



Finite Element Method as a Useful Modern Engineering Tool to Enhance Learning of Deformation Concepts

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Incorporating the Finite Element Method with Photoelasticity as a Useful Modern Engineering Tool to Enhance Learning of Deformation Concepts

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Abstract

As an advanced modern engineering tool, the Finite Element Method (FEM) has been widely adopted in current undergraduate engineering curricula, especially in the discipline of mechanical engineering. However, the usage of FEM as a tool integrated into other fundamental engineering classes, such as statics and dynamics, fluid and thermal, and mechanics of materials, is not as common as one might suppose. Including, this present-day engineering tool is proposed to assist the teaching of deformation concepts in mechanics of materials. Due to the inherent complexity of FEM, a small finite element analysis (FEA) program, mini-FEA, developed by Professor Paul S. Steif at Carnegie Mellon University about fifteen years ago, is used to illustrate the concepts and quickly show how it works. For complex geometry, ANSYS Mechanical APDL programs were created by the instructor so that the requirements of student interaction with the program are minimal, and to keep their focus on deformation concepts. The mini-FEA allows the instructor to provide a quick illustration of deformation concepts as well as the basic steps in implementing FEM. The concepts of deformation mechanics are then demonstrated by graphical illustrations from both FEM and the traditional photoelasticity method. The purpose of this paper is to study the effectiveness of integrating FEM and discover how FEM further enhances students' learning in comparison with the traditionally used photoelasticity method. From the survey feedback, the effectiveness of the FEM model in enhancing student learning is clearly seen. Assessment of this approach and results of teaching strategies are presented.

Introduction

Engineering educators are constantly challenged on how best to incorporate fast advancing technologies. One of these modern technology advancements is the development of modern Computer Aided Engineering (CAE) software and applications. To adopt these advanced engineering tools effectively into today's engineering curricula is important.

The Finite Element Method (FEM) is a numerical method, but it is by far the most widely used and the most successfully commercialized engineering tool. FEM as a CAE tool and its adoption into the undergraduate engineering curricula has become prominent especially in mechanical engineering (ME) curriculum. The significance and development of FEM in ME undergraduate curriculum has been addressed as early as in the 1980s.^[1-4] Due to the fast development of computer capacities and user-friendly commercialized FE programs, FEM has become well established with time^[4-7].

Most recently, efforts have been made to include the FEM into teaching methodologies in low level undergraduate courses^[8-11] such as statics and mechanics of materials. For such low level engineering classes, the FEM is mainly used for demonstration of concepts in the subject matter

rather than the understanding of the program itself. Because of this, specialized FEA software such as mini-FEA^[12] have been developed with simplicity and ease of application in mind. A question that remains is how such a simple FEA program can be integrated into a traditional engineering mechanics class and be used efficiently and effectively as a teaching tool.

Mechanics of Materials (MOM) class is one of the core courses in the ME discipline. Lab work is conducted each week during the quarter when the class is offered in order to enhance learning. One of the key labs conducted during the quarter is the photoelasticity lab. The main goal of this lab is to use this visualization tool to understand some key concepts of MOM rather than the underlying principles of photoelasticity.

In a similar manner, any present-day commercially available FE software allows numerical results to be post-processed graphically. Even some notable web-based FE programs such as mini-FEA have the capacity for users to visualize results in an explicit manner through striking graphical presentations. This observation poses important questions such as how FEM can be adopted into the traditionally taught MOM class. In this study, the authors included the FEM as one of the labs during the quarter, parallel to the traditional lab conducted using the photoelasticity method, to illustrate deformation concepts graphically. The effectiveness of including this component of teaching, in comparison with the photoelasticity method alone, is studied through the feedback information from students after they were exposed to both visualization tools.

Methods

FEM Programs

Both the web-based FE software, mini-FEA and the commercial FE package, ANSYS Mechanical APDL, were adopted for the class. While the mini-FEA program allows a quick demonstration of FE protocols and how it can be used to solve a class of MOM problems, the ANSYS Mechanical APDL allows handling of complex geometrical configurations of specimens and solving much more complicated problems. Since the key purpose in using these tools is to assist students' learning curve on important deformation concepts, the understanding and complex handling of the ANSYS package is minimized so students can focus their attention and not be side-tracked by the complexity in using the software itself. However, mini-FEA does not have the disadvantage of complex handling difficulties that the ANSYS APDL has and allows a quick introduction of common implementation steps that all FE packages share.

As articulated in the initial construction of the mini-FEA program^[11], the goal of this program is aimed to attack only a small class of problems addressed in classical mechanics of materials using the basic theory of linear elasticity through FEM. This is very different from any commercial general purpose FEA package such as ANSYS, which is developed to address diverse problems in various disciplines, from solid mechanics to fluid and heat flow problems, from linear to nonlinear, from statics to dynamics, from regular simple geometry to irregular complicated three-dimensional geometrical configurations. Because of the complicated features in those commercial programs, they are by nature hard to grasp in a short time. Therefore, the expectation from the mini-FEA program is very unique and the limitation of this program is also very apparent. For example, from the geometrical aspect, only rectangular geometries are available in mini-FEA and only 2-D problems can be solved. The theoretical background of this

program is based on 2-D linear elasticity theory only. Because of this, the interface of this program is extremely simple and easy to master even for those who have no previous experience with an FEA program and don't know at all about how it works. It is so simple that it may be referred to as an FEA calculator.^[15] In addition, the program was written using the Java programming language and it is web-based, freely available to everyone. Although the mini-FEA program can only be used to solve a small class of problems, it is very clear and specific and can effectively address hard-to-grasp deformation concepts for first time learners in the subjects of elasticity and mechanics of materials.

