Five Simplified Integrated Methods of Solution (SIMS) for the Ten Types of Basic Planar Vector Systems in Engineering Mechanics

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Dr. Malladi earned his PhD (Mechanisms) at Oklahoma State University, USA in 1979, MTech (Machine Design) at Indian Institute of Technology (IIT), Madras in 1969, and BE (Mechanical) at Osmania University, India in 1965. He was on the faculty of Applied Mechanics Department, IIT Madras from 1968 till 1973 when he left for US. He received a Republic Day Award for "Import Substitution" by Government of India in 1974 for developing a hydraulic vibration machine at IIT Madras, for Indian Space Research Organization (ISRO), Tumba. In US he worked for the R&D departments of Computer, ATM and Railway Industry. He then resumed teaching at several US academic institutions. He spent two summers at NASA Kennedy Space Center as a research fellow. He received awards for academic, teaching and research excellence. His teaching experience ranges from KG to PG.

After his return to India, Dr. Malladi taught his favorite subject "Engineering Mechanics" at a few engineering institutions and found a need to 1. simplify the subject 2. create a new genre of class books to facilitate active reading and learning and 3. reform academic assessment for the sure success of every student to achieve "Equality", that is "Equal High Quality" in their chosen fields of education.

The results of his efforts are 1. the following "landmark" paper 2. a class-book titled "Essential Engineering Mechanics with Simplified Integrated Methods of Solution (EEM with SIMS) and 3. material for software development for Improved Assessment Score (IAS), so that even an initially failing student achieves grade A, irrespective of the tier of school one attends.

Dr. Malladi has a simple and practical vision for future Unified Education with Diversity for the world, to develop the four quadrants of each child' brain in four languages and four subjects, namely Math, Management, Design and Technology with their applications. For example application of the study of Geometry of atoms results in Physics or Chemistry. Application of Design will result in Art or Engineering.

Dr. Malladi is willing to offer free workshops and short courses world wide in his Essential Engineering Mechanics, and free consultation on his Simplified Secular Education and Meditation, provided his "single" travel, lodging and alcohol-free pure vegetarian food expenses are met.

Dr. Malladi is a naturalized citizen of USA, and holds Overseas Citizen of India (OCI) Registration. He has US Federal Health Insurance valid anywhere. However he did not use his insurance for decades.

Dr. Malladi lives happily at his birth-place Raja-mahendra-varam, an East Godavary river-town, in the State of Andhra Pradesh, India, with his India born and educated daughter-in-law and US born and Cornell educated son. They both help him in Malladi Academy Activities.

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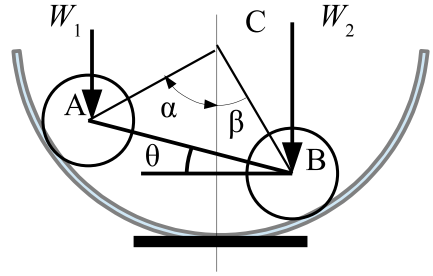
1. Inspired Introduction:

The inspiration for this paper had originated from a classic Timoshenko Problem [1] with six unknowns, that was asked in an Engineering Mechanics Semester University Exam in India during 2015. The author's students (400 in 5 sections of 80 each) at an affiliated college to Jawaharlal Nehru Technological University, Kakinada, found it to be very tough. After realizing that this problem was often repeated in previous exams by several universities, the author did some research on it and other problems considered tough for an average student. These problems were solved in a standard textbook [2] and the like, in more number of steps than the author felt were needed. The author could solve all of them in lesser number of steps. But, he still felt their solutions could be further simplified. He then recalled the Perpendicular Component Equation, he developed upon students' request for simplification, while teaching a course in Kinematics at Tuskegee University, USA, in early 1990s. With its extensions, he could solve for the six unknowns of the Timoshenko problem, using six equations, each with only one unknown. The author named these methods that yield equations with only one unknown / variable, as Five Simplified Integrated Methods of Solution (SIMS), in 2018 while working as a professor at KL University in India. Work related to SIM3 was presented by the author, as "Vector Rotation Theorem" at Mathematical Association of America's MathFest 2016. The new genre and format called IILA (Integrated Instruction, Learning and Assessment), of the author's forthcoming class-book was originally conceived by him at the Indian Institute of Technology, Madras in 1970s and implemented at Tuskegee University in 1990s. This class-book is titled "Essential Engineering Mechanics with Simplified Integrated Methods of Solution (EEM with SIMS)".

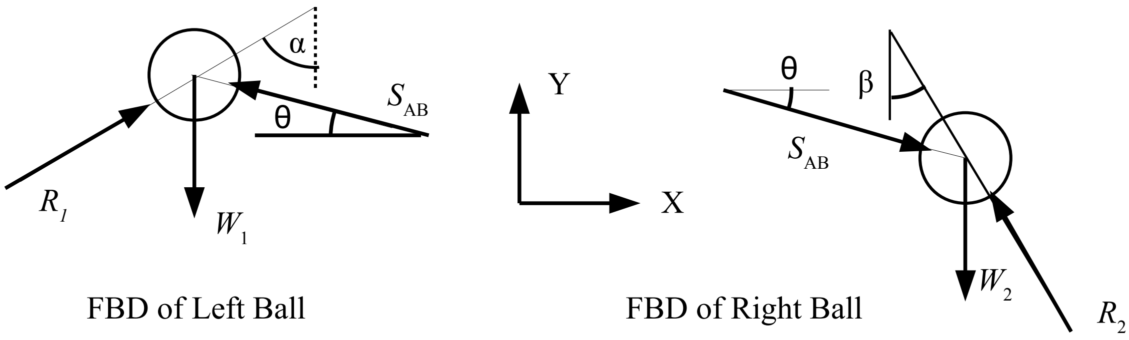
2. Timoshenko Problem and Typical Solution as in Reference [2].

A rigid bar AB with rollers of weights $W_1 = 75$ N and $W_2 = 125$ N at ends is supported inside a circular ring in a vertical plane as shown in the system space diagram. Radius of the ring and the length AB are such that the radii AC and BC make a right angle at the centre of the ring C. Neglecting friction and the weight of the bar AB, ascertain (find) equilibrium configuration defined by angle $[(\alpha - \beta) / 2]$ that AB makes with the horizontal. Find the contact (normal) reactions R_1 and R_2 at A and B and the axial force S_{AB} in the bar AB.



System Space Digram

Solution to Timoshnko Problem as in reference [2] with comments in square brackets by this paper's author. Minor modifications are made due to changes in format.



Solution: $(\alpha + \beta) = 90^o$ ---- (1) **[Equation with 2 unknowns, from data]**

From the condition of inclination of AB.

$\theta = (\alpha - \beta) / 2$ or $(\theta + \beta) = (\alpha - \theta)$ -- (2) **[This equation with 3 unknowns is to be derived first and then used]**

Consider the FBD of Left Ball

$\sum F_X = 0;\ S_{AB} \cos\theta = R_1 \sin\alpha$ -- (3) **[Equation with 4 unknowns]**

$\sum F_Y = 0;\ R_1 \cos\alpha + S_{AB} \sin\theta = W_1 = 75$ -- (4) **[Equation with 4 unknowns]**

From Equations (3) and (4)

$75 = S_{AB} \sin\theta + S_{AB} (\cos\theta / \sin\alpha) \cos\alpha$ leads to

$75 = S_{AB} \cos(\alpha - \theta) / \sin\alpha$ --- (5) **[R_1 Eliminated to form equation with 3 unknowns]**

Consider the FBD of Right Ball

$\sum F_X = 0;\ S_{AB} \cos\theta = R_2 \sin\beta$ -- (6) **[Equation with 4 unknowns]**

$\sum F_Y = 0;\ R_2 \cos\beta - S_{AB} \sin\theta = W_2 = 125$ -- (7) **[Equation with 4 unknowns]**

From Equations (6) and (7)

$125 = S_{AB} (\cos\theta / \sin\beta) \cos\beta - S_{AB} \sin\theta$ leads to

$125 = S_{AB} \cos(\theta + \beta) / \sin\beta$ --- (8) **[R_2 Eliminated to form equation with 3 unknowns]**

From Equations (8) , (5), (2) and (1) **[Four equations solved to get α]**

$(125 / 75) = [\cos(\theta + \beta) / \sin\beta] * [\sin\alpha / \cos(\alpha - \theta)]$

$(5 / 3) = \sin\alpha / \sin\beta = \sin\alpha / \sin(90^o - \alpha) = \sin\alpha / \cos\alpha$

or **$\tan\alpha = 5/3$** or **$\alpha = 59.04^o$**

So, from (1) and (2) **$\beta = 30.96^o$ and $\theta = 14.04^o$**

From Equations (5), (3) and (6) **$S_{AB} = 90.95$ N,**

$R_1 = 102.89$ N and $R_2 = 171.51$ N

Compare the above complicated solution with the author's simplified solution in Example 18, in which α is found by applying SIM5 to Type 9 FBD with three unknowns, in the very first step of the solution and Equation (2) is derived by applying SIM4.

3. A Brief History Leading to 21st Century "Essential Engineering Mechanics"

17th Century: Isaac Newton, a native of England developed his three laws of motion for a particle moving in a Straight Line / Single Dimension (1D). The subject was known as Mechanics.

18th Century: Bernard Lamy, a native of France known by his famous Lami's Theorem, developed Parallelogram Law of Forces for two forces acting in a plane (2 Dimensional Space) on a Particle at rest.

19th Century: J.W. Gibbs, a native of USA and O. Heaviside, a native of UK, independently developed Vectors for 3 Dimensional Space (3D) to study the Laws of Electro-magnetism found by J.C. Maxwell, a native of Scotland.

20th Century: Stephen Timoshenko, a native of Russia, applied Mechanics to Engineering problems and as a professor at Stanford University, USA came to be known as the Father of Modern Engineering Mechanics (EM) with the publication of his Classic book on EM in 1937 with D.H. Young. However they did not use any Vector Notation in their book. This book is still in print in India, being widely used with its later editions.

J.L. Meriam pioneered the use of Vector Notation in Engineering Mechanics in US with his textbook in 1952, followed by F. Beer and R. Johnston Jr. in 1956, Irving H. Shames in 1959 and many others, with their later editions now [3] [4] [6]. However, the author found that the use of cartesian vector notation and cross products in solving the planar problems did not get popular in India. Timoshenko's formulae, $\Sigma X = 0$, $\Sigma Y = 0$ and $\Sigma M = 0$ are in wide use.

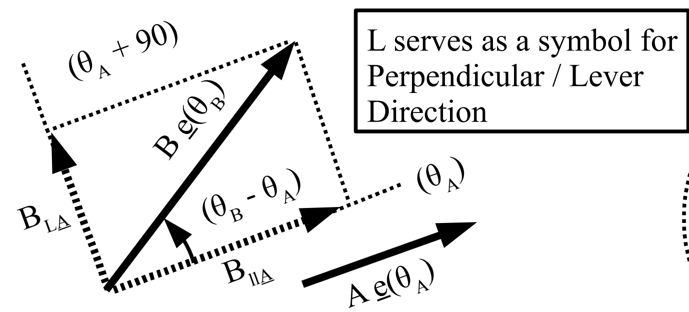
21st Century: Over the last four centuries, the subject had accumulated vast amount of information as well as several methods of solution for a given problem. It is usually taught in USA as Sophomore Statics in one semester followed by Dynamics in the next semester. But Engineering Institutions in India are teaching the subject as a single semester course. in the freshman year. This trend is spreading to other nations. The author felt the need for writing a book on "Essential Engineering Mechanics" for a single semester course with a fresh and efficient Genre. Using a New Polar Vector Notation with unified features that can be used for both 2D and 3D, he developed Five SIMS (Simplified Integrated Methods of Solution).

The five SIMS retain the traditional methods for finding the magnitude and direction of a vector using its X,Y component equations, but employ polar angles in stead of relative acute angles. This is to remove the confusion about sines and cosines, most students have, with the direct use of relative acute angles given in the problem description, in the solution steps. The concept behind SIM3 or Perpendicular Component Equation (LCE) with only one unknown, also known as direct method, is not new [6] [7]. But it is not used widely, because of the confusion students often have, in using sines and cosines of relative acute angles. Perpendicular Components found by using polar angles clear the above confusion. Perpendicular Component (LC) of apolar force vector to its polar arm vector, facilitates easy computation of moment of a force about its moment center, without considering their X, Y components. The author found that there are essentially Ten Types of Basic Planar Vector Systems in Engineering Mechanics. These will be discussed in later sections.

