



## Flipping the Classroom to Address Cognitive Obstacles

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## Flipping the Classroom to Address Cognitive Obstacles

Engineering students have difficulty transitioning to post-calculus courses, in part, because how calculus is expected to be known at the end of calculus differs from how it is expected to be used in later courses like differential equations<sup>1</sup>. Our objective was to identify cognitive obstacles for engineering students as they transition from calculus to differential equations and to adapt the differential equations course to address them. The purpose of this paper is to describe a way of exploiting the flipped classroom and the theory of cognitive obstacles in order to increase coherence in the mathematics classroom. The instructional design, data collection, and analysis all centered around the question: Do students' perceptions of the flipped classroom reflect coherence in the course?

### Theoretical Background

*Flipped Classroom:* Though implementation of the flipped classroom varies from instructor to instructor, the model moves lecture content to be work done at home as preparation for class while class time is used for problem solving. Typically, instructors accomplish the “flip” by using instructional technology to deliver lectures or learning modules out-of-class through lecture capture videos, online quizzes, educational videos (i.e. TED talks), and other online resources. Instructional difficulties may arise from this model: a failure to address student misconceptions, poorly designed activities that require only recall, disconnect between active-learning components and other material<sup>2</sup>, and persistence of traditional lecture during class time<sup>3</sup>. These difficulties, coupled with the poor alignment between in-class and out-of-class materials, can leave students with a feeling of incoherence in their learning. That is, the flipped classroom instructional model is itself a potential source of incoherence in the curriculum.

*Cognitive Obstacle:* A cognitive obstacle<sup>4</sup> is a way of thinking about a mathematical structure or object that is appropriate in one situation and inappropriate in another. They are a source of conceptual incoherence in the mathematics curriculum. For example, one well-documented cognitive obstacle that arises in differential equations learning is the *function-as-solution dilemma*<sup>5</sup>. Students may see solutions to mathematical problems as numbers, which is appropriate in algebra. However, in differential equations, when the solution is often a family of functions, this understanding of solution as a number is inappropriate and can serve as an obstacle to an understanding of equilibrium solutions and many other situations in differential equations. Another example of a documented cognitive obstacle is viewing rate of change as a quantity instead of as an algebraic symbolic object<sup>6</sup>. This cognitive obstacle inhibits students' capabilities in understanding the concept of derivative<sup>7</sup>. We wrote learning modules to help students make sense of various cognitive obstacles in order to make in-class time available for deeper discussions. As such, the modules were designed to highlight prior mathematical knowledge, and connect it to the current use of that mathematics as a way of helping the students see appropriate uses in both calculus and differential equations.

Using what research has produced from studies of the flipped classroom and acknowledging cognitive obstacles, we designed the flipped classroom model to address two types of coherence: *coherence within curriculum* and *coherence within instruction*. We addressed coherence in the curriculum by improving connections among mathematical content; we addressed coherence within instruction by synchronizing out-of-class materials with in-class

material. In the present study, we designed a survey for students to gauge perceived instructional coherence.

### **Classroom Context**

The 85 students who were participants in this study were mostly sophomores enrolled in an engineering program at a large Midwestern university. They were enrolled in one section of a differential equations course specifically designed to meet the mathematical needs of engineering majors. The course was centered around mathematical modelling of paradigmatic engineering problems. Whereas many differential equations courses increase the analytic or algebraic difficulty of problems as the semester progresses, our course increased the complexity of the situation being modeled. At the beginning of the semester, students modeled pollution seeping into a dam and made assumptions like the distribution of pollutant, the variation of pollutant levels throughout the dam, the rate of water entry, etc. These assumptions make the real situation accessible to students to model mathematically. As they gained in their abilities to translate physical situations to mathematics, we allowed the complexity of the problems to grow. In the pollutant and dam problem, we assumed at the end that the density of pollutant changes over time and space. The resulting course is highly valued by the engineering faculty and the students remark after each semester that the course is the first time they see how mathematics fits into engineering.

Because our goal is to focus on transforming physical engineering situations into mathematical problems, many of the problems that are proposed during class are long and some take multiple lecture sessions to work through. In the past, students expressed that they didn't remember pieces of pre-calculus, calculus, or other science courses that were essential to grasping the new mathematics introduced in differential equations. Additionally, students express discomfort with not seeing a whole concept or problem in one lecture period. In response to these requests, and because we understand that the majority of learning in mathematics courses occurs outside of the classroom while students are working on homework and studying, we drew on technology to provide students access to resources anytime and anywhere. In the past we used pencasts as a way to provide exemplars to students<sup>8</sup> and used lecture capture<sup>9</sup> so that students might view or re-view lecture content. This study of students' perceptions of *coherence within instruction* when implementing a flipped classroom model was based on the idea of *coherence within curriculum*. It is the next step in our series of technology adaptations in support of differential equations for engineering students.

