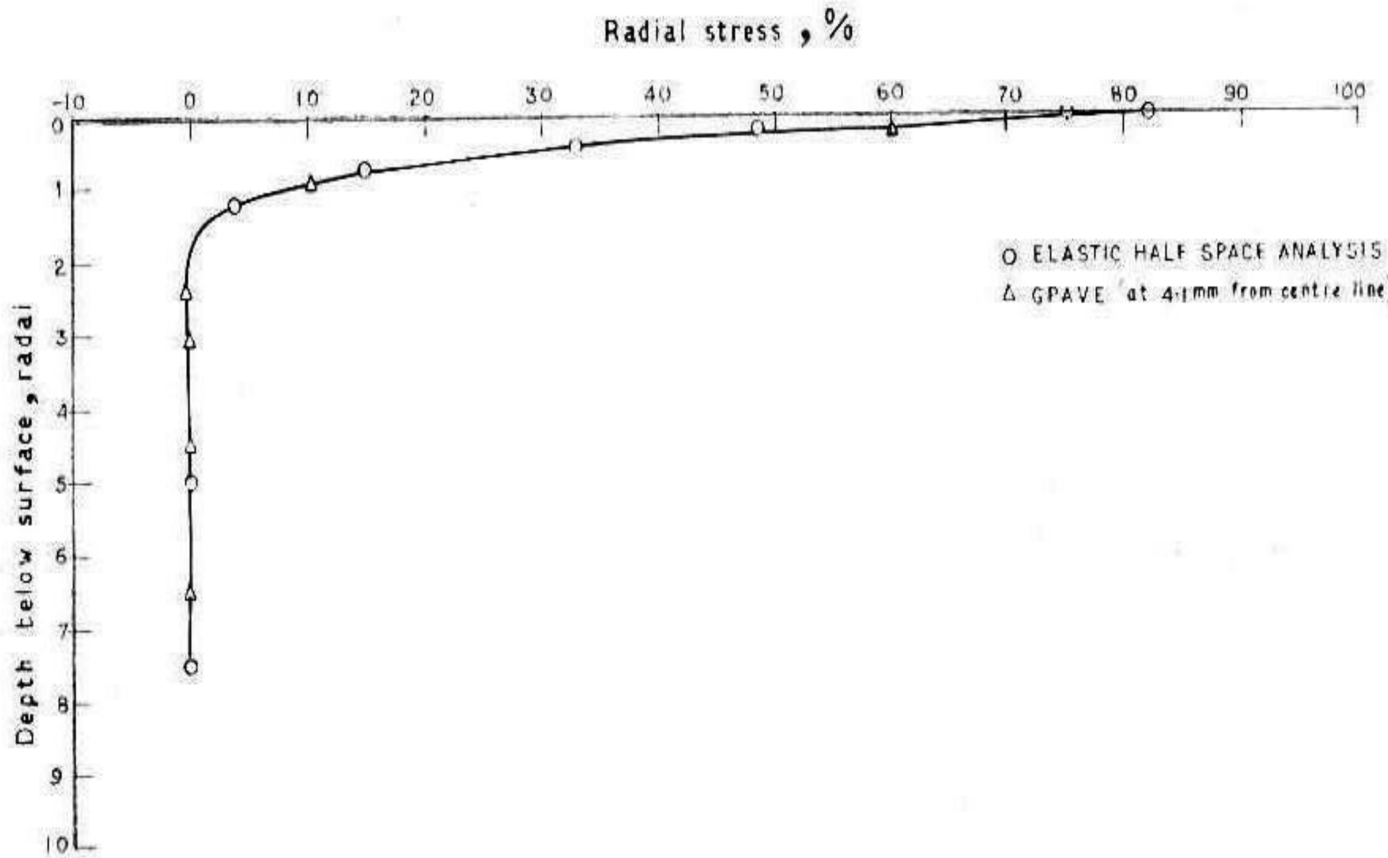


FIG.3 COMPARISON OF RADIAL STRESS AT THE CENTRE LINE OF LOAD (GPAVE AND ELASTIC HALF SPACE ANALYSIS)