Formulation and computation of the direction of an optical source using multiple detectors

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Abstract

Detection of optical source and its direction is necessary in applications such as solar panel alignment systems, object-tracking systems, and in robotic control. A simple system of three directional optical detectors mounted on one side of a plane to look in preset directions can be adequate to compute the direction of an unknown optical source.

Formulation is based on the directional response of optical detectors (sensors). In general, output signal of a detector is largest in a specific direction and tapers off as the source makes an angle with the direction of maximum output. Detectors using lenses have the direction of maximum output aligned with the normal axis of the lens. Manufacturers of the detectors provide this information in the data books as well as it can be obtained experimentally.

The output of each detector can be written as function of the angular distance between its preset direction and the unknown source direction. The spherical coordinate system is most suitable for this type of problems. Three detector system is selected which results in a set of three simultaneous equations. There are three unknown, two angular coordinates (θ, ϕ) and one related to the peak signal level. When three measured output signal values and the preset angles are available, a small computer program can generate the unknown angles.

In this paper the system set up, formulation, and computation of the direction of the optical source are presented. A relationship between the number of bits of the Analog-to-

digital converter (ADC) and corresponding angular resolution of estimation is also presented.

Introduction

Detection of self-radiating optical sources and their direction is necessary in applications such as solar panel alignment systems, object-tracking systems, and robotic controls. Both infrared and visible-light devices can be used for such applications. Solar systems with an aligning system can theoretically increase the output by 41.2% compared to that of fixed mounted panels. The information about the direction of the source is the primary parameter necessary for alignment. For tracking a moving object, we will need to mount a source of optical signal on it.

Formulation

Optical detectors that use lenses have directional response[1]. The output peaks when the light source is placed along the axis normal to the lens. The output reduces as the source is moved away from the direction of maximum signal. The actual output voltage, V_{out} depends on the peak signal voltage, V_{peak}, and the angular distance, ψ.

$$V_{out} = V_{peak}\ f(\psi) \tag{1}$$

where $f(\psi)$ is a function either provided by the manufacturer of the optical detectors[2] (as shown in Fig.1), or can be practically obtained.

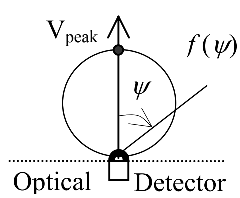

Fig. 1 Directive response of optical detectors

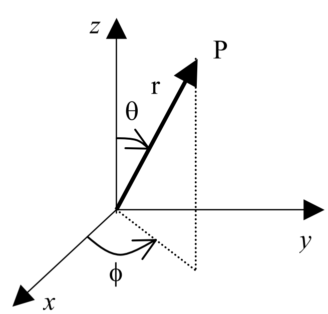

Fig. 2 Spherical coordinate system

The spherical coordinate system, as shown in Fig. 2, shows three parameters for a vector. The angular distance, ψ_{AB} between two vectors $\vec{A}$ (r, θ_{A}, ϕ_{A}) and $\vec{B}$ (r, θ_{B}, ϕ_{B}) is given by:

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right| \cdot \left|\vec{B}\right| \cdot \cos\psi_{\mathrm{AB}} \tag{2}$$

which can be simplified further using vector calculus[3] to obtain the ψ_{AB}:

$$\cos\psi_{\mathrm{AB}} = \sin\theta_{\mathrm{A}} \sin\theta_{\mathrm{B}} \cos(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}) + \cos\theta_{\mathrm{A}} \cos\theta_{\mathrm{B}} \tag{3a}$$

and
$$\psi_{\mathrm{AB}} = \cos^{-1}(\sin\theta_{\mathrm{A}} \sin\theta_{\mathrm{B}} \cos(\phi_{\mathrm{B}} - \phi_{\mathrm{A}}) + \cos\theta_{\mathrm{A}} \cos\theta_{\mathrm{B}}) \tag{3b}$$

If we preset the looking directions of the three detectors as (θ_1, ϕ_1), (θ_2, ϕ_2), and (θ_3, ϕ_3), respectively, then the angular distance, ψ between unknown direction (θ_{x}, ϕ_{x}) and each detector can be written as:

$$\begin{aligned}
\cos\psi_{x1} &= \sin\theta_x \sin\theta_1 \cos(\phi_1 - \phi_x) + \cos\theta_x \cos\theta_1 \\
\cos\psi_{x2} &= \sin\theta_x \sin\theta_2 \cos(\phi_2 - \phi_x) + \cos\theta_x \cos\theta_2 \\
\cos\psi_{x3} &= \sin\theta_x \sin\theta_3 \cos(\phi_3 - \phi_x) + \cos\theta_x \cos\theta_3
\end{aligned} \tag{4}$$

We will arrange three detectors on a horizontal plane. The x-axis will be aligned with the north, y-axis with the west and z-axis with the zenith directions. Fig. 3 shows the plan of the detectors' positions. It is assumed that the source illuminates the three detectors equally.