### **Methods**

*Technology:* Prelecture assignments consisted of completing a module designed to address a cognitive obstacle related to the next lesson's content. The modules were built in Articulate Storyline, a program which allows for many different kinds of student response: fill in the blank, multiple choice, multiple selection, short answer, and essay answer. Once built, the modules resembled power point presentations, but with occasional questions inter-mixed among mathematics content slides. Websites and picture files could also be embedded in the slides.

The Storyline files were uploaded to the course webpage and linked to the grade book. Students had between 10AM on the morning the day before lecture until 1AM the evening before lecture to complete the modules. After that time, the modules would become unavailable. Multiple choice, multiple select, and short answer questions were graded automatically by the system and directly sent to the students' grades spreadsheet after the

student finished the entire module. Through Articulate Storyline, point values for each question can be assigned and the number of attempts possible can also be assigned. The points assigned for each module ranged between 4 and 10 points and altogether counted as a part of the students' homework grades. Short answer and essay question answers were viewed by the instructor and the researchers to inform future modules and the lecture for the next day, but were not graded. An example of an essay question is shown below in Figure 1. Though students were permitted to work together, only individuals' scores were recorded. On average 90% of the students completed each of the modules across the semester.

The screenshot shows a question interface for 'PreLecture1'. The question asks: 'What do you think is happening based on the data given in the table? What trends do you notice and how did you notice them?'. To the right of the question is a table with two columns: 'Months' and 'Amount Owed'. Below the question is a large text input field with the placeholder text 'type your text here'. At the bottom right of the interface is a 'SUBMIT' button. A speaker icon is visible at the bottom left.

Months	Amount Owed
0	80000.00
1	79919.13
2	79837.45
3	79754.96
4	79671.64
5	79587.48
6	79502.49
7	79416.64
8	79329.94

Figure 1. Screenshot of a question from the first module.

The modules asked questions about pre-calculus and calculus content in ways consistent with how those concepts needed to be used in upcoming in-class activities. For example, since students have difficulty thinking about ways to measure change in a data set<sup>6,10</sup> (e.g., absolute change, relative change, average change, etc.), out-of-class materials asked students to consider appropriate ways to measure change in a data set while in-class activities centered on how measurement of change is used to derive difference equations. The link between the out-of-class material and the upcoming in-class activity was explicitly described, and the final question on that out-of-class activity was the first question used during the in-class time. The pre-class materials provided a link from students' prior mathematical knowledge to how that knowledge was expected to be used in the upcoming activities. Although not all out-of-class materials had the end and start connection, each module ended with an explanation of how or why the material covered would be used during the next day.

*Surveys:* We developed two types of surveys to study students' perceptions of curricular and instructional coherence. One type of survey given was a large survey on students' overall perceptions of the course and the technology use in the course. It was given online during the third week and during the last week of the course. It was developed as a part of a larger project within the mathematics department made possible through an Impact Grant funded by our university. The online pre-survey included three questions related to the modules:

1. I expect pre-class multi-media materials to prepare me to participate in class activities (group discussion, problem solving, etc.)
2. As a result of the out-of-class material, I expect to be confident in my understanding of the concepts that each module covered.
3. I expect the in-class activities to be clearly coordinated with the pre-class material.

The second type of survey was an in-class paper survey given four times throughout the semester at the end of class on a Friday. They focused on whether the students found the modules helpful for understanding course material. These surveys were modelled after previously designed surveys used for analyzing how students used pencasts<sup>5</sup> as well as Powers, Bright, and Bugaj's<sup>11</sup> instrument for use of pencasts.

The first in-class survey asked the following questions related to the prelectures:

1. Did you complete the prelectures this week? (Circle one) All Some None
2. Were the lectures related to the prelectures this week?
3. If they were related, give an example of something from a prelecture that you felt was useful in a lecture.