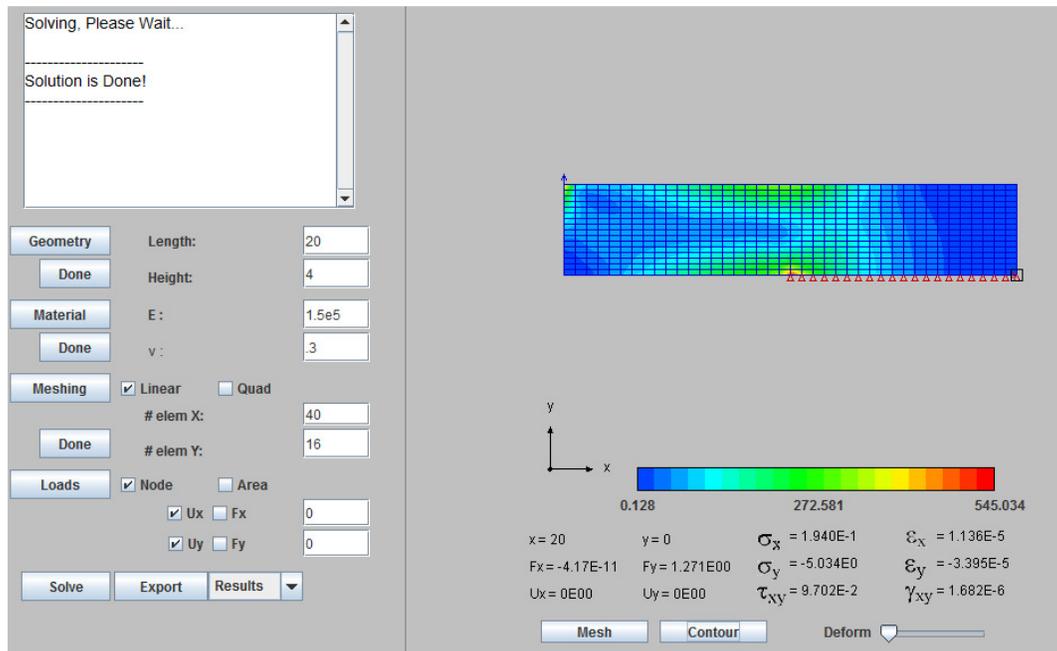


Fig. 1 The mini-FEA interface and illustration of a contour plot^[16].

Figure 1 illustrates the graphical user interface of the mini-FEA program on the web.^[12] This one-page user interface contains all the basics of an FE program. Analogous to the implementation steps in ANSYS Mechanical APDL, a few command blocks may be readily identified.

- First, the basic definition of an FE application may be given under the *preprocessor* command block in ANSYS. This preprocessor requires information for the domain geometry, the material properties and an element type. The domain is then required to be broken into small pieces, which is called discretization or meshing. All of this functionality is clearly seen in the mini-FEA program interface. Since mini-FEA only handles 2-D linear elastic problems, only two property parameters, the elastic modulus E and Poisson's ratio ν are necessary as inputs. In addition, there are only two element types available in the program and only rectangular geometry can be defined. Therefore the mini-FEA GUI is very specific, simple and clear in comparison with the ANSYS GUI.
- Second, the *solution* command block in ANSYS can be used to define boundary conditions and apply loads. Then an analysis type and a solver can be chosen to find the numerical solutions. In mini-FEA, the solution processor is clearly given by two clickable buttons, "Load" and "Solve", respectively.

- Third, the *post-processor* command block in ANSYS can be used to retrieve desired results after the solution phase is accomplished. Post processing in ANSYS GUI is often not so straightforward, since complicated interfacing may be necessary in order to get what a user is seeking. On the other hand, in mini-FEA, the post-processor command is executed through a few clickable buttons and one slider, i.e., “Results”, “Explore”, “Contour” and the “Deform” slider, which is very user-friendly, simple and straightforward.

While solving problems using the mini-FEA program, students are directed to a clear picture about how a sophisticated FE package works. Not only is there a clear analogy between the ANSYS GUI and the mini-FEA GUI, but also the mini-FEA possesses the power to demonstrate essential elastic deformation behaviors and typical finite element steps and pitfalls in the same way as it does in ANSYS, but with much less effort needed in the initial setup of the problem and thus it is better suited for classroom use. To illustrate, some key features of the mini-FEA program may be summarized from the existing work by Steif.^[11,12, 14-16]

- Mini-FEA can be used to solve any 2-D plane linear elastic problem with rectangular domain. Key quantities at any node or a cluster of nodes can be retrieved with ease including displacements U_x and U_y , normal stresses σ_x and σ_y , shear stress τ_{xy} , normal strains ϵ_x and ϵ_y , and shear strain γ_{xy} . The von Mises stress can also be obtained from the “Results” button. A contour plot may be obtained for any stress component. The deformed and undeformed configurations can be superimposed and magnified by moving the slider to the right. Reactions at any constraint can be shown on the interface.
- The program assumes model geometry with unit thickness. Actual thicknesses can be considered through a proper treatment^[15].
- Typical problems in mechanics of materials can be solved including axially loaded bars, simple shear and beam bending problems. Statically determinate and statically indeterminate problems can be easily handled as well. Once discretized solutions are obtained, some other concepts may be readily explored. Taking a beam problem as an instance, the curvature of a beam can be calculated using the displacement solutions at relevant nodal points.
- Some common FE pitfalls can be illustrated readily including proper vs. improper constraints, over constraints, stress singularity, mesh sensitivity and convergence, etc.
- Some key concepts, assumptions, and their significance used in mechanics of materials can be demonstrated with clarity in mini-FEA. For example, St. Venant’s principle can be easily illustrated for a bar under a concentrated load.

All these key features can be demonstrated in class or during lab time readily from the pre-defined problems given on the website^[13]. When mini-FEA is used in the lab, simulation cases can be created easily based on the specimens used in the photoelasticity lab provided that the specimens are rectangular in shape.

ANSYS Mechanical APDL can be used to simulate all the cases that the mini-FEA program does. However the complicated GUI interface results in greater student difficulty in order to get the same case modeled that can be created using mini-FEA within seconds. This is NOT practical for beginning FE users. However, for cases such as a plate with a central hole, for the study of stress concentration, mini-FEA cannot handle such cases and ANSYS must be used. In

order to avoid side-tracking students from handling the software, ANSYS APDL simulation input files were pre-created and ready for experiments during the lab hours. The simulation cases are based on the specimens used in the photoelasticity lab. The ANSYS input files are case-specific and only require students to run the program and follow the pop-up windows to enter some key information such as the magnitude of certain loading conditions. Then the simulation codes direct students to the post-processing stage of the software. Students can either visualize the results directly through the graphics window or manipulate the graphics and extract results with much less effort in the ANSYS post-processing stage.