4. Origin for Simplified Integrated Methods of Solution (SIMS)

During early 1990s, while teaching Kinematics at Tuskegee University, the author's students complained at the "**complex**" notation to find velocities and accelerations in mechanisms either graphically or analytically using existing methods found in a typical US Textbook [8].

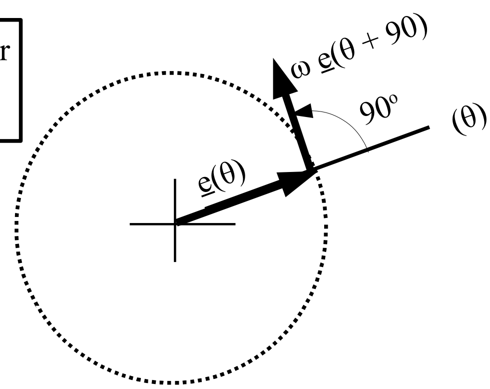
The author, then wrote a "Vector Loop Equation (VLE)" using chain rule for a four bar mechanism with his new "**simplex**" notation $\underline{e}(\theta)$ for Radial Unit Vector as shown in Figures A and B, θ being the polar angle. Velocity Vector Equation was found by differentiating the VLE with the notation $\underline{e}\,(\theta + 90)$ for the Transverse / Perpendicular (L as symbol) Unit Vector. Then Perpendicular Component Equation (**LCE**) to a Vector with an unknown (**say** ω_4) **or to the unit vector** $\underline{e}(\theta_4 + 90)$ **or line**$(\theta_4 + 90)$ was written. In this LCE, the LC of the vector with ω_4 was zero and thus ω_4 got eliminated. Then this LC Equation with unknown ω_3 was directly solved. This procedure was repeated by LCE to Vector with ω_3 and then ω_4 was solved. Four other SIMS were developed by 2018 and named, to solve all the Ten Types of Basic Planar Vector Systems each forming an equation with only one unknown.



$\mathbf{B_{ll\underline{A}} = B\cos(\theta_B - \theta_A)}$; $\mathbf{B_{L\underline{A}} = B\sin(\theta_B - \theta_A)}$

LC of $\underline{A}$ to $\underline{A}$ = $A_{L\underline{A}} = A\sin(\theta_A - \theta_A) = 0$ ... SIM3

Figure A. Parallel (ll) and Perpendicular (L) Components of $\underline{B}$ to $\underline{A}$ or line(θ_A)



$$\mathbf{(d/dt)\,[\underline{e}(\theta)] = \omega\,\underline{e}(\theta + 90)}$$

Figure B. Rotating unit vector and its time derivative. ω is angular velocity in radians / second.

Velocity Analysis of a Four bar mechanism:

Equating the Vector Loop Equations for Diagonal $\underline{AC}$:

$\underline{AC} = \underline{AD} + \underline{DC}$; $\underline{AC} = \underline{AB} + \underline{BC}$, we have

$$r_1\,\underline{e}(\theta_1) + r_4\,\underline{e}(\theta_4) = r_2\,\underline{e}(\theta_2) + r_3\,\underline{e}(\theta_3) \quad \text{--- (1)}$$

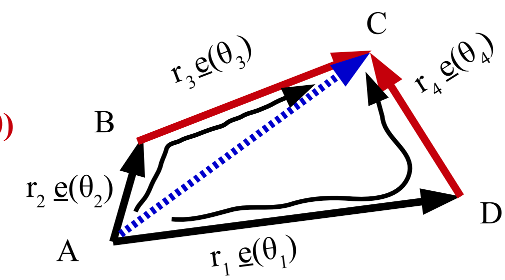


Figure C. Vector Loops for $\underline{AC}$

Velocity Vector Equation: [Time Derivative of (1)]

$$0 + r_4\omega_4\,\underline{e}(\theta_4 + 90) = r_2\omega_2\,\underline{e}(\theta_2 + 90) + r_3\omega_3\,\underline{e}(\theta_3 + 90) \quad \text{--- (2)}$$

LCE to line(θ_4 + 90) is

$$0 = r_2\omega_2\sin(\theta_2 - \theta_4) + r_3\omega_3\sin(\theta_3 - \theta_4) \quad \text{--- (3)}$$

LCE to line(θ_3 + 90) is

$$r_4\omega_4\sin(\theta_4 - \theta_3) = r_2\omega_2\sin(\theta_2 - \theta_3) \quad \text{---- (4)}$$

ω_3 and ω_4 can be directly computed from (3) and (4).

5. Simplified Integrated Methods of Solution (SIMS)

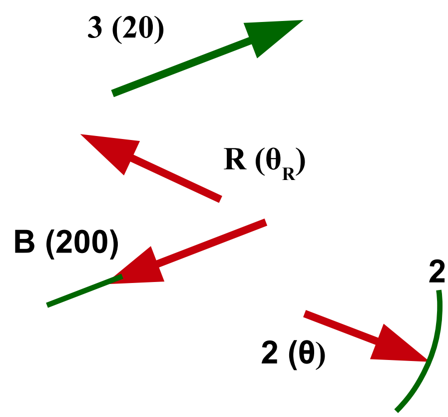
Traditional set of equations $\Sigma X = 0$, $\Sigma Y = 0$ and $\Sigma M = 0$, for a general 2D system will sometimes result in nonlinear and convoluted set of equations requiring several intermediate steps. as shown in section 2's typical textbook solution for the Timoshenko problem. To simplify, all vectors are represented in polar form. Five Simplified Integrated Methods of Solution (SIMS) are developed and applied progressively to solve the ten types of basic planar vector systems that occur in statically determinate engineering mechanics problems. SIMS will yield scalar equations each with only one unknown for solving basic vector systems with two or three unknowns. For use in SIMS, polar vectors are classified as below. Solutions to examples start by identifying the type and forming the vector system equations.

1. Vector with known magnitude and direction
Known Vector Example: $\underline{A} = 3\ \underline{e}(20)$

2. Vector with unknown magnitude and direction
Unknown Vector Example: $\underline{R} = R\ \underline{e}(\theta_R)$

3. Vector with unknown magnitude
Line Vector Example: $\underline{B} = B\ \underline{e}(200)$

4. Vector with unknown direction
Arc Vector Example: $C = 2\ (\theta)$

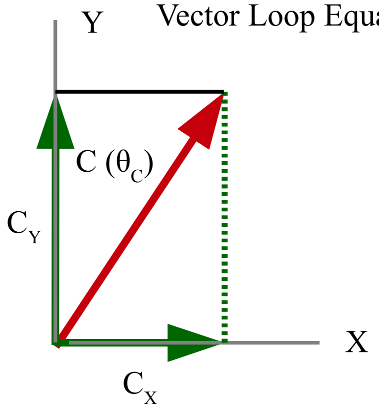


Vector System Equation (VSE) relates the Vectors in a System.
Vector Loop Equation (VLE) is a form of Vector System Equation with its Left Hand Side (LHS) having a single vector with an unknown direction.

SIM1: By squaring and adding the X, Y or Orthogonal Components of a Vector Loop Equation with two unknowns, its Left Hand Side (LHS) vector's unknown direction is eliminated. The resulting equation has only one unknown magnitude or direction.

SIM2: The unknown direction of a vector is found by an inverse trigonometric function of the vector's known X, Y components.

If the Vector System Equation is not already so, it is to be rearranged as a Vector Loop Equation to apply SIM1 and SIM2, for finding the unknowns.

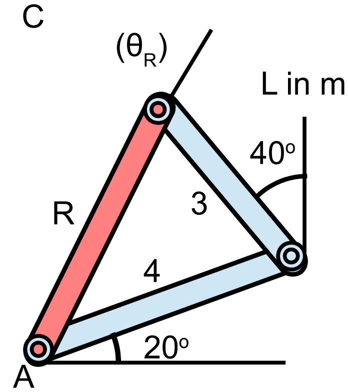


Elimination and Finding of a Direction in VLE.
$C\ \underline{e}(\theta)$ as the LHS, with unknown direction (θ_C) is related to Right Hand Side (RHS) Vectors.

$C \cos(\theta_C) = C_X = \Sigma$ RHS Cosine Components
$C \sin(\theta_C) = C_Y = \Sigma$ RHS Sine Components

SIM1: $C^2 = [C_X^2 + C_Y^2]$ **[θ_C is eliminated]**
SIM2: $\theta_C = [C_Y / |C_Y|]\ \cos^{-1}[C_X/C]$ **[θ_C is found]**

6. Type 1 System, T1[R, θ_R]: An unknown vector is related to two or more known vectors. The unknowns magnitude and direction are found by applying SIM1 and SIM2 to the Vector Loop Equation.



System Space Diagram

Example 1: Find the magnitude and direction of the member AC of the truss ABC shown.

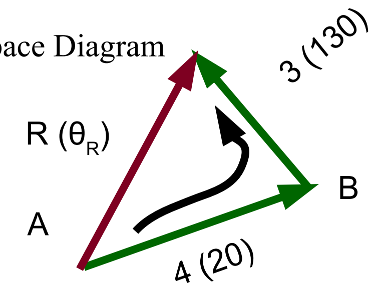
Solution: System is T1. Vector Loop Equation to draw the triangle ABC.

<u>AC</u> = <u>AB</u> + <u>BC</u>

R <u>e</u>(θ_R) = 4 <u>e</u>(20) + 3 <u>e</u>(130)

R_X = 4 cos(20) + 3 cos(130) = 1.83

R_Y = 4 sin(20) + 3 sin(130) = 3.67



Vector Loop Diagram

SIM1: Squaring and Adding X, Y Component Equations

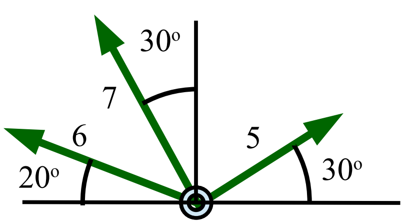
$R^2 = [R_X^{\,2} + R_Y^{\,2}]$ **Answer1 R = 4.1 m**

SIM2: Direction by Inverse Cosine Function

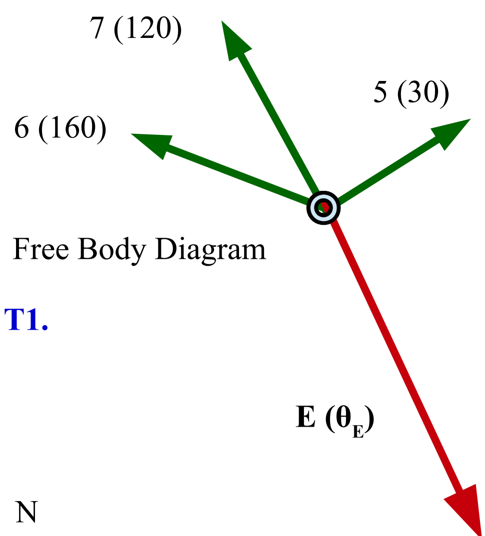
(θ_R) = $[R_Y / |R_Y|] \cos^{-1} [R_X/R]$, **Answer2 ($\theta_R$) = (63.5)**

Example 2: Find the equilibrant of the forces on a particle shown, in Newtons.

F in N



System Space Diagram



Free Body Diagram

Solution: (Given forces + Equilibrant) form FBD T1.