The second in-class survey asked these questions:

1. Did you complete the prelectures this week? (Circle one) All Some None
2. Were the lectures related to the prelectures this week? Yes No Maybe
3. What was useful from the prelectures this week?
4. Give an example of something that we could use to improve the prelectures:

The questions on the in-class survey were altered for the second version to answer questions generated from the first. For example, we added the fourth question above in hopes of eliciting specific pieces of the prelecture that were useful for the students. We also added the fourth question above because students would write on the first survey, "They were kind of useful this week." We wanted to know not only what specifically students found useful, but what they felt needed improvement.

## Results

*Online Survey:* The pre-survey was taken by 58 students from the differential equations course (91% returned). The post-survey was taken by 20 students (31% returned) from the differential equations course. One possible reason for this gap in sample size might be due to the post survey being given just before finals. In the future, we will give this survey two weeks before the end of the semester to ensure a better response rate. Table 1 shows student responses from the pre-survey given during the second week of the semester.

Table 1. Results from the online pre-survey.

Question (5-point Likert scale)	Mean (SD)
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<b>I expect pre-class multi-media materials to prepare me to participate in class activities (group discussion, problem solving, etc.)</b>	3.67 (0.95)
<b>As a result of the out-of-class material, I expect to be confident in my understanding of the concepts that each module covered.</b>	3.16 (1.03)
<b>I expect the in-class activities to be clearly coordinated with the pre-class material.</b>	3.88 (0.89)

The qualitative responses from the online pre-survey were mixed with negative responses linked to technological difficulties. For example, in the beginning of the semester, we were still learning how to best design the modules and so in the first few weeks, there were errors which resulted in correct answers being given no credit. These problems were rectified as quickly as possible and students had a more positive response to the modules after the technology problems were fixed.

Out of the 20 students who took the post survey, 65% reported that course design and instruction met or exceeded their expectations. Table 2 shows the results from the questions related to the online modules. The questions from Table 1 and Table 2 that are matched are in bold font and in the same order. The non-matched questions from Table 2 were added to provide further input on the modules and how the students used them.

Table 2. Results from the online post-survey

Question (5-point Likert scale)	Mean (SD)
<b>The prelectures prepared me to participate in class activities.</b>	3.60 (0.67)
<b>As a result of the prelectures, I was confident in my understanding of the concepts that each section covered.</b>	3.10 (1.94)
I preferred the prelectures to traditional reading assignments	3.80 (0.80)
The prelectures prepared me to do my homework	2.80 (1.01)
<b>The in-class activities were clearly coordinated with the pre-class material.</b>	3.90 (0.97)

For the in-class surveys, the results were generally positive. The lowest rating was for Survey 2, which corresponded to the most technology problems and a reorganization of class time. The reorganization of class time occurred because a student asked a valuable question during discussion, and the professor chose to follow that student's thought through. It was a good experience for all of the students, but the prelecture before that class meeting was no longer linked to the material during the class session.

Table 3. In-class survey results

	Survey 1 (n=59)	Survey 2 (n=62)	Survey 3 (n=53)	Survey 4 (n=55)
Completed all the prelectures	85%	82%	72%	81%
Prelecture was related to the lecture	85%	74%	85%	96%

Analysis suggests that students found the pre-class materials related to the in-class materials. Thus the flipped classroom model we implemented supported coherence between prerequisite material to differential equations. When asked to give a specific example of from prelectures that was useful during the lecture on the in-class survey, students highlighted the usefulness of a review of implicit differentiation. Overall, student responses suggested that aligning in-class and out-of-class materials through a focus on cognitive obstacles preserved coherence often reported to be lacking in the flipped classroom model. Additionally, although the surveys were designed to assess students' perceptions of instructional coherence, in the in-class qualitative responses, students expressed satisfaction with connections made between past mathematics and current mathematics which indicates a perception of curricular coherence.

One negative aspect of the software was the lack of mathematical representation allowed. There is no equation editor and Articulate Storyline does not accept LaTeX representations. We got around this by asking the students to type  $\exp(x+6)$  to represent  $e^{x+6}$ . Still, the mathematical representations were occasionally too complicated and we had to ask students to bring in responses to be collected during lecture the following day. In the future, we would like to find software choices that are capable of handling and displaying advanced mathematics.

We will continue to adopt and adapt new technologies in order to increase mathematical coherence in our flipped model. We urge other instructors considering a flipped classroom model to first question what the goals of their courses are and then choose technology and instructional activities which align with those goals in order to prevent instructional incoherence. Our future work will concentrate on determining whether cognitive obstacles are being addressed and eliminated through this implementation of the flipped classroom paradigm.

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