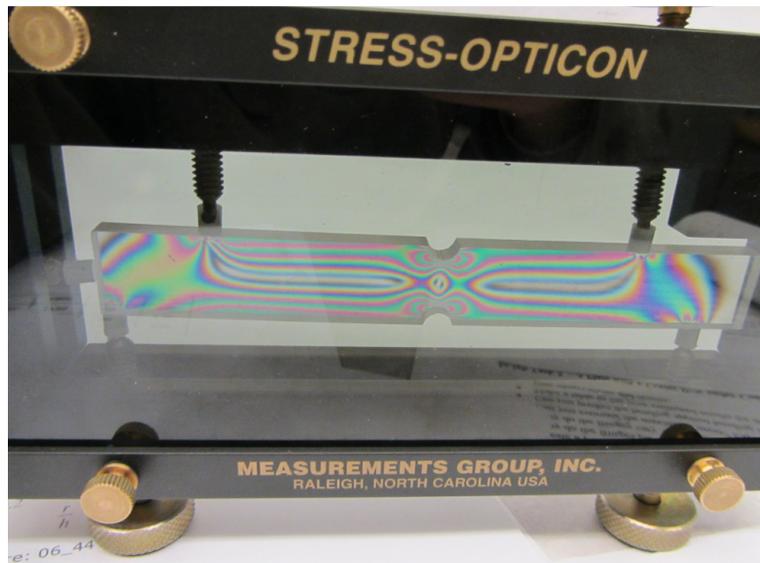


Fig. 2 Illustration of the fringe pattern on a loaded sample in the Stress-Opticon Kit ^[17] .

The Stress-Opticon Kit

The Stress-Opticon (SO) is the kit used in the photoelasticity lab. The kit is a unique pocket-size “photoelastic laboratory” designed for qualitative demonstration of photoelastic stress analysis, mechanics principles, stress concentration and the behavior of structural elements ^[17] . The apparatus is shown in Fig. 2. When a specimen is loaded inside the kit, the polarizers allow visualization of the fringe patterns that correspond to the pattern of stress distribution in the loaded sample.

Assessment of Results and Discussion

During the quarter, the photoelasticity lab was conducted one week and the FEM lab was conducted the following week. Each lab was about 3-hours. At the beginning of the photoelasticity lab, the concept of photoelasticity and the stress-opticon working principle, the stress-optic law, was briefly introduced. Then students were grouped to perform a few case studies using the SO kit. The cases include seven in-lab tasks with the first four tasks parallel to those used in the FEM lab. Hence, our discussion focuses on students’ responses to those first four in-lab questions.

The in-lab case study tasks for the photoelasticity lab are listed in the Appendix section A1. A typical student’s work is discussed below. Student’s effort to accomplish the tasks demonstrates

from good to excellent assessment results. Survey results are discussed in the subsequent sections for students' learning from their FEA work versus their photoelasticity work.

In Task1, students were required to load a column bar under compression in the SO frame. As the load increases, students were able to observe the color change at the loaded end region, the color uniformity away from the loading point and color variation at the end opposite from loading. Then students were directed to explain Saint Venant's principle and be prepared to find a way to predict the applied load. In the process, students learn the difficulty of color identification and the associated uncertainty of results. A typical student's graphical observation is shown below in Fig. 3. And Table 1 presents the predicted loads based on the color observed by matching the given isochromatic fringe characteristics table in order to find the appropriate fringe order for a certain load. This is very different from the FEA approach although certain concepts become clear to students by analogy from both methods such as the concept of stress concentration and Saint Venant's principle. Since the underlying principle about how it works is so different, when it comes to quantitative analysis, students may have different experiences regarding the level of difficulty and confidence in the results thus obtained from different approaches.



Fig. 3 A color map for an axially loaded bar in the Stress-Opticon Kit.

Table 1 Force estimation for an axially loaded bar by finding the fringe number.

Load #	N_f	Color Match	$\sigma = KN_f$ (psi)	P (lbs)
1	0.28	Grey	44.8	7.39
2	0.60	Pale Yellow	96	15.84
3	1.00	Purple	160	26.40
4	1.08	Deep Blue	12.8	28.51
5	1.39	Green Yellow	222.4	36.70
6	1.63	Orange	260.8	43.03
7	2.00	Purple	320	52.81

Where: $K = \text{specific stress-optic factor} = f/t = 40/0.22 = 182 \text{ psi/fringe}$; $N_f = \text{Fringe number}$

In Task2, students were required to investigate a beam subject to pure bending using the photoelasticity SO kit. When load gradually increases, students could observe easily how the rainbow stripes appear parallel to each other in the middle section of the beam starting from very few stripes to many stripes as shown in Fig. 4. Students were required to explain why those stripes are parallel to each other and what it means when the density of stripes increases. Then students would develop a method to predict the bending moment by analyzing bending stress through counting the fringe numbers. A typical student's work is given in Table 2. The bending moment was evaluated using the flexural formula by using the estimated bending stress, which was determined using the fringe numbers.

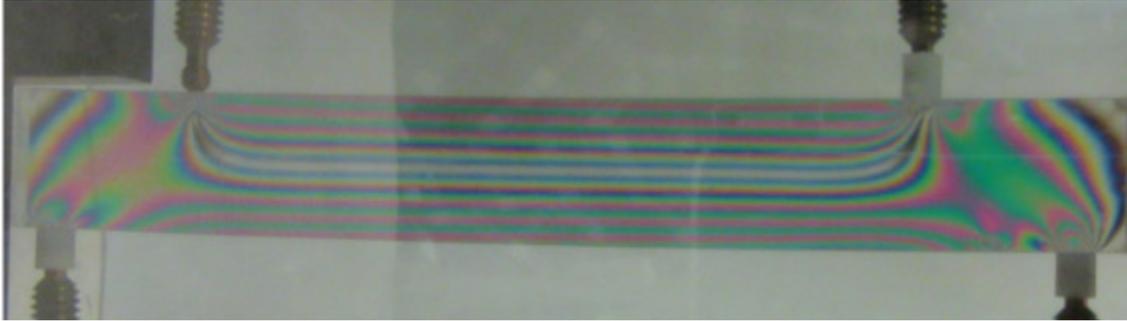


Fig. 4 A color map for a beam under pure bending in the Stress-Opticon Kit.