Σ<u>F</u> = 5 <u>e</u>(30) + 7 <u>e</u> (120) + 6 <u>e</u>(160) + E <u>e</u>(θ_E) = 0

Vector Loop Equation:

E <u>e</u>(θ_E) = - 5 <u>e</u>(30) - 7 <u>e</u> (120) - 6 <u>e</u>(160)

E_X = - 5 cos (30) - 7 cos (120) - 6 cos (160) = 4.808 N

E_Y = - 5 sin (30) - 7 sin (120) - 6 sin (160) = - 10.614 N

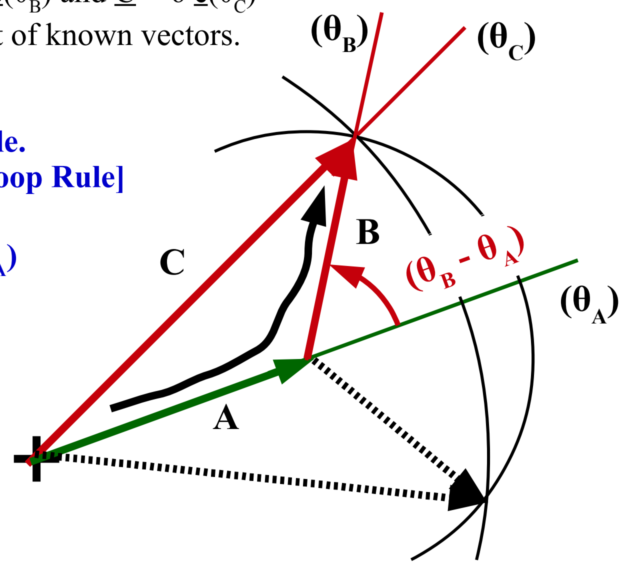
SIM1: $E^2 = [E_X^{\,2} + E_Y^{\,2}]$, **Answer1 E = 11.652 N**

SIM2: $\theta_E = [E_Y / |E_Y|] \cos^{-1} [E_X / E]$, **Answer2 θ_E = (- 65.63) = (294.7)**

7. Type 2 System, T2[θ_1, θ_2]: Two Vectors with unknown directions are related to the resultant of a set of known vectors. The two unknowns are found by applying SIM1 and SIM2 to the Vector Loop Equation with one of the unknown directions in the LHS vector.

Example 3. Find the polar angles of $\underline{B} = 3\ \underline{e}(\theta_B)$ and $\underline{C} = 6\ \underline{e}(\theta_C)$ the vector Δ, with $\underline{A} = 4\ \underline{e}(20)$, the resultant of known vectors.

Solution: System is T2.
Vector Loop Equation to draw the triangle.
$C\ \underline{e}(\theta_C) = A\ \underline{e}(\theta_A) + B\ \underline{e}(\theta_B)$ ... [Vector Loop Rule]
Orthogonal Components
Parallel (||) & perpendicular (L) to line(θ_A)
C || to line(θ_A) $= A + B\cos(\theta_B - \theta_A)$
C L to line(θ_A) $= 0 + B\sin(\theta_B - \theta_A)$



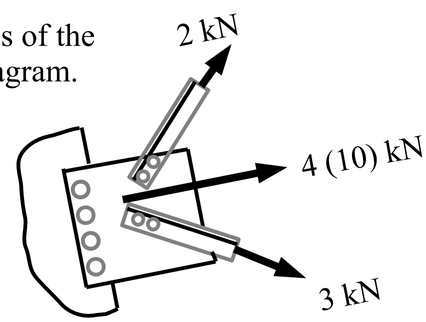
Vector Loop Diagram
(Vector Triangle)

SIM1: Squaring and adding,
the Polar Cosine Law is:
$C^2 = A^2 + B^2 + 2AB\cos(\theta_B - \theta_A)$
$(\theta_B - \theta_A) = \cos^{-1}[(C^2 - A^2 - B^2)/2AB]$
$(\theta_B - 20) = \cos^{-1}[(6^2 - 4^2 - 3^2)/2*4*3] = \pm 62.72^o$
Choosing $(\theta_B - \theta_A) = +62.72^o$

Answer1 $(\theta_B) = (82.72)$

With θ_B = (82.72), $C_X = 4.139$, $C_Y = 4.344$
SIM2: $(\theta_C) = [C_Y/|C_Y|]\cos^{-1}[C_X/C]$, Answer2 θ_C = (46.383)

Example 4. For the resultant 4 (10) kN, find the polar angles of the component forces 2 kN and 3 kN as in the System Space Diagram.

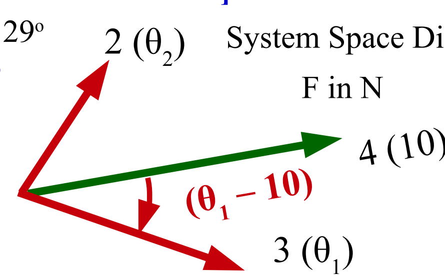


System Space Diagram
F in N

Solution: System is T2. Vector System Equation
$3\ \underline{e}(\theta_1) + 2\ \underline{e}(\theta_2) = 4\ \underline{e}(10)$ [Known Vectors on RHS]
Vector Loop Equation
$2\ \underline{e}(\theta_2) = 4\ \underline{e}(10) - 3\ \underline{e}(\theta_1)$ .... [Note the '–' sign]

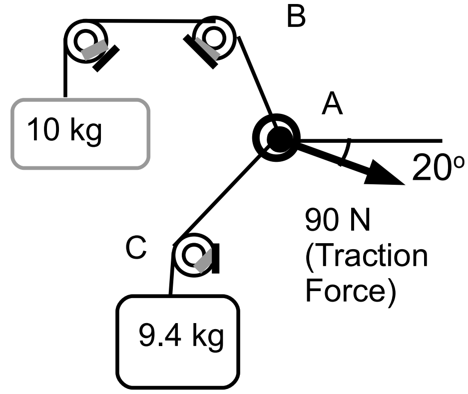
SIM1: $2^2 = 4^2 + 3^2 - 2*4*3\cos(\theta_1 - 10)$ --- [Polar Cosine Law]
$(\theta_1 - 10) = \cos\text{-}1[(2^2 - 4^2 - 3^2) / (-2*4*3)] = \pm 29^o$
In Vector System Diagram, $(\theta_1 - 10) = -29^o$
$\theta_1 = (-19)$ **Answer1 $\theta_1 = (341)$**

With $\theta_1 = (341)$
$2_X = 1.10$; $2_Y = 1.67$
SIM2: $\theta_2 = \cos^{-1}[2_X/2] = \pm 56.6$
As 2_Y is positive Answer2 $\theta_2 = (56.6)$



Vector System Diagram

Example 5: Determine the polar angle (θ_1) of the cable AB in the 2nd quadrant and corresponding (θ_2) of the cable AC for the Equilibrium of Pulley A of the tractive apparatus configuration shown. Assume smooth pulleys.



System Space Diagram

Solution: FBD is T2. Equilibrium Equation.

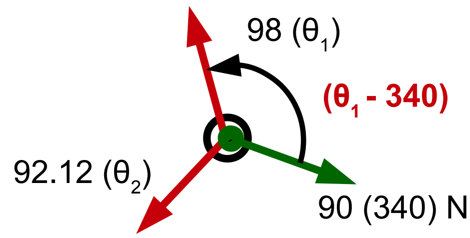
98 e (θ_1) + 92.12 e (θ_2) + 90 e (340) = 0

VLE: 92.12 e (θ_2) = - 90 e (340) - 98 e (θ_1)

SIM1: $92.12^2 = 90^2 + 98^2 + 2*90*98\cos(\theta_1 - 340)$

$(\theta_1 - 340) = +121.50$ from FBD [2nd Quadrant]

θ_1 = (461.5 – 360), Answer1 θ_1 = (101.5)



Free Body Diagram

With **θ_1 = (101.5)**

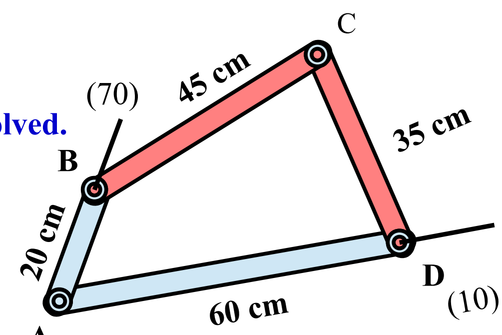
92.12_X = - 65.03 N; 92.12_Y = - 65.25 N

SIM2: $\theta_2 = \pm \cos^{-1} [A_X/A]$,

As A_Y is negative, θ_2 = (- 134.9 + 360)

Answer2 θ_2 = (225.1)

Example 6. Find the polar angles of the links 3 and 4 for the given four-link mechanism position.



Vector Space Diagram

Solution: T1 Loop BAD and T2 Loop BCD are solved.

In Δ BAD, <u>BD</u> = <u>BA</u> + <u>AD</u>

<u>S</u> <u>e</u>(θ) = - 20 <u>e</u>(70) + 60 <u>e</u>(10)

S_X = 52.25 cm; S_Y = - 8.38 cm; **<u>S</u> = 52.92 <u>e</u>(350.9)**

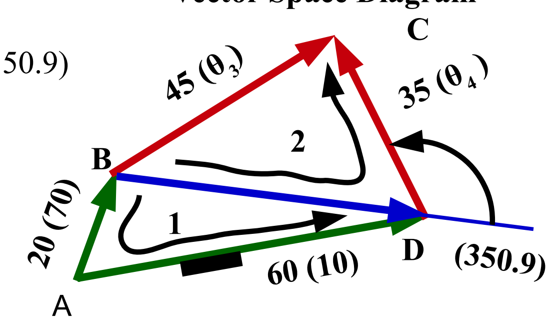
Vector Loop Equation for <u>BC</u> in ΔBDC:*

45 <u>e</u>(θ_3) = 52.92 <u>e</u>(350.9) + 35 <u>e</u>(θ_4)

SIM1: $45^2 = 52.92^2 + 35^2 + 2*52.92*35\cos(\theta_4 - 350.9)$

$\cos(\theta_4 - 350.9) = -0.54$; $(\theta_4 - 350.9) = +122.68$

θ_4 = (473.58 – 360), Answer1 θ_4 = (113.6)



Vector Loop Diagram

VLE for <u>BC</u> with θ_4 = (113.6)

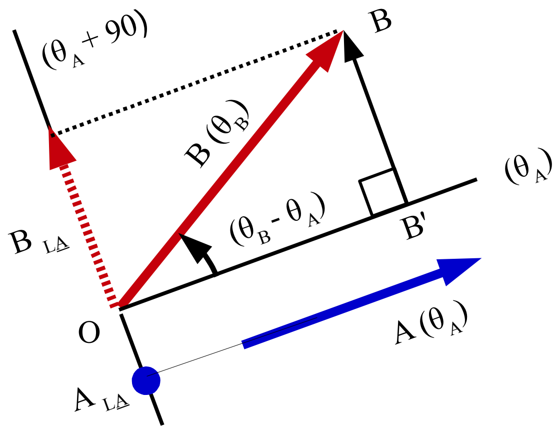
45 <u>e</u>(θ_3) = 52.92 <u>e</u>(350.9) + 35 <u>e</u>(113.76)

45_X = 38.24 cm; 45_Y = 23.7 cm

SIM2: $\theta_3 = [A_Y / |A|] \cos^{-1}[A_X/A]$, Answer2 θ_3 = (31.8)

***Father of Modern Kinematics, Ferdinand Freudenstein's Design Equation for Four-Link Mechanisms (1954), named after him, is derived by writing VLE: <u>BC</u> = <u>BA</u> + <u>AD</u> + <u>DC</u> with <u>BA</u> = - 1 e(θ_{AB}) and applying SIM1 [9] [10].**

8. SIM3 / Perpendicular Component Equation (LCE)



Eliminating the Magnitude of a Vector with Known Direction.

A (θ_A) and B (θ_B) are two vectors in a plane. Line (θ_A) is Parallel (||) to A (θ_A). Line (θ_A + 90) and BB' are both Perpendicular (L as symbol) to A (θ_A). OB'B is a Right Triangle.

LC of B (θ_B) to A (θ_A) or line(θ_A)

$B_{L\Delta}$ = B'B = B sin ($\theta_B - \theta_A$)

LC of A (θ_A) to A (θ_A) or line(θ_A)

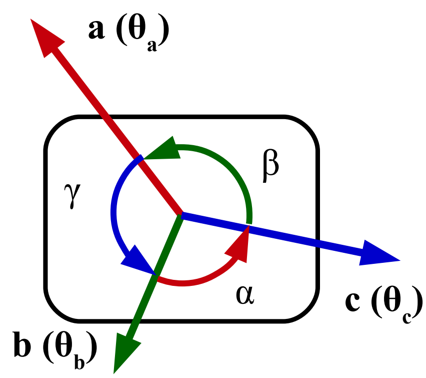
$A_{L\Delta}$ = A sin ($\theta_A - \theta_A$) = 0 SIM3

SIM3 / Perpendicular Component Theorem / Polar Sine Law:
The Perpendicular Component Equation to a Vector with known direction, in a system, eliminates that Vector's magnitude.
In Practice **SIM3 is "Subtracting the Line Angle and Taking Sine."**
SIM3 is applied twice in solving System Type 3 with two vectors, each of unknown magnitude and Type 4 with one vector of unknown magnitude and another of unknown direction.