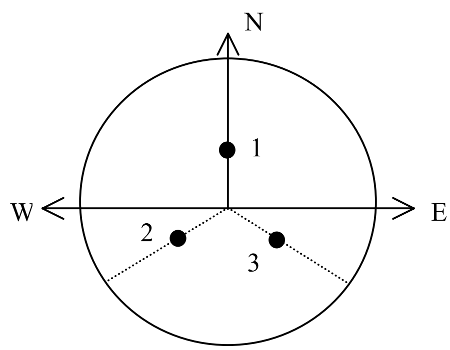

Fig. 3 Plan of three detectors' positions

The detector output is a function of the angular distance between the unknown direction and the direction of individual detector and can be written as:

$$\begin{aligned}
\mathrm{V}_1 &= \mathrm{V}_{\mathrm{peak}}\ f(\psi_{x1}) \\
\mathrm{V}_2 &= \mathrm{V}_{\mathrm{peak}}\ f(\psi_{x2}) \\
\mathrm{V}_3 &= \mathrm{V}_{\mathrm{peak}}\ f(\psi_{x3})
\end{aligned} \tag{5}$$

The function $f(\psi)$ will depend on the construction of the detector and the lens. The simplest and common function is:

$$f(\psi) = \cos\psi \tag{6}$$

Assuming the function in Eq. (6) for all the three detectors, we can simplify Eq. (5) as:

$$\begin{aligned} V_1 &= V_{peak} \cos\psi_{x1} \\ V_2 &= V_{peak} \cos\psi_{x2} \\ V_3 &= V_{peak} \cos\psi_{x3} \end{aligned} \tag{7}$$

Eq. (4) and (7) can be combined to obtain:

$$\begin{aligned} V_1 &= V_{peak} [\sin\theta_x \sin\theta_1 \cos(\phi_1 - \phi_x) + \cos\theta_x \cos\theta_1] \\ V_2 &= V_{peak} [\sin\theta_x \sin\theta_2 \cos(\phi_2 - \phi_x) + \cos\theta_x \cos\theta_2] \\ V_3 &= V_{peak} [\sin\theta_x \sin\theta_3 \cos(\phi_3 - \phi_x) + \cos\theta_x \cos\theta_3] \end{aligned} \tag{8}$$

Eq. (8) has three unknowns, three measured values, six preset values, and can be solved as simultaneous equations.

Computations

We will select three preset directions for the detectors as:

$$\begin{aligned} &\theta_1 = 30^{o}, \phi_1 = 0^{o}, \\ &\theta_2 = 30^{o}, \phi_2 = 120^{o} \\ &\theta_3 = 30^{o}, \phi_3 = -120^{o} \end{aligned} \tag{9}$$

Substituting the preset values in Eq. (9) in Eq. (8), we obtain a set of final equations as:

$$\begin{aligned} V_1 &= V_{peak} [0.5\sin\theta_x \cos\phi_x + 0.866\cos\theta_x] \\ V_2 &= V_{peak} [-0.25\sin\theta_x \cos\phi_x + 0.433\sin\theta_x \sin\phi_x + 0.866\cos\theta_x] \\ V_3 &= V_{peak} [-0.25\sin\theta_x \cos\phi_x - 0.433\sin\theta_x \sin\phi_x + 0.866\cos\theta_x] \end{aligned} \tag{10}$$

Eq. (10) contains three unknowns, V_{peak}, θ_x, and ϕ_x. A scientific calculator can be used to find the values. A simple program can be written to solve these equations and implemented in the data acquisition (DAQ) system.

Angular Resolution

The angular resolution of estimation can be found from mapping the θ and ϕ for various detector output voltage combinations. The total number of combinations for n-bit system will be (n^3-1) for detector output between 0 and V_{peak}.

Based on the computations made using MathCAD for different number of bits, peak value of the resolution, $\Delta\psi$, was determined and is presented in Table 1.

TABLE 1 NUMBER OF BITS OF ADC AND ANGULAR RESOLUTION

Number of bits, n of ADC	Resolution $\Delta\psi$
4	4.526^o
6	1.031^o
8	0.259^o
10	0.047^o
12	0.016^o
16	0.001^o

Scope and Limitations

The above formulations are based on the assumption of the presence of only one point source such as the Sun. The average solar disk as seen from the earth is about 0.5^o. An 8-bit ADC will be adequate for solar panel alignment applications. Larger number of detectors, though redundant, can improve the reliability and precision of the estimation. In addition with larger number of detectors it will be possible to determine the spread of the disk of an optical source. One limitation of this set up is that it is not capable of detecting multiple sources with equally brightness shining at the same time.

Conclusions

In this paper a method of estimating the direction of a single point optical source using three optical detectors is presented. The angular resolution of the estimation improves with the number of bits of the ADC used in the measurement. A simple program can be incorporated in the DAQ system utilized in students' projects. Larger numbers of detectors can improve the reliability of estimation.

References

[1] A.R. Jha, *Infrared Technology Applications to Electro optics, Photonic devices, and sensors*, John Wiley, New York, 2000, pp 356-357.

[2] *Optoelectronic Data Book*, Sharp Corp, 1992.

[3] Erwin Kreyszig, *Advanced Engineering Mathematics*, 8th Edition, John Wiley, New York, 1998.

MARCUS D. JOHNSON is a senior in electrical engineering technology and will graduate in May 2004. He is member of Tau Alpha Pi and student member of IEEE PVAMU chapter. He was selected for UNCF/Dell Minority Corporate Scholars Program for summer internships. He intends to go to graduate school.

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