Table 2 Bending moment estimation using fringe numbers.

Load #	N_f	Fringe (contour # from top or bottom to center)	$\sigma = KN_f$ (psi)	M (lb-in)
1	1.0	1	182	3.77
2	2.0	2	364	7.54
3	2.5	2.5	455	9.43
4	3.0	3	546	11.31
5	3.5	3.5	637	13.20
6	4.0	4	728	15.09
7	5.0	5	910	18.86

Where: $K = \text{specific stress-optic factor} = f/t = 40/0.22 = 182 \text{ psi/fringe}$; $N_f = \text{Fringe number}$

Task3 requires students to estimate the stress concentration factor K when the plate with a center hole is subjected to uniaxial compression. For this case, the K value can be estimated based on the geometry alone and a textbook graph. The student arrived at a value of $K=2.1$ by $K=K(2r/D)=K(0.375/0.75) = K(0.5) =2.1$. Then the student loaded the plate by increasing the load gradually so that observations could carefully be made about how the stress (color) pattern changes. The stress at the stress concentration location and the remote stress using the fringe counting method could be estimated. As a result K of 2.4 was determined, which is close to the value given previously.

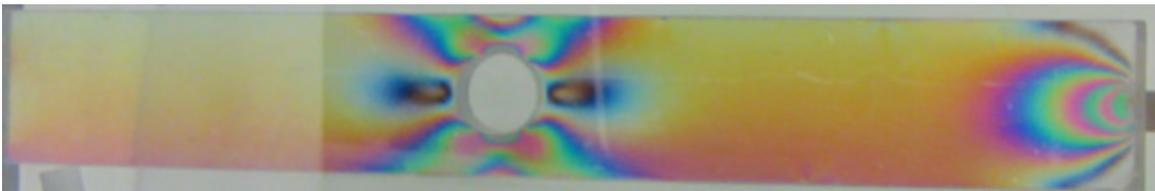


Fig. 5 A color map for a plate with a center hole under compression in the Stress-Opticon Kit.

Task4 requires students to estimate the stress concentration factor K when the notched plate is subjected to pure bending. For this case, the K value can also be estimated based on the geometry alone and a textbook graph. In this case the student came up to a value of $K=1.45$ by $K=K(r/h,b/h)=K(0.1315/0.51, 0.25/0.51) =1.45$. Then the plate is loaded gradually so that the change of the stripe patterns can be observed. Then using the fringe counting method, the student estimated the stress at the stress concentration location which is at the bottom of the notch and the stress at the other location of interest as the normalizer. The estimated value of K was 1.43, which is very close to the value given previously.

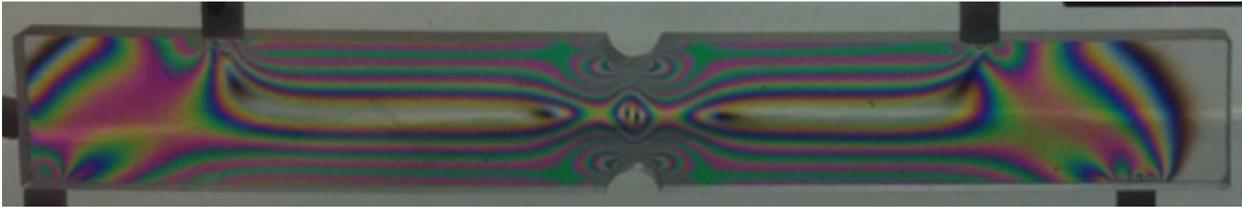


Fig. 6 A color map for a notched plate under pure bending in the Stress-Opticon Kit.

During the FEM lab, students were guided by the instructor to have a brief understanding about how the software, namely mini-FEA and ANSYS APDL work. The main objectives of this lab were then articulated, which included:

- Solve for the stress field of a loaded object
- Solve for the SCFs for a plate with a central hole and for a notch plate
- Contrast the difference between the two visualization tools of FEM and photoelasticity for stress analysis

Then students were grouped to perform the following in-lab tasks, parallel to what was done in the photoelasticity lab. Since we are concerned the most with student responses regarding learning through FEA versus learning through photoelasticity, we give the students' assessment results that are most relevant.

In Task1, students were directed to learn the basics about implementing an FEA program. Then they were required to complete specific tasks such as finding the uniaxial stress at a point, elongation/contraction of a certain segment in the body to enhance their concept learning and observe results graphically, similar to what they saw in the previous photoelasticity lab. Then the following questions are given in Task1 e):

e) Near the applied load, what do you observe about the color that indicates stresses? What do you observe of the color away from the loading point, say at the middle of the column? What is the name of this phenomenon?

A typical color map that students could easily get from the FEA post-processing graphics is given as follows.



Fig. 7 A Typical FEA Post-Processing Graph for a plate under compression to illustrate Saint Venant's Principle.

From such typical graphics, students observe how the color becomes uniform or a single color away from the applied point load, which indicates a constant axial stress state. Then it becomes an easy task for them to answer the aforementioned questions and to expect a longer learning retention.

The last question in Task1 is a survey question, which was given as:

f) Recall what happened in your photoelasticity lab about this same problem. Which tool gives you more confidence about stress results?

This same question was also given at the end of Task 2 and Task 3. Our summary of students' responses to this question is given after a brief discussion of Task 2 and 3.

In Task2, students were required to investigate the stress field of a cantilever beam under pure bending. Mini-FEA was used for both Task1 and Task2. Bending stress results at multiple locations were required to be extracted from the FEA post-processing graphics as illustrated in the following figure. Based on the FEA post-processing interface, students can readily extract the stress results and then compare them with their analytical solutions based on the beam bending flexural formula that students learned from their MOM class. All major results are tabulated and students' work assessment results are in general excellent.