9. Proving Lami's Theorem and Sine Law Applying SIM3 / LCE / Polar Sine Law.

Lami's Theorem:

If three coplanar concurrent forces acting on a body keep it in equilibrium, then each force is proportional to the sine of the angle between the other.



Vector System Diagram

Proof: Given a $\underline{e}(\theta_a)$ + b $\underline{e}(\theta_b)$ + c $\underline{e}(\theta_c)$ = 0

SIM3 - LCE to line(θ_c)

a sin($\theta_a - \theta_c$) + b sin($\theta_b - \theta_c$) + c sin(0) = 0

a sin(β) + b sin(- α) = 0

a sin(β) = b sin(α)

a / sin α = b / sin β

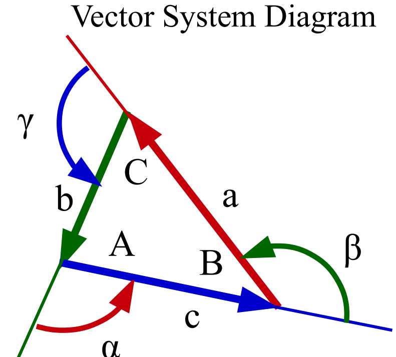
SIM3 - LCE to line(θ_a)

b sin($\theta_b - \theta_a$) + c sin($\theta_b - \theta_a$) + a sin(0) = 0

b sin(γ) + c sin(- β) = 0

b sin(γ) = c sin(β)

b / sin β = c / sin γ



Vector Loop Triangle

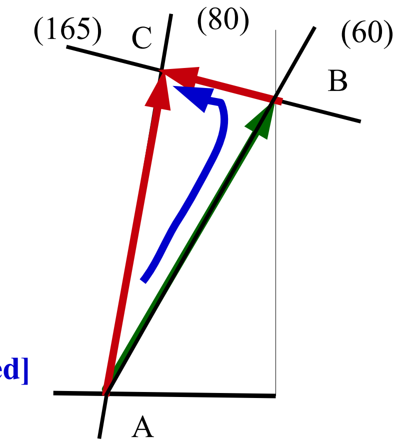
Combining the above we have **Lami's Theorem:**

a / sin α = b / sin β = c / sin γ; As A = (180 – α) etc.

Sine Law for Δ ABC: a / sin A = b / sin B = c / sin C

10. Type 3 System, T3[A, B]: Two Line Vectors each with unknown magnitude are related to known vectors. Each unknown magnitude is found by applying SIM3 to one line vector at a time. This is the most widely occurring type in Systems and Free Body Diagrams.

Example 7: In the triangle ABC shown, find the AC and then vector CA, with AB = 10 m. CA will be used for calculating moments in SIM5 during later sections.



Vector Loop Diagram

Solution: System is T3. Vector Loop Equation

AC = AB + BC

AC e(80) = 10 e(60) + BC e(165)

SIM3 / LCE to line(165):

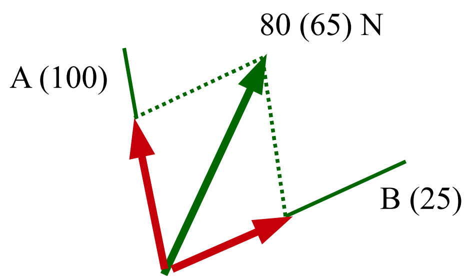
[Subtracting the Line Angle and Taking Sine]

AC sin(80 – 165) = 10 sin(60 – 165) + 0 --- **[BC is eliminated]**

AC = 9.7 m. **AC = 9.7 e(80); CA = - 9.7 e(80)**

or CA = 9.7 e(80 + 180); Answer CA = 9.7 e(260)

Example 8. Find the Components of the force vector along directions shown. [Resolve the vector along line(100) and line(25) / find the unknown magnitude component vectors A e(100) and B e(25)]



Vector System Diagram

Solution: System is T3. Vector System Equation

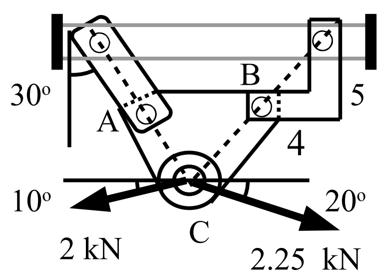
A e(100) + B e(25) = 80 e(65)

SIM3 / LCEs to line(25) and line(100):	**Answers**
A sin(100 – 25) + 0 = 80 sin(65 – 25) ;	**A = 53.24 N**
0 + B sin(25 – 100) = 80 sin(65 – 100);	**B = 47.5 N**

Example 9: For the loaded structure shown, determine the forces on Pins A and B.

$\theta_B = \tan^{-1}[5/4]$

= **(51.34)**



System Space Diagram

Solution: FBD is T3. Vector Equilibrium Equation

A e (120) + B e (51.34) + 2 e (190) + 2.5 e (340) = 0

SIM3 / LCEs to line(51.34) and line(120)

A sin (120 – 51.34) + 2 sin (190 – 51.34)

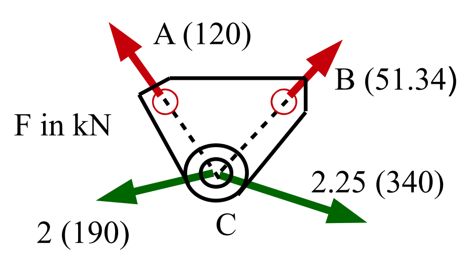
+ 2.5 sin (340 – 51.34) = 0

Answer1 A = 1.125 kN

B sin (51.34 – 120) + 2 sin (190 – 120)

+ 2.5 sin (340 – 120) = 0

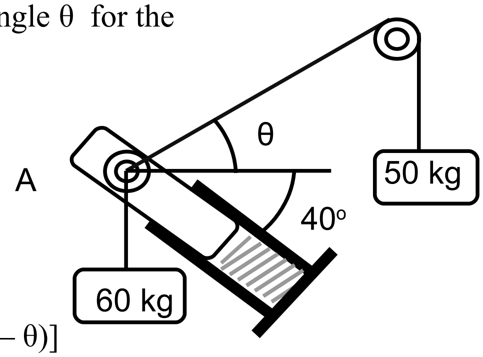
Answer2 B = 0.430 kN



Free Body Diagram

11. Type 4 System, T4[θ, A]: Two Vectors, one with unknown magnitude and another with unknown direction, are related to known vectors. SIM3 is applied first to the line vector with unknown magnitude, to find the unknown direction of the arc vector and then to the arc vector with its solved direction to find the unknown magnitude of the line vector.

Example 10. In the mechanism shown, find the polar angle θ for the higher compressive force P in the spring.



System Space Diagram

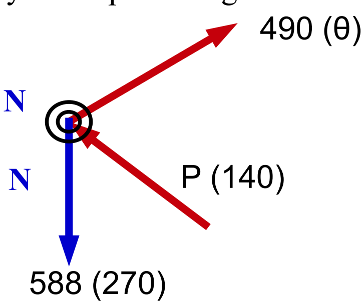
Solution: T4 FBD. Equilibrium Equation is
P e (140) + 490 e (θ) + 588 e (270) = 0

SIM3: LCE to line(140)
0 + 490 sin(θ –140) + 588 sin 270 – 140) = 0
sin (θ –140) = - 0.92;
(θ –140) = (- 66.8) and (246.8) since [sin θ = sin (180 – θ)]
(θ_1) = (73.2) and (θ_2) = (26.8)

LCEs to line(θ) with solved θ_1 and θ_2 are:
P_1 sin (140 – 73.2) + 0 + 588 sin(270 -73.2) = 0, **P_1= 184.9 N**

P_2 sin (140 – 26.8) + 0 + 588 sin(270 – 26.8) = 0, **P_2 = 571.0 N**

Answer: **θ = (26.8) for P_{higher} = 571.0 N**



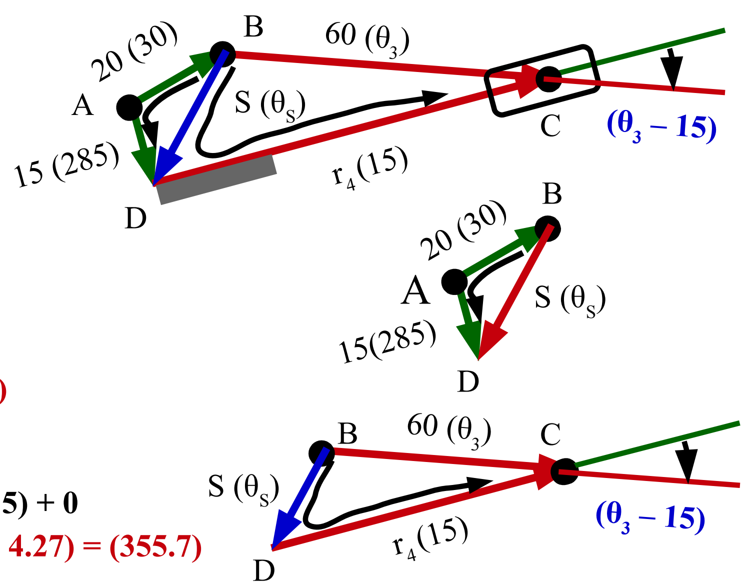
Free Body Diagram

Example 11. A crank slider mechanism with eccentricity, is shown as a vector loop diagram. Find the Direction θ_3 of the connecting rod and the slider displacement r_4 from D.

Solution: Loop BAD is T1
BD = BA + AD
S e(θ_S) = - 20 e (30) + 15 e(285)
S_X = - 14.91 cm, S_Y = - 24.49 cm,
S = 28.67 e (238.7)

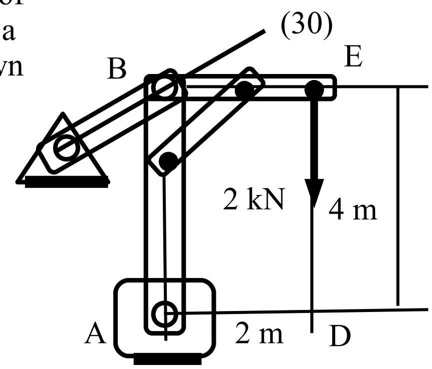
For T4 Vector Loop BDC:
BC = BD + DC
60 e(θ_3) = 28.67 e(238.7) + r_4 e(15)

SIM3 / LCE to line(15):
60 sin(θ_3 – 15) = 28.67 sin(238.7 – 15) + 0
(θ_3 – 15) = -19.27, Answer1 θ_3 = (- 4.27) = (355.7)
SIM3 / LCE to line(θ_3 = 355.73):
Answer2 r_4 = 77.13 cm



Vector Loop Diagrams

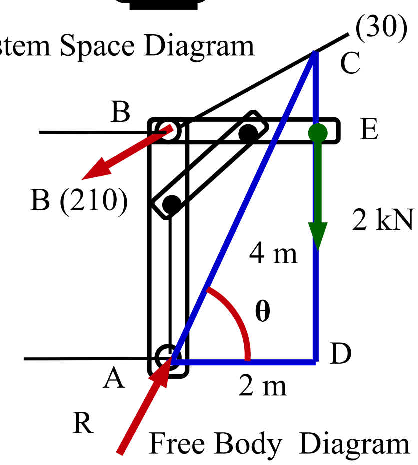
12. Type 5 System, T5{θ, A, B]: T5 is FBD with a pin reaction of unknown magnitude A and direction θ, reaction of unknown magnitude B and a known force / weight. Thus it is a three force system with three unknowns. The unknown direction is found by SIM2 in the geometry of concurrency



System Space Diagram

Example 12. For the structure shown on right find the pin reaction direction, reaction and the tension in link at B.

Solution: FBD is T5 with three forces. The point of Concurrency C is obtained by the PALs of B and 2 kN
From the Right Triangle EBC.
EC = 2 tan(30) = 1.155



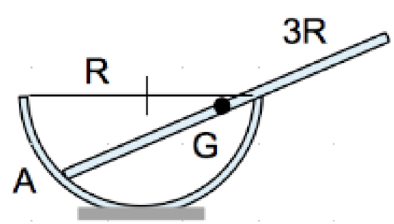
Free Body Diagram

In the Right Triangle ADC, AD = 2 m DC = 5.155 m
<A = θ = $\tan^{-1}$[5.155/2] **Answer1 θ = (68.8)**

Vector Equilibrium Equation:
R e(68.8) + B e(210) + 2 e(270) = 0
By SIM3: Answers R = 2.765 kN, B = 1.155 kN
Note: This problem can also solved by first taking moments about A. Then the FBD is T7, discussed later.

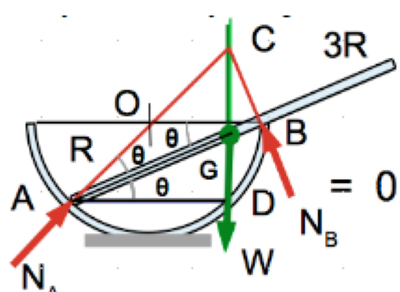
13. SIM4: Expressions that are functions of the same unknown for a common variable are equated and the unknown is solved.

14. Type 6 System, T6[θ, A_θ, B_θ,]: T6 is FBD with a force with known magnitude and direction and two normal reactions with unknown magnitudes A and B that are functions of an unknown angle θ. The angle is solved by SIM4. It is a three force system with three unknowns.



System Space Diagram

Example 13. A uniform rod of length 3R, weight W rests in a smooth bowl of radius R as shown. Find the angle the rod makes with the horizontal.



Free Body Diagram

Solution: FBD is T6. Concurrent Force Geometry.
In Right Δs ADC and ADG AD is common.
SIM4: [AC = 2R, AG = 1.5R]
$2 R \cos 2\theta = 1.5 R \cos\theta$ => $2 (2 \cos^2\theta - 1) = 1.5 \cos\theta$
=> $4 \cos^2\theta - 1.5 \cos\theta - 2 = 0$
$\cos\theta = [1.5 + (1.5^2 - 4{*}\ 4.{*}{-2})^{1/2}] / [2{*}4] = 0.92$;
From Diagram Answer 1 θ = (23.07)

In ΔAOB, OA = OB = R
So A = <B = θ
in ΔBOC, <C = <B = (90° – θ)
So OC = OB = R

In FBD: N_A e(46.1) + N_B e(113.1) + W e(270) = 0
By SIM3: Answers N_A = 0.426 W & N_B = 0.753 W

15. Moment of A Force in XY plane.:

Moment is the measure of turning, bending or twisting effect of a Force in the XY plane about the Z axis that appears as moment center C. A is any point on the Polar Action Line (PAL) of the Force.

Moment M about C of Force through A:

$M_C{}^A$ = Lever Arm of CA * Force F = r_L F

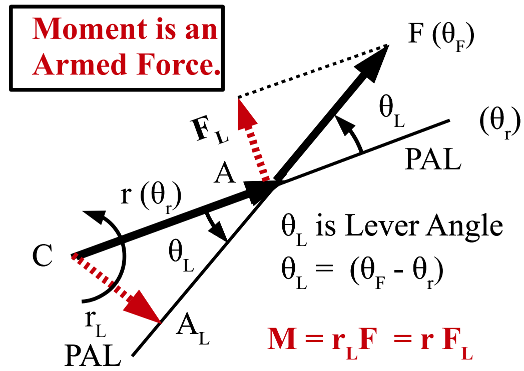
= $[r \sin(\theta_L)]$* F

or $M = [r \sin(\theta_F - \theta_r)]$*F

$M_C{}^A$ = Arm CA * Lever Force = r F_L

= r * F $\sin(\theta_L)$

or $\boxed{M = r\,F \sin(\theta_F - \theta_r)}$

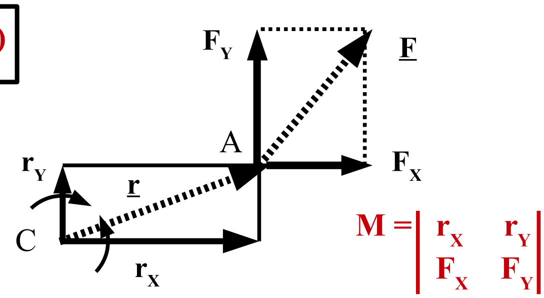


Polar Angle /Action Lines (PALs) are drawn to show θ_L , r_L and F_L. Counter Clockwise Turning is Positive.

$M_C{}^A$ = [X Lever] F_Y + [Y Lever] F_X

= $r_X F_Y - r_Y F_X$ or $M_Z = \begin{vmatrix} r_X & r_Y \\ F_X & F_Y \end{vmatrix}$

for 3D, $M_X = \begin{vmatrix} r_Y & r_Z \\ F_Y & F_Z \end{vmatrix}$ $M_Y = \begin{vmatrix} r_Z & r_X \\ F_Z & F_X \end{vmatrix}$



Moment of A Force

Example 14. Find $M_C{}^G$ of W_G, given W_G = 4 kN and Geometry on right.

Solution: CG is Composite Arm given by Vector Loop Equation

CG = CA + AG [Chain Rule]

CA = = - AC e(75). So CA = 9.5 e(255) m

AG = 5 e(50), W_G = 4 e (270) kN

$M_C{}^G = (M_C{}^A + M_A{}^G)$ of W_G; $M = r\,F \sin(\theta_F - \theta_r)$

$M_C{}^A$ = 9.5 * 4 sin(270 – 255) = 9.835 kNm

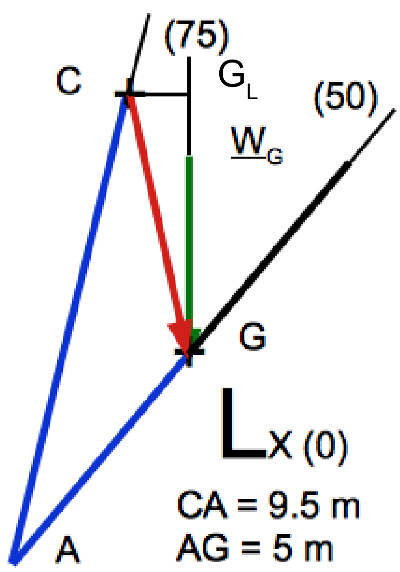
$M_A{}^G$ = 5.0 * 4 sin(270 – 50) = - 12.856 kNm

Adding **Answer $M_C{}^G$ = - 3.021 kNm**

Alternately: Note Composite Lever Arm for W_G is

$(CG)_L$ = 9.5 sin(270 – 255) + 5 sin(270 – 50) = - .755 m

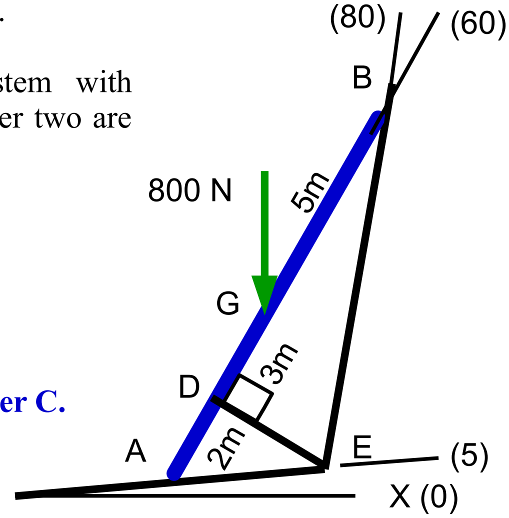
$M_C{}^G = (CG)_L W_G$ = - 3.021 kNm (Check)



Composite Armed Force

16. SIM5: Moment Equation about a point called Canonical Moment Center, that eliminates all unknowns except one, is obtained and solved. It is applied to solve T7 to T10.

17. Type 7 System, T7[F, A, B]: A multi-force system with three unknowns, of which one is a force and the other two are usually contact reactions, solved by SIM3 and SIM5.



System Space Diagram

Example 15: A 800 N (W) ladder, resting on a floor with friction angle 10° and wall of 5° and inclined as shown, is restrained by a perpendicular cable. Find the Tension in the cable DE and the reactions.

Solution: FBD is T7 with Canonical Moment Center C.

CA is is first found in ΔABC formed
by PALs (Polar Action Lines) of Reactions at A and B intersecting at Canonical Moment Center C.

Vector Loop Equation for ΔABC
AC $\underline{e}$(85) = 10 $\underline{e}$(60) + BC $\underline{e}$(165)

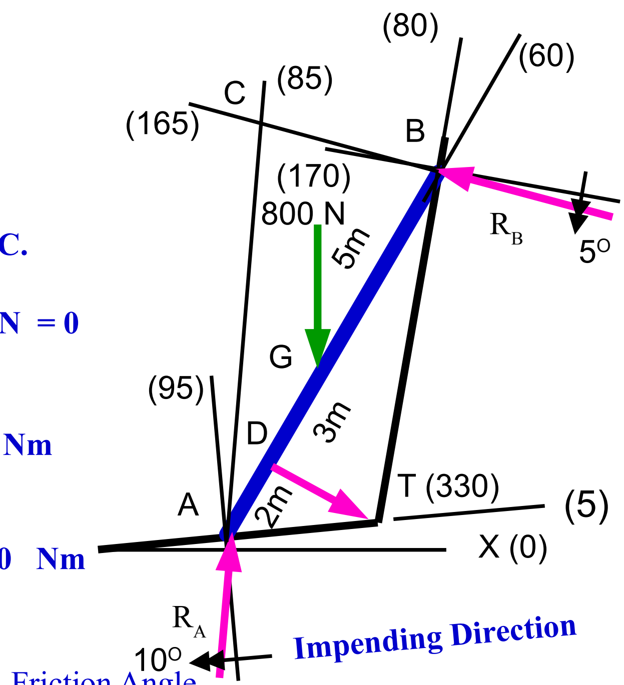
SIM3: LCE to BC (165)
AC sin(85 - 165) = 10 sin(60 – 165) + 0
AC = 9.81m = CA, **θ_{CA} = (85 + 180) = (265)**

SIM5: Canonical Moment Center is point C.
$\Sigma M_C = M_C^D$ of T + M_C^G of W = 0
($M_C^A + M_A^D$) of T + ($M_C^A + M_A^G$) of 800N = 0

M_C^D = (9.81) (T) sin (330 – 265) + (2.0) (T) sin (330 – 60) = 6.89 T Nm

M_C^G = (9.81) (800) sin (270 - 265) + (5) (800) sin (270 – 60) = -1316.0 Nm

ΣM_C = 6.89 T - 1316.0 = 0 ;
Answer1 T = 191.0 N



Free Body Diagram

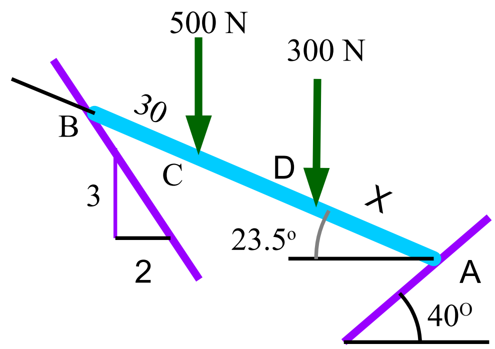
Equilibrium Equation with solved T = 191 N
R_A $\underline{e}$(85) + R_B $\underline{e}$(165) + 191 $\underline{e}$(330) + 800 $\underline{e}$(270) = 0
or R_A $\underline{e}$(85) + R_B $\underline{e}$(165) = - 191 $\underline{e}$(330) - 800 $\underline{e}$(270)

SIM3: LCEs to R_B (165) and R_A (85)
lead to Answers R_A = 835 N; R_B = 267 N

Traditional solution involves six forces with inclined normal and frictional forces and three simultaneous equations.