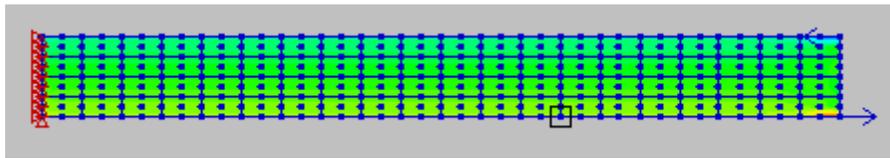


Fig. 8 Bending of a plate using miniFEA.

In Task3 and Task4, students were required to study the concept of stress concentration using ANSYS FEA programs. These problems were given such that students' attention was primarily focused on the post-processing steps. In Task3, a plate with a center hole under compression was investigated with the typical post-processing stress graphics given in Fig. 9. The plate in the simulation was analogous to one of the actual plates with a center hole in the photoelasticity lab, both in size and properties. While students still remembered how they observed the stress patterns for an identical plate in their photoelasticity lab, they also might recall the frustrations they experienced when they had to come up with quantitative solutions for the stress concentration. They might feel better about how this FEA program could help them to learn the basic concepts almost the same way as they observed using the SO kit, but with less frustration. By so doing, they obtain more accurate results in comparison with those they worked out using the analytical method from their textbook.

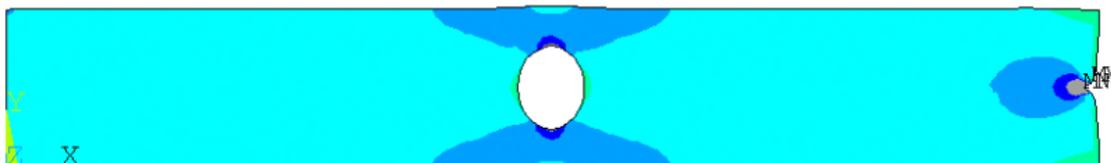


Fig. 9 FEA Post-processing graphics for a plate with a center hole under compression in Task3.

Similarly, in Task4, a notched plate used in the previous photoelasticity lab was simulated using the FEA program. A typical post-processing graphic is given in Fig. 10. Students could still recall the colorful stripes as well as the stress patterns at the nearby stress concentration location that they observed for such a plate when loaded under bending in the previous photoelasticity lab. In like manner, the FEA program post-processing graphics provides similar impressive results. As an outcome, students could retrieve these results of interest more confidently and observe how strikingly close the solutions match their calculations based on the textbook method.

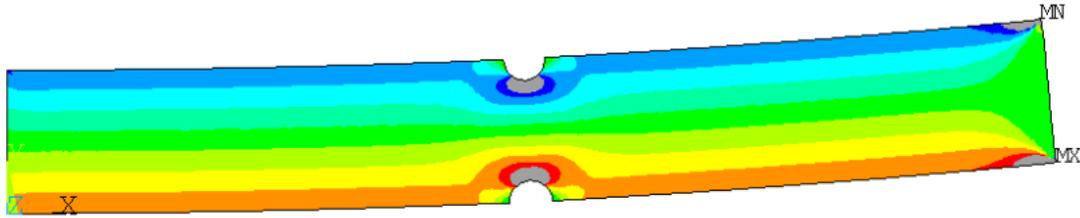


Fig. 10 FEA Post-processing graphics for a notched plate under bending in Task4.

Although both methods of photoelasticity and FEA may introduce errors with different mechanisms, FEA works under clear assumptions with more definite answers for students to grasp while photoelasticity provides more direct and quick results. However, the quantitative interpretation of the later method can be more subjective because the counting of colors is often difficult. Typical students' responses as summarized below do reflect this argument.

Quote: "This tool of analysis gives me more confidence because the analysis is much more precise. In the photoelasticity lab, we estimated stress based on the color range, which can be vague according to the interpretation of color."

Quote: "I trust the FEA results more than the photoelasticity results. I am more confident in the FEA results."

Quote: "The FEA method for this problem is more accurate since the program is looking at very specific points. For photoelasticity lab you eyeball the points which has high degree of error. I am more confident with SCF results since it is calculated and the computer helps to analyze the stress which is more accurate than eyeing it."

Quote: "Stress opticon is the tool we used last week. FEA gives more confidence about stress results."

Quote: "The photoelasticity lab showed us a more precise color demonstration and I think it will allow us to see the stress better. The program gives a better confidence although the photoelastic lab might give a more realistic result."

The FEA in-lab work was ended with the following survey questions:

*Overall what do you think about photoelasticity vs. FEA for your learning of MOM concepts?
Which one do you like the most so far? Why?*

Typical responses from students are given below. From their responses, we may see that both tools are appreciated but they do have some bias towards either one depending on the purpose of its usage.

Quote: "Overall the FEA methods appeared to be more reliable for providing accurate data and analysis. The photoelasticity method was a better visual representation of the stress distribution, but because of the interpretation/guessing required, it did not hold up as well in computing actual values. I would prefer to use the FEA results for analysis, but the photoelasticity method for observation (more tactile approach/interactive)."

Quote: "Both methods are helpful at clarifying the concepts learned in class. However, I like the FEA better because it is more accurate and gives you more information. For example, in the photoelasticity experiment, we always had a dilemma of which color was the right one, here we don't have that problem."

Quote: "Having the chance to work with the photoelasticity and FEA. I think that both of them are cool to learn the concepts with but when it comes to the FEA I think I have more visual determination and how the stress changes through it. I like the FEA because it is more accurate precise and visual to learn from."

Quote: “I think both tools are very valuable. The computer program is easier to use so it is more practical. But I think I enjoy the photoelasticity learning because it is more realistic in nature.”

Quote: “Photoelasticity is more realistic but the FEA is more accurate for calculating the different values. The FEA gives us real numbers to work with and that helps us have more confidence in the answer, the photoelastic lab is only based off of inspection at colors and opinion. Both are good but the FEA is better in a real application.”

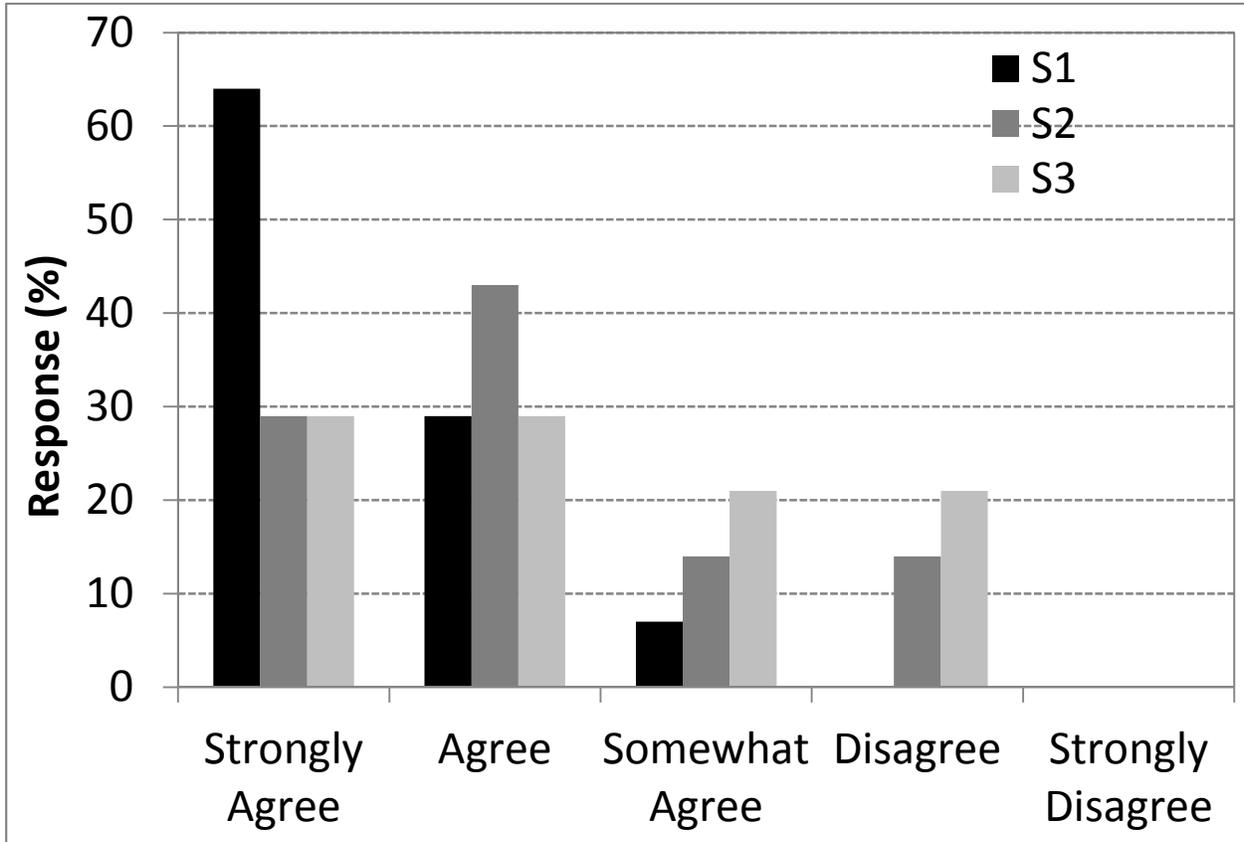


Fig. 11 Survey Results for the Statements in A4.

A different approach was taken for the second time of teaching by including the FEA activities at the end of the photoelasticity lab in order to enhance the effectiveness of learning. The FEA method and the survey of its effectiveness of learning were given to only one of the lab sessions with about 14 students participating. Cases illustrated using the FEA method were similar to what was done previously but with much less time spent. The attached survey questions listed in A3 and A4 were then given to students for their response. The survey results were strikingly similar to what was observed the first time for the survey questions listed in A3. For example, the survey results for stress predictions for Case I to III show that on average about 70% students indicate that they were more confident in using the FEA method and about 22% thought both methods were the same. For the illustration of other concepts graphically such as the Saint-Venant’s principle, stress concentration, bending stress stripe patterns etc., most students, approximately 62%, regarded both methods the same.

In the second part of the survey questions, A4, students were asked to provide their own opinion to each of the three statements S1-S3 on a scale from Strongly Agree to Strongly Disagree. The results of their response are summarized in Fig. 8. It can be seen that almost all students either

strongly agree or agree to Statement 1 given in A4, which indicates that both FEA and photoelasticity methods are useful tools for visualizing some key fundamental concepts in MOM. Notice that there was no disagreement to Statement 1. Statement 2 says that the FEA method illustrates stresses more precisely and provides a more solid grasp on the concepts of stress concentration, Saint-Venant's principle, uniaxial tension/compression and bending stress etc. For this statement, we have more than half, >70% either strongly agree or agree, only about 12% somewhat agree and about 12% do disagree. Lastly, Statement 3 is essentially to test if FEA is the more preferred tool over photoelasticity, in illustrating concepts. The response again leans toward FEA for most students, but for this statement, there is a relatively bigger portion of disagreement at ~21%.

Concluding Remarks

In summary, FEM is a valuable tool and can be integrated as a component into today's MOM class. If it is appropriately used, it can provide not only an alternative method, to the traditional photoelasticity method, to illustrate some key concepts of MOM graphically, but also allows a quick introduction of the basic implementation steps in using a FE package. To use the FEM effectively, however, one needs to realize when, where and for what amount of usage it shall be given in order to have the most effect for students' learning process.

Future Work

Although FEM can be used as an advanced visualization tool to illustrate some key concepts in MOM, its advantage may be weakened due to its complexity. In order to use this tool more effectively, the author's next effort is to develop user friendly graphic interfaces for case-specific studies so that students can design experiments without struggling with the software handling difficulties.

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Appendix

A1. Selected PHOTOELASTICITY In-Lab Case Study Questions (Fall 2013)

In-lab Task 1 – Column Loading (a Bar under Compression)

- Observe and explain Saint Venant's Principle
- What do you observe as you load the specimen gradually with the increase of load?
- Can you obtain the stress? Demonstrate how the uniaxial stress varies with load using the fringe order estimation.
- Can you predict the applied load? Demonstrate how the load can be estimated.
- Make a table to list your estimated results for the previous two questions. Discuss your observations and results.