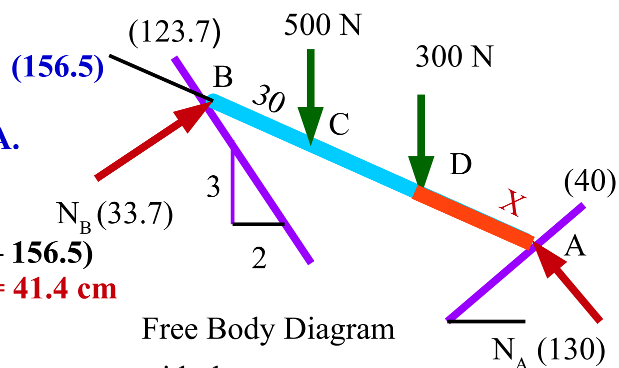
18. Type 8 System, T8[X, A, B]: A multi-force system with three unknowns, of which one is a straight distance and the other two are usually contact reactions, solved by SIM3 and SIM5.

Example 16. A smooth rod AB of length 100 cm is in equilibrium at an angle 23.5^{O} from the horizontal with the 300N load at a distance X from A. Find X by equations with one unknown.



Solution: FBD is T8. Solved by SIM3 and SIM5
Equilibrium Equation
$N_A\, \underline{e}\,(130) + N_B\, \underline{e}\,(33.7) + (500 + 300)\, \underline{e}\,(270) = 0$

SIM3: LCEs to N_A (130) & NB (33.7)
$N_B = 517$ N $\quad N_A = 670$ N

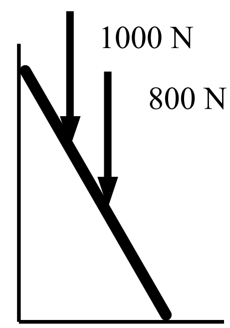


Free Body Diagram

SIM5: Canonical Moment Center is Point A.
$\Sigma M_A = M_A^{D} + M_A^{C} + M_A^{B} = 0$

(X) 300 sin(270 -156.5) + (70) 500 sin(270 – 156.5)
+ (100) 517.4 sin(33.7 – 156.5) = 0, $\quad$ **X = 41.4 cm**

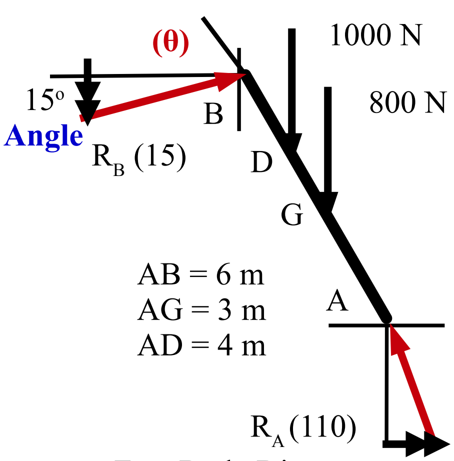
19. Type 9 System, T9[θ, A, B]: A multi-force system with three unknowns, of which one is a direction and other two are usually contact reactions, solved by SIM3 and SIM5.



Example 17. A uniform ladder 6 m long, weighing 800 N rests on a wall of friction angle 15° and floor of 20°. Find the angle from the horizontal ground at which the ladder will slip if a man of 1000 N weight reaches 4 m from the lower end of the ladder.

Solution: FBD is T9. Solved by SIM3 and SIM5
$R_A\, \underline{e}(110) + R_B\, \underline{e}(15) + (800 + 1000)\, \underline{e}(270) = 0$

SIM3: LCEs to line(110) and line(15)
Answers1&2 $\quad R_B = 618$ N, $\quad R_A = 17.45$ N $\quad$ **Friction Angle**

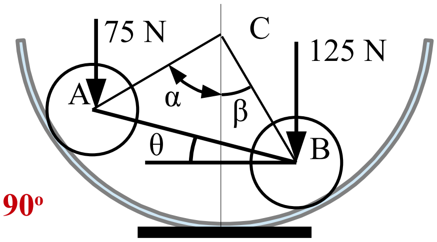


Free Body Diagram

SIM5: Canonical Moment Center is point A.
$\Sigma M_A = M_A^{G}$ of 800N + M_A^{D} of 100N + M_A^{B} of $R_B = 0$
(3) (800) sin (270 – θ) + (4) (1000) sin (270 – θ)
+ (6) (618) sin (15 – θ) = 0;
- 6400 cos θ + 960 cos θ – 3582 sin θ = 0

3582 sin θ = - 5440 cos θ; Answer 3 $\quad$ θ_{AB} = (123.4)

Example 18. Timoshenko Problem of section 2, using all the 5 SIMS to solve the 6 Unknowns.



Space Diagram

Solution: FBD A is T9. Canonical Moment Center is C

SIM5: $\Sigma M_C = M_C^A$ of 75N + M_C^B of 125N = 0

$(R \sin\alpha)\ 75 - (R \cos\alpha)\ 125 = 0$ **$\alpha + \beta = 90^o$**

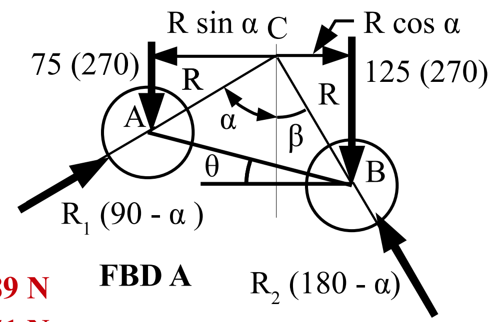
$75 \sin\alpha = 125 \cos\alpha;$ **Answer1 $\alpha = 59.04^o$**

$\alpha + \beta = 90^o$, $\beta = 90 - 59.04$, **Answer 2 $\beta = 30.96^o$**

SIM4: Equating expressions for the same unknown

In Δ ABC <A = <B; So $(\alpha - \theta) = (\beta + \theta)$

$(\alpha - \beta) = 2\,\theta;\quad \theta = (\alpha - \beta)/2.$ **Answer 3 $\theta = 14.04^o$**



FBD A

Equilibrium Equation

$R_1\ \underline{e}(30.96) + R_2\ \underline{e}(120.96) + (200)\ \underline{e}(270) = 0$

SIM3: LCEs to line(120.96) and line(30.96)

Answer 4 $R_1 = 102.89$ N

Answer 5 $R_2 = 171.51$ N

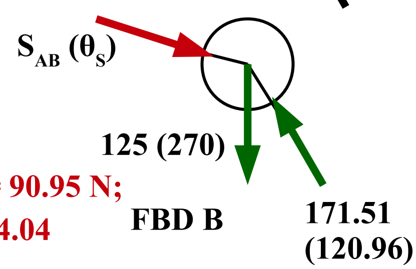
T1 FBD B: Solved by SIM1 and SIM2

$S_{AB}\ \underline{e}(\theta_S) + 171.51\ \underline{e}(120.96) + 125\ \underline{e}(270) = 0$

$S_{AB}\ \underline{e}(\theta_S) = -\ 171.51\ \underline{e}(120.96) - 125\ \underline{e}(270)$

$S_{ABX} = 88.2$ N, $S_{ABY} = -\ 22.07$ N, SIM1&2: Answer 6 $S_{AB} = 90.95$ N;

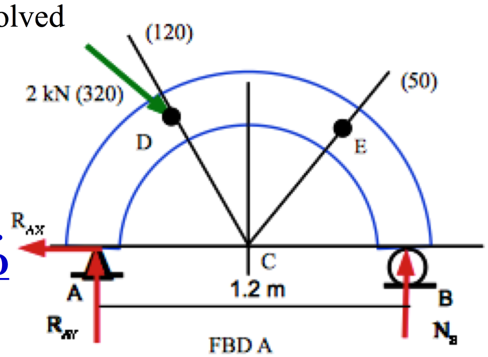
$\theta_s = (\ -14.04) = (345.96)$ (Check for $\theta = 14.04^o$) $\theta\ = 14.04$



FBD B

20. Type 10 System, T10[C, A, B]: A multi-force system with three unknowns, of which one is a Couple Moment and the other two are usually contact / internal reactions, solved by SIM3 and SIM5.

Example 19. Find the axial force, shear force and bending moment at E for the system shown.



FBD A

Solution: FBD1 is T7. Canonical Moment Center is A.

$\Sigma M_A = M_A^B$ of N_B + M_A^D of 2KN = 0; $\underline{AD} = \underline{AC} + \underline{CD}$

$(1.2)\ (N_B) + [\ (.6)\ (2) \sin(320 - 0)$

$+ (.6)\ (2) \sin(320 - 120)] = 0$ $N_B = 0.985$ kN

FBD 2: T10 is solved by SIM3 and SIM5

$A\ \underline{e}\ (320) + S\ \underline{e}\ (230) + 0.985\ \underline{e}\ (90) = 0$

SIM3 / LCEs to line(230) and line(320)

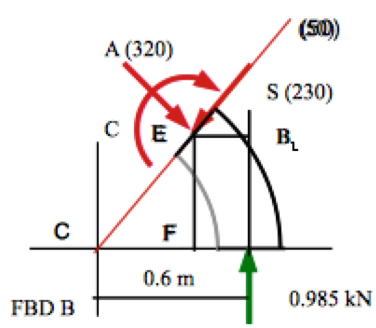
Answers $A = 0.633$ kN; $S = 0.755$ kN

SIM5: Canonical Moment Center is E

$\Sigma M_E = C + M_E^{B'}$ of 0.985 kN = 0

$M + (0.6 - 0.6 \cos 50^O)\ (0.985) = 0;$

Answer $C = 0.211$ kNm



FBD B

21. Summary:

A. FIVE SIMS: Equations with Only One Unknown

SIMS for Planar Vector Systems with two unknowns.

SIM1: By squaring and adding the X, Y or Orthogonal Components of a Vector Loop Equation with two unknowns, its Left Hand Side (LHS) vector's unknown direction is eliminated. The resulting equation has only one unknown, magnitude or direction.

SIM2: The unknown direction of a vector is found by any inverse trigonometric function of the vector's known X and Y components.

SIM3: The Perpendicular Component Equation (LCE) to a vector with known direction, in a system, eliminates that vector's magnitude. In practice **SIM3 is "Subtract the Line Angle and Take Sine."** LCE has only one unknown, magnitude or direction.

SIMS for Planar Vector Systems with Three Unknowns:

SIM4: Expressions that are functions of the same unknown for a common variable are equated and the unknown, usually an angle, is solved.

SIM5: Moment Equation about a Canonical Moment Center, usually an intersection of reactions, that eliminates all unknowns except one, is obtained and solved.

B. The Ten Types of Basic Planar Vector Systems in Engineering Mechanics

Systems are classified and solved in three groups containing Knowns and Unknowns occurring as Arms and Displacements [r, X, S], Forces [R, A, B, F], Angles [θ, φ] and Moments [M, C].

			SIMS Applied
Group A	Systems with **Two Unknowns**		
Vector	Types:	T1[R, θ_R], T2[θ_1, θ_2]	SIM1 and SIM2
Equations		T3[A, B], T4[θ, A]	SIM3
Group B	Systems with **Three Unknowns**		
Geometric	and **Three Forces only**		
& Vector	Types:	T5[$\theta_{R,}$ R, F]	SIM2 and SIM3
Equations		T6[θ, A_θ, B_θ,]	SIM4 and SIM3
Group C	Systems with **Three Unknowns**		
Vector &	and **Many Forces**		
Moment	Types:	T7[F, A, B], T8[A, B, X]	SIM3 and SIM5
Equations		T9[θ, A, B], T10[A, B, M]	SIM3 and SIM5

SIM3 is used in the solution of 8 of the 10 types of basic planar vector systems.