In-lab Task 2– A Beam under Pure Bending

- Create a pure bending scenario, load gradually from small to big.
- Why are the fringes parallel to each other?
- Why do the fringes vary with uniform spacing?
- Can you estimate the maximum stress? Explain
- Can you predict the applied bending moment? Explain.
- Make a table to list your estimated results for the previous two questions. Discuss your observations and results.

In-lab Task 3 – A Plate with a Center Hole under Compression

- Based on the given geometry of the sample, estimate the stress concentration factor K when the plate is loaded under uniaxial compression or tension.
- Load up the sample gradually. What do you observe? How does the stress pattern change at the top and bottom points on the circle, what about left and right corresponding points on the circle?
- Can you estimate the magnification of stress at the stress concentration locations based on the fringe pattern under each specific load?
- Can you estimate the stress concentration factor based on the photoelastic method?
- Discuss any estimated results from the previous two questions.

In-lab Task 4 – A Notched Bar under Pure Bending

- Based on the given geometry of the sample, estimate the stress concentration factor K when the sample is loaded under pure bending.
- Load up the sample gradually. What do you observe? How does the stress pattern change at the notch corners?
- Can you estimate the magnification of stress at the stress concentration locations based on the fringe pattern under each specific load?
- Can you estimate the stress concentration factor based on the photoelastic method?
- Discuss any estimated results from the previous two questions.

A2. Selected FEA In-Lab Case Study Questions (Fall 2013)

In-lab Task 1 – Column Loading (a Bar under Compression)

Use the openware at <http://engineering-education.com/miniFEA/> to complete this exercise. Follow your lab instructor's lead about how this simulation can be done.

Step 1 – build your model by the given information below

- **Geometry:** Length = 146 mm , Height = 20mm, Thickness = 6 mm (note that this software assumes unity thickness) ; the origin of coordinates are set at the bottom left corner by default.
- **Material:** $E = 3.2(10^3) \text{ N/mm}^2$, $\nu = 0.4$.
- **Mesh:** 30 x 6 Linear Elements.
- **Loads:**
- **Left end:** all nodes do not displace horizontally ($U_x = 0$); To solve the problem, you should let the displacements in the vertical direction on the left end to be zero as well, i.e., $U_y = 0$ at one of the nodes at least.
- **Right end:** Apply a horizontal concentrated compressive force P at the middle node

Step 2 – Fill up the following tables as much as possible based on your FEA analysis

(note: keep the given units: stress will be in MPa, elongation in mm) and then provide answers to the questions asked.

Table 1: Stress component σ_x on point A (x,y) = A(73, 10); Compare FEA results and predictions based on simple uniaxial compression for the loads given. (note: take the thickness to be unity for analytical solutions in order to be comparable with the FEA results)				
	P = 1000 N	P = 2000 N	P = 4000 N	notes
σ_x (FEA)				
σ_x (Analytical)				

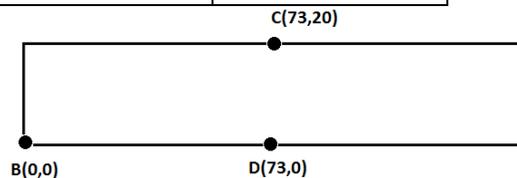


Table 2: Displacements from FEA at three points when P = 2000 N			
	B	C	D
U_x			
U_y			

Table 3: Elongation/contraction of segments BD and CD; compare FEA and axial loading prediction when P = 2000 N (note: take the thickness to be unity for analytical solutions in order to be comparable with the FEA results)		
	FEA	Axial Loading Prediction
$\delta_{DB} = U_x^D - U_x^B$		
$\delta_{CD} = U_y^C - U_y^D$		

Answer the following questions:

- Derive here the uniaxial prediction for stress here, then fill up table 1
- Would you expect the FEA stresses σ_x to agree reasonably well at the point A(73,10)? Why or why not does it agree well with the axial prediction? If the actual thickness is considered for the prediction, how can you correct the FEA results to be more realistic?

- c) Derive here δ_{BD} and δ_{CD} based on the theory of a linear elastic axially loaded bar.
- d) Does your analytical solution of elongation of the bar agree with your FEM results? Why or why not? If the actual thickness is considered for the prediction, how can you correct the FEA results to be more realistic?
- e) Near the applied load, what do you observe about the color that indicates stresses? What do you observe of the color away from the loading point, say at the middle of the column? What is the name of this phenomenon?
- f) Recall what happened in your photoelasticity lab about this same problem. Which tool gives you more confidence about stress results?

In-lab Task 2– A Beam under Pure Bending

Use the openware at <http://engineering-education.com/miniFEA/> to complete this exercise. Follow your lab instructor’s lead about how this simulation can be done.

Step 1 – Build your model by the given information below

- **Geometry:** Length = 146 mm , Height = 20mm, Thickness = 6 mm (note that this software assumes unity thickness) ; the origin of coordinates are set at the bottom left corner by default. (Note: *take the thickness to be unity for analytical solutions in order to be comparable with the FEA results.*)
- **Material:** $E = 3.2(10^3)$ N/mm², $\nu = 0.4$.
- **Mesh:** 20x4 Quadratic Elements.
- **Loads:**
- **Left end:** : All nodes at left end $x = 0$ (not just those shown in Figure) do not displace horizontally or vertically ($U_x = 0, U_y = 0$).
- **Right end:** $F_x = 1000$ at $(x,y) = (146,0)$; $F_x = -1000$ at $(x,y) = (146,20)$

Step 2 – Analyses and Comparison with FEA Results

- Enter FEA stress σ_x at points A(73,0), B(73,5),C (73, 10), D(73,15), and E(73,20) into table.
- Use simple bending to predict stresses at these points, and enter values into the table. Remember how y is defined in the bending stress formula $\sigma = -My/I$.

Table: Stress Comparison

	(73,0),	(73,5),	(73,10),	(73,15),	(73,20),
σ_x (FEA)					
σ_x (-My/I)					

Show the individual terms for evaluating $\sigma_x = -My/I$

Answer the following questions:

- a) Would you expect the FEA stresses σ_x to agree reasonably well at those points shown in the table? Why or why not does it agree well with the bending prediction? If the actual thickness is considered for the prediction, how can you correct the FEA results to be more realistic?
- b) Recall what happened in your photoelasticity lab with this same problem. Which tool gives you more confidence about stress results?

In-lab Task 3 – A Plate with a Center Hole under Compression

This problem will be demonstrated using ANSYS codes given by your lab instructor. Students are expected to provide discussion of observations and results.

Step 1 – build the model by the given information below

- **Geometry:** Length = 146 mm , Height = 20mm, Thickness = 6 mm, center circle diameter = 10mm.
- **Material:** $E = 3.2(10^3)$ N/mm², $\nu = 0.4$.
- **Mesh:** fine mesh
- **Loads:**
- **Left end:** all nodes do not displace horizontally ($U_x = 0$); To solve the problem, you should let the displacements in the vertical direction on the left end to be zero as well, i.e., $U_y = 0$ at one of the nodes at least.
- **Right end:** Apply a horizontal concentrated compressive force P at the middle node

Step 2 – Fill up the following table based on the FEA analysis

(note: keep the given units: stress will be in MPa, elongation in mm) and then provide answers to the questions asked.

Table 1: Stress component σ_x on point A (x,y) = A(73, 15) and point B(x,y)=B(73,20); Compare FEA results and predictions based on the geometrical configuration alone				
	P = 1000 N	P = 2000 N	P = 4000 N	notes
σ_x^A (FEA)				
σ_x^B (FEA)				
σ_x^{avg}				
$K_1 = SCF = \sigma_x^A / \sigma_x^B$				
$K_2 = SCF = \sigma_x^A / \sigma_x^{avg}$				

Note: σ_x^{avg} is the average normal stress for an axially loaded member at the location of stress concentration but without stress concentration effect considered.

Answer the following questions:

- What do you observe for the results of SCF under different loading magnitude? How does the SCF from the FEA results compare to that based on the geometry only? Why or why not do they agree with each other? Which evaluation method, K_1 vs. K_2 , is desired and correct for obtaining the accurate SCF? Why is the other not correct?
- Recall what happened in your photoelasticity lab with this same problem. Which tool gives you more confidence about the SCF results?

In-lab Task 4 – A Notched Bar under Pure Bending

This problem will be demonstrated using ANSYS codes given by your lab instructor. Students are expected to provide discussion of observations and results.

Step 1 – build the model by the given information below

- Geometry:** Length = 146 mm , Height = 20mm, Thickness = 6 mm, notch diameter = 6 mm (half-circle notch, $b = r = 3\text{mm}$)
- Material:** $E = 3.2(10^3)$ N/mm², $\nu = 0.4$.
- Mesh:** fine mesh
- Loads:**
- Left end:** : All nodes at left end $x = 0$ (not just those shown in Figure) do not displace horizontally or vertically ($U_x = 0, U_y = 0$).
- Right end:** $F_x = 1000$ at (x,y) = (146,0); $F_x = -1000$ at (x,y) = (146,20) , or other loading condition as asked.

Step 2 – Fill up the following table based on the FEA analysis

(note: keep the given units: stress will be in MPa, elongation in mm) and then provide answers to the questions asked.

Table 1: Stress component σ_x on point A (x,y) = A(73, 3) and point B(x,y)=B(30,3); Compare FEA results and predictions based on the geometrical configuration alone				
	$F_x = 1000$ N	$F_x = 2000$ N	$F_x = 4000$ N	notes
σ_x^A (FEA)				
σ_x^B (FEA)				
σ_x^M				
$K_1 = SCF = \sigma_x^A / \sigma_x^B$				
$K_2 = SCF = \sigma_x^A / \sigma_x^M$				

Note: σ_x^M is the bending stress at the location of stress concentration but without stress concentration effect considered.

Answer the following questions:

- What do you observe for the results of SCF under different loading magnitude? How does the SCF from the FEA results compare to that based on the geometry only? Why or why not do they agree with each other? Which evaluation method, K_1 vs. K_2 , is desired and correct for obtaining the accurate SCF? Why is the other not correct?
- Overall what do you think about photoelasticity vs. FEA for your learning of MOM concepts? Which one do you like the most so far? Why?

A3. FEA vs. PHOTOELASTICITY for understanding MOM concepts – Student Survey I

Case I – Column Loading (a Bar under Compression)

Circle the method that provides you more confidence about the compressive stress prediction under certain loading conditions

Photoelasticity	FEA	Both the same
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Circle the method that illustrates Saint-Venant’s Principle better

Photoelasticity	FEA	Both the same
-----------------	-----	---------------

Case II – A Beam under Pure Bending

Circle the method that provides you more confidence about the bending stress prediction.

Photoelasticity	FEA	Both the same
-----------------	-----	---------------

Circle the method that illustrates Saint-Venant’s Principle better

Photoelasticity	FEA	Both the same
-----------------	-----	---------------

Circle the method that illustrates better the characteristics of pure bending stress, i.e., uniform spacing between straight line color contours.

Photoelasticity	FEA	Both the same
-----------------	-----	---------------

Case III – A Plate with a Center Hole under Compression

Circle the method that provides you more confidence about the value of SCF estimated.

Photoelasticity	FEA	Both the same
-----------------	-----	---------------

Circle the method that illustrates the concept of stress concentration better

Photoelasticity	FEA	Both the same
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A4. FEA vs. PHOTOELASTICITY for understanding MOM concepts – Student Survey II

The purpose of this survey is to help both students and instructors understand the effectiveness of these visualization tools on some fundamental physics addressed in MOM.

Statement 1: Both FEA and photoelasticity are useful tools for visualizing some key fundamental concepts in MOM such as stress concentration, Saint-Venant’s Principle, uniaxial tension, compression and bending stress.

Statement 2: Although both FEA and photoelasticity are useful tools, FEA illustrates more precisely and provides a more solid grasp on the concepts of stress concentration, Saint-Venant’s Principle, uniaxial tension, compression, and bending stress.

Statement 3: If I am allowed to choose one of the two methods, FEA vs. photoelasticity, to demonstrate the concepts of stress concentration and Saint-Venant’s Principle, I will definitely choose FEA.

Statements →	1	2	3
Strongly Agree			
Agree			
Somewhat Agree			
Disagree			
Strongly Disagree			