22. Discussion and Conclusions:

The Five SIMS are developed to achieve simplicity and systematic uniformity in solving problems in Engineering Mechanics as well as Kinematics and Dynamics of Mechanisms and Machines. SIMS may not provide simpler solutions than the ones current textbooks provide for simple problems like in examples 1 to 5, for which well known trigonometric formulae or Lami's Theorem can be applied. But the position analysis of a four-bar mechanism is not simple. Soni's textbook [10] and the like provide lengthy solutions using complex numbers and quadratic equations of tangents with half angles. However SIMS provide novel solution with SIM1 applied in two steps to this problem. Step 1 as T1 solution represents Pythagorus Theorem using squared sums of X and Y components to find a resultant. Step 2 as T2 solution is the vector adaption of the algebraic identity for $(A \pm B)^2$ as $C^2 = A^2 + B^2 \pm 2\,AB\cos(\theta_B - \theta_A)$ for Polar Vector Equation $C\,\underline{e}(\theta_C) = A\,\underline{e}(\theta_A) \pm B\,\underline{e}(\theta_B)$ with (θ_C) and (θ_B) as unknowns. Thus SIM1 adapts itself as "Generalized Pythagorus Theorem" for T1 system and as "Polar Cosine Law" for a Vector Triangle / Parallelogram for T2 system. SIM2 is applied to find the direction by any inverse trigonometric function of known X , Y components of a vector.

While Sine Law can be applied to a Triangle and Lami's Theorem to three forces in equilibrium, one has to resort to $\Sigma F_X = 0$, $\Sigma F_Y = 0$ for solving multi-force concurrent T3 FBD as in Example 8. But SIM3 can be readily applied to a system with any number of forces or vectors, even if they are not concurrent as shown in Example A2 (Appendix) with two unknown magnitudes. SIM3 again provides a simple solution for the position analysis of a Crank Slider with Eccentricity in Example 11, which otherwise becomes complex. Thus SIM3 can be considered as a "Polar Sine Law". SIM3 is the most applied one being employed in eight of the ten solution schemes of the ten types of basic vector systems.

SIM4 presents the most general way, to eliminate one of the unknowns in two simultaneous equations by equating the expressions of one unknown in terms of the other as in T6 FBD Example 13. Two FBDs with two common unknowns can be solved by SIM4, by first eliminating the uncommon unknown reaction in each by SIM3. In section 4, Vector Loops for diagonal $\underline{AC}$ of a four bar, are equated and differentiated for velocity analysis by SIM3.

SIM5 is the most efficient way to eliminate all unknowns except one, by forming the Moment Equation, about a Canonical Moment Center as in T7 to T10 FBD Examples 15 to 19, in which SIM3 and SIM5 are used in an efficient order to solve for all unknowns.

Thus solutions are simplified by choosing a set of FBDs of proper type and in right order as in Example 18, with all Five SIMS applied so, to solve the Timoshenko Problem.

When one starts the solution with the $\Sigma X = 0$ and $\Sigma Y = 0$, lengthy procedures may result as in the reference [2] solution to Timoshenko Problem presented in section 2. Short SIMS solutions to usually lengthy traditional ones are presented in Appendix Examples A1 to A8.

Polar Angles are used in AutoCAD drafting and much earlier in surveying as "Azimuths" with North as the reference axis. Kinematic and Dynamic Analyses of Planar Mechanisms including Balancing of Rotors are carried out in terms of Polar Coordinates.

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So it is good to use polar angles in Mechanics, to simplify solutions. The author, who once taught Euclidian Geometry to US 9th graders, invites discussion on including vector loops and SIMS in that course. Then instead of teaching "Mechanics" as Physics in senior high school and again as "Engineering Mechanics" in college, "Essential Engineering Mechanics" can be taught at the senior high school level. The author's forthcoming one semester class-book "Essential Engineering Mechanics" in about 240 pages with a self and peer assessment booklet, for each unit, and software support for unit and semester exams, is meant for 1st year engineering students, free from any pre-requisites, with the following six units.

1. Systems with Two Unknowns
2. Systems with Three Unknowns
3. Friction and Spatial Systems
4. First and Second Moments of Elements
5. Kinematics and Kinetics of Bodies
6. Work-Energy and Impulse-Momentum

The class-book introduces "System Space Diagram" as a figure with data, "System Loop Diagram" for the vector geometry of structures and mechanisms and "System Vector Diagram" for systems with force vectors, "Vector System Equation" for general usage and "Vector Loop Equation" for the application of SIM1 and SIM2 and SIM4.

Hundreds of students of the author, after a week of initial hesitation, responded positively to SIMS approach. Similar is the reaction of faculty when presented with supporting slides and solutions for the class-book reading material and problems. The author is grateful to the reviewers for advising him to address their comments that added immense value to the paper. The author gracefully invites further comments and discussion on this paper from the readers.

23. References:

[1] Timoshenko, S.P. and D.H. Young, *Engineering Mechanics,* 4th edn., McGraw Hill International Book Company, Singapore, 1984

[2] Bhattacharyya, B., *Engineering Mechanics*, 2nd edn., Oxford University Press, New Delhi, 2014

[3] Meriam, J.L and L.G Kraige, *Engineering Mechanics: Statics and Dynamics,* 5th edn, John Wiley and Sons (Asia) Pvt. Ltd., Singapore 2002.

[4] Beer, F.P. and E.R. Johnston, *Vector Mechanics for Engineers,* 7th edn., Tata McGraw Hill Publishing Company, New Delhi, 2004.

[5] I.H. Shames, *Engineering Mechanics,* 3rd edn., Prentice Hall Incorporated Pvt. Ltd., New Delhi, 1995

[6] Singer, F.L., *Engineering Mechanics,* 3rd edn., Harper and Row Publishers, New York City, 1975.

[7] Hibbeler, R.C., *Engineering Mechanics,* 12th edn., Prentice Hall Incorporated, Upper Saddle River, 2010.

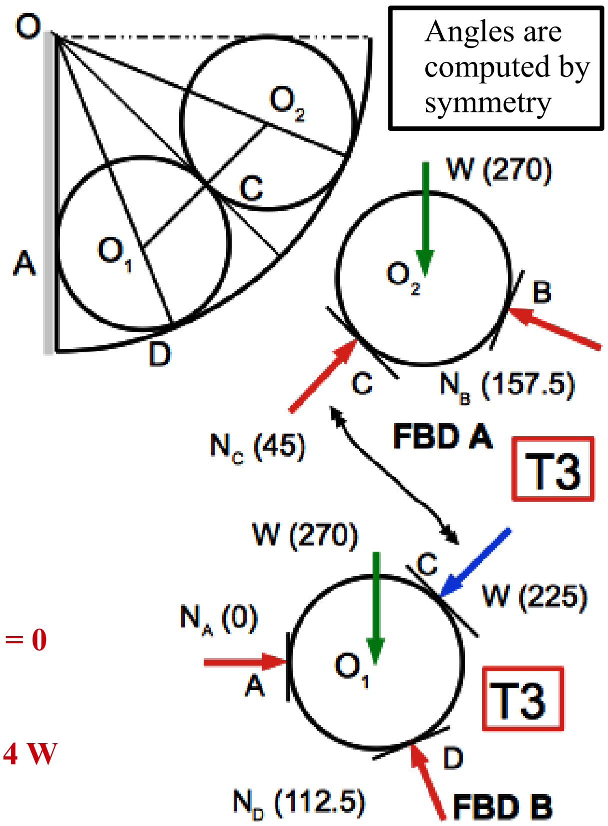
[8] Soni, A.H., *Mechanism Synthesis and Analysis,* McGraw Hill Book Company, New York, 1974.

[9] Freudenstein, F., *Design of Four Link Mechanisms*, PhD Thesis, Columbia University, USA, p VI-15, 1954.

[10] Ghosal, A., *Freudenstein Equation - Design of Four–Link Mechanisms*, Resonance, pp. 704 – 705, August 2010.

24. Appendix for Examples on Contacting and Connected Bodies and Dynamics.

Example A1. Contacting Bodies:
Two identical smooth cylinders each of weight W and radius R are placed in a quarter circular channel such that they fit in as shown. Determine the reactions at contact points C and A.



Solution A: FBD A is T3. It is solved for N_C.

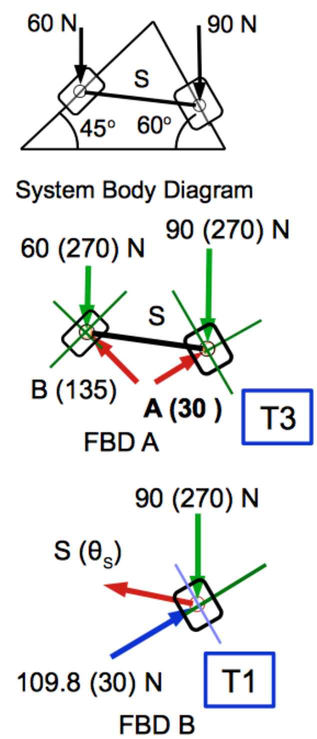
Equilibrium Equation:
$N_B\ \underline{e}(157.5) + N_C\ \underline{e}(45) + W\ \underline{e}(270) = 0$

SIM3: LCE to line(157.5):
Answer 1 $N_C = W$

Solution B: FBD B is T3. It is solved for N_A.
Equilibrium Equation:
$N_D\ \underline{e}(112.5) + N_A\ \underline{e}(0) + W\ \underline{e}(225) + W\ e(270) = 0$

SIM3: LCE to line(112.5):
Answer 2 $N_A = 1.414\ W$

Example A2. Connected Bodies:
Two collars connected by a weightless rod, rest on smooth rods of a vertical frame as shown. Find the force S in the connecting rod AB and its direction.



Solution: T3 FBD A and T1 FBD B are solved.

T3 FBD Equilibrium Equation:
$A\ \underline{e}(30) + B\ \underline{e}(135) + (60 + 90)\ \underline{e}(270) = 0$
or $A\ \underline{e}(30) + B\ \underline{e}(135) = -\ 150\ \underline{e}(270)$

SIM3 / LCE to line(135): $A = 109.8$ N

T1 FBD B Equilibrium Equation:
$S\ \underline{e}\ (\theta_{\underline{S}}) + 109.8\ \underline{e}\ (30) + 90\ \underline{e}\ (270) = 0$
or $S\ \underline{e}(\theta_{\underline{S}}) = -109.8\ \underline{e}(30) - 90\ \underline{e}(270)$

$S_X = -95.1$ N; $S_Y = 35.2$ N
SIM1&2 : **Answers $S = 101.4$ N; $\theta_S = (159.7)$**

Solution by any other method involves more steps.

Example A3.

Joint Analysis of a Truss with one FBD.
SN is the Tensile force in Member N

Joint A: S1 e(0) + S4 e(63.4) + 3.5 e(90) = 0
LCEs to S4 (63.4) and S1 (0)

S1 = 1.75 kN;
S4 = - 3.91 kN

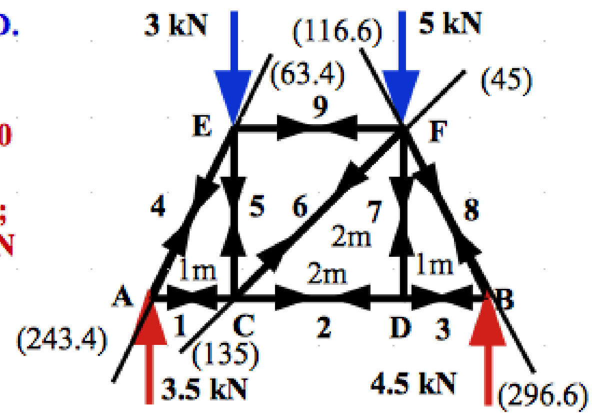
Joint E: S5 e(270) + S9 e(0)
-3.91 e(243.4) + 3 e(270) = 0
LCEs T9 (0) and T5(270)

S5 = 0.5 kN
S9 = - 1.77 kN

Joint B: S3 e(180) + S8 e(116.6) + 4.5 e(90) = 0
LCEs to T8 (116.6) and T3 (180)

S3 = 2.25 kN
S8 = - 5.03 kN

Joint D:By inspection S7 = 0
and S2 = S3 = 2.25 kN



FBD of the Truss

Joint C: S6 e(45) + 2.25 e(0)
+ 1.75 e(180) + 0.5 e(90) = 0
LCE to S2 (0)

S6 = - 0.707 kN

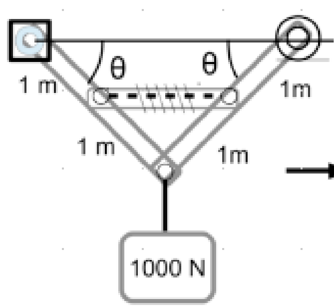
Example A4.

This problem is solved by Hibbeler in his Engineering Mechanics textbook 14 ed. on Virtual Work.
Page 590, EX: 11.4

He solves it with
1. coordinates of points,
2. their displacements
and finally 3. Virtual Work equation. It is a lengthy procedure.

Here applying SIM5 twice, one for each FBD, two equations are obtained each with only one unknown!

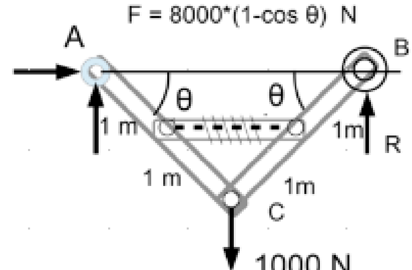
Example: The mechanism in the figure supports a cylinder of weight 1000N. Determine θ for the equilibrium if the spring has an uncompressed length of 2m when θ = 0. Neglect mass of the members. The spring constant k = 4000 N /m.



Solution:

Spring force F = 4000*(2 – 2 cos θ)

F = 8000*(1-cos θ) N



FBD 1

FBD 1

$\Sigma M_A = -2 \cos\theta * 1000 + 4 \cos\theta * R = 0$

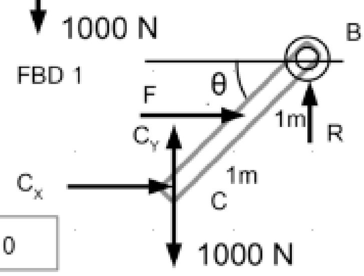
R = 500 N

FBD 2

$\Sigma M_C = 2 \cos\theta * R - 1 \sin\theta * F = 0$

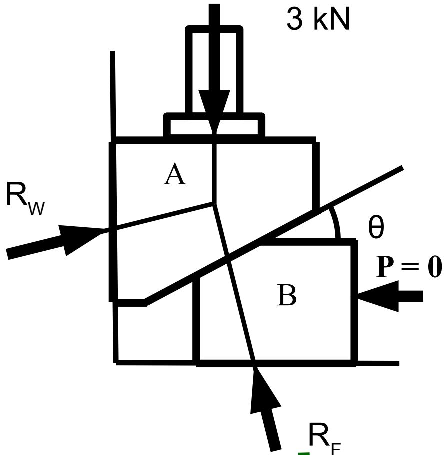
$\boxed{1000 \cos\theta - 8000 \sin\theta + 8000 \sin\theta \cos\theta = 0}$

Hibbeler's solution is: θ = 34.9° by trial and error.



FBD 2

Example A5. Block A supports a pipe column and rests as shown on Wedge B. The coefficient of friction of all surfaces of contact is 0.25. If P = 0, a) find the angle θ for which sliding is impending. , and b) the corresponding force exerted by the vertical wall.



Solution: Two Force FBD 1

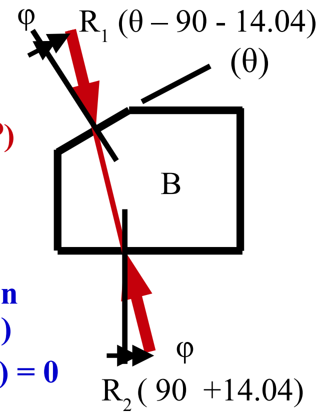
$\mu = 0.25$ so $\varphi = 14.04^o$

Equal and Opposite Forces

Direction 1 = (Direction 2 + 180°)

$(\theta - 90 - 14.04) = (90 + 14.04 + 180^o)$

$\theta = (28.08)$



FBD 1

T3 FBD 2: Equilibrium Equation

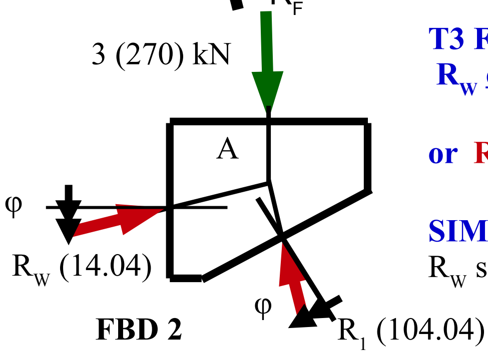
$R_W\ \underline{e}(14.04) + R_1\ \underline{e}(\theta + 90 - 14.04) + 3\ \underline{e}(270) = 0$

or $R_W\ \underline{e}(14.04) + R_1\ \underline{e}(104.04) = -3\ \underline{e}(270)$

SIM3: LCE to line(104.04)

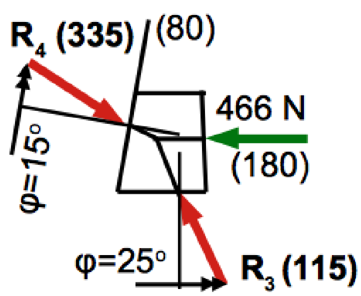
$R_W \sin(14.04 - 104.04) = -3 \sin(270 - 104.04) = 0$

$R_W = 0.728$ **kN**

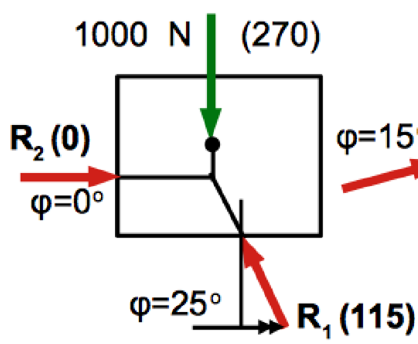


FBD 2

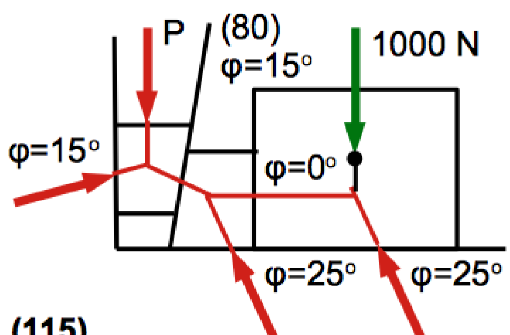
Example A6. Find P for the Impending Motion of the 1000 N Block as shown.



FBD B



FBD A



Force Flow Diagram

T3 FBD A: Equilibrium Equation.

$R_1\ \underline{e}(115) + R_2\ \underline{e}(0) + 1000\ \underline{e}(270) = 0$

SIM3: LCE to line(115). **R2 = 466 N**

T3 FBD B: Equilibrium Equation

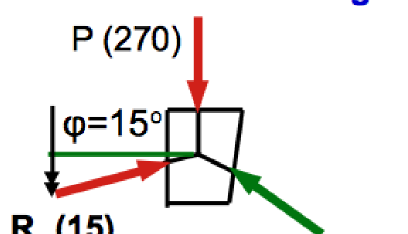
$R_3\ \underline{e}(115) + R_4\ \underline{e}(335) + 466\ \underline{e}(180) = 0$

SIM3: LCE to line(115). **R4 = 657 N**

T3 FBD C: Equilibrium Equation

$R_5\ \underline{e}(15) + P\ \underline{e}(270) + 657\ \underline{e}(155) = 0;$

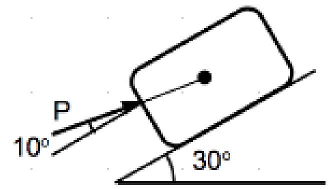
SIM3: LCE to line(15). **Answer** **P = 437 N**



FBD C

Solution to this problem by any other method will involve more steps.

Example A7. A mass of 10 kg is being moved up an incline as shown. The angles of friction are 15° and 12° for static and kinetic states. Find P for Impending motion up. What is the body's acceleration if P is increased by 50 N over its impending value.



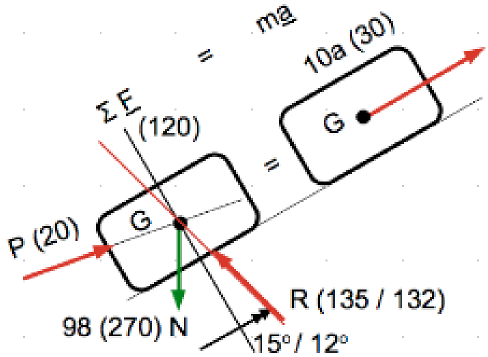
Solution: FBD is T3 for Impending Motion
Equation of Equilibrium ΣF = 0
R e(135) + P e(20) + 98 e(270) = 0

SIM3: LCE to line(135); Answer 1 P = 76.5 N

Equation of Motion: ΣF = ma
with P_{New} = P + 50 = 126.5 N
R e(132) + (126.5) e(20) + 98 e(270) = 10a e(30)

SIM3: LCE to line(132):

10a = 52.9 N
Answer 2 a = 5.29 m/s²

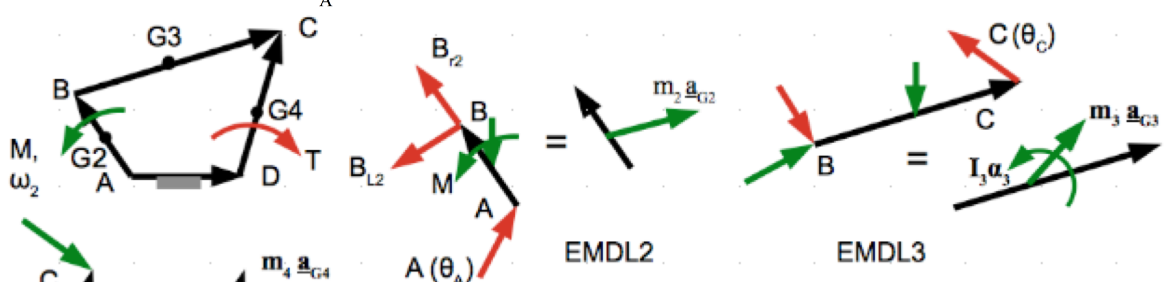


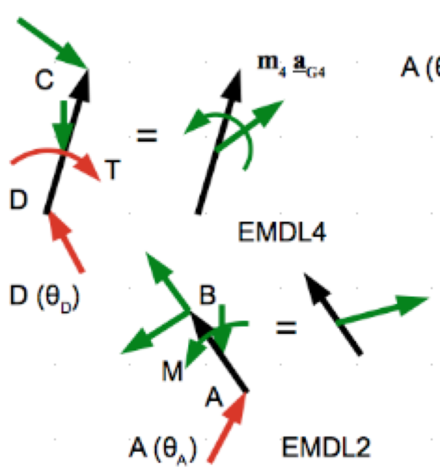
(Impending motion arrow and static / kinetic friction angles)

Equation of Motion Diagram

Example A8. Given the position, velocity and acceleration of the four bars with constant input angular velocity ω_2, input torque M, Link masses m_2, m_3 and m_4, and their weight acting at G_2, G_3 and G_4. find SIMS equations for output torque T and all the pin reactions.

Solution: SIM5 and SIM1&2 are applied as below. In the 1st Equation of Motion Diagram pin reaction at B is resolved in radial and perpendicular directions to L2, to eliminate the radial component and A (θ_A) by SIM5 / Moment about A.





Four-bar Equations of Motion with One Unknown

1. EMDL2	SIM5	$\Sigma M_A = M_A^{G2}$ of $m_2\underline{a}_{G2}$	B_{L2}
2. EMDL3	SIM5	$\Sigma M_C = I_3\alpha_3 + M_A^{G3}$ of $m_3\underline{a}_{G3}$	B_{r2}
3. EMDL3	SIM1	$\Sigma \underline{F} = m_3\,\underline{a}_{G3}$	C
4. EMDL3	SIM2	$\Sigma \underline{F} = m_3\,\underline{a}_{G3}$	θ_C
5. EMDL4	SIM5	$\Sigma M_D = I_4\alpha_4 + M_A^{G4}$ of $m_4\,\underline{a}_{G4}$	T
6. EMDL4	SIM1	$\Sigma \underline{F} = m_4\underline{a}_{G4}$	D
7. EMDL4	SIM2	$\Sigma \underline{F} = m_4\underline{a}_{G4}$	θ_D
8. EMDL2	SIM1	$\Sigma \underline{F} = m_2\underline{a}_{G2}$	A
9. EMDL2	SIM2	$\Sigma \underline{F} = m_2\underline{a}_{G2}$	θ_A