Free Body Diagrams of Gear Trains

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Abstract

Many senior students in the author's machine elements course have difficulties in drawing a FBD (Free Body Diagram) correctly, which is the first step in force and stress analysis of a mechanical system. One of the challenges to those students is that even though the principles to draw a FBD are the same for every mechanical system (A FBD is a sketch of a mechanical system cut free of its surroundings to shows all the external forces acting on the system), each problem seems different.

Textbooks provide limited examples, especially in the case of gear sets and gear trains when three dimensional FBDs are necessary. For students who have not mustered basic concepts in the FBD, they need additional sample problems and concise rules to enhance their understanding. To this end, the author has developed a course note with simple rules to determine the direction of gear force components (radial, tangential, and axial) with extensive two and three dimensional FBDs of gear sets and gear trains.

One of the reasons for lack of helpful examples in the past is that it is cumbersome to construct complex FBDs of gears with a CAD (Computer Aided Design) software package for publication. With the advancement of CAD, the task to drawing FBDs becomes manageable for this supplementary course note.

This paper gives several examples from the author's course notes on different gear sets (spur, helical, bevel, and worm gears) and gear trains (compound and planetary gear trains) to determine the rotation direction of gears and force direction. Correctly drawn FBDs help the design of shafts (determining the shear force and bending moment) and bearings (determine the reaction force at bearings). The notes with the extensive graphics are especially helpful to visual learners.

Background

Gear sets are used to transmit rotary motion and power from one shaft to another. The magnitudes and directions of the tangential, radial and axial components of gear forces are important because they act on the shafts that the gears are mounted on and contribute to the forces acting on the bearings that support the shafts. Since the conditions of static equilibrium will be used to determine bearing reactions, correct directions for the gear forces acting on a shaft must be established.

Since machine design textbooks [l,2,3,4] typically include equations to determine the magnitudes of the gear forces, the material presented here is for determination of only the gear force directions. Thomas and Hillsman [5] proposed simple word rules for the determination of gear force directions. However, there are insufficient figures to illustrate these rules. This paper will discuss the author's attempt to address this problem.

Force on a spur gear

When two spur gears are meshed shown in Fig. 1, and the left one is the driver, the contact point moves along a line as the gears rotate, as shown in Fig. 2. The line of action is sometimes called the pressure line. The force pushing the driven gear is shown in Fig. 3, and will always be along this line of action. The type of force is bearing (pushing) force, applying pressure to the mating tooth. From the principal of force transmissibility in statics, we know that any point along the line of action will still create the same torque about the gear. Therefore, we can use a fixed contact point (pitch point) to simplify the representation of gear force during the engagement, as in Fig. 4. The pitch point is the intersection of the pressure line and the center line of gears. The angle between the line tangent to both pitch circles and the line of action is called the pressure angle.

Therefore two pitch circles can be used to represent two gears, as shown in Fig. 5. The gear force can be resolved to two components – tangential component which is used to transmit the power, and the radial component, which will cause bending of the gear shaft, as shown in Fig. 6. The radial component of the gear force is in the radial direction directed to the gear center and applied at the pitch point. This is because the gear force is a compressive force, always pushing into the gear teeth.

The tangential component of the driving gear will be in the direction to balance the input torque, $\Sigma T = 0$, and therefore, $T_{in} + F_t \frac{d}{2} = 0$. Consequently, the tangential component will be opposite to the direction of the input torque.

The tangential component of the driven gear will be in the opposite the direction of that of the driving gear; so that $\Sigma F = 0$. The direction of rotation of the driven gear will be determined by this force, as this force turns the driven gear.

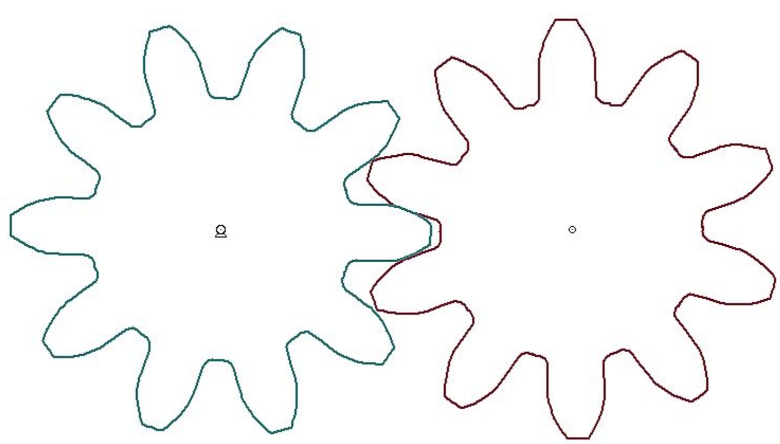

Figure 1 Two meshed gears - the left one is the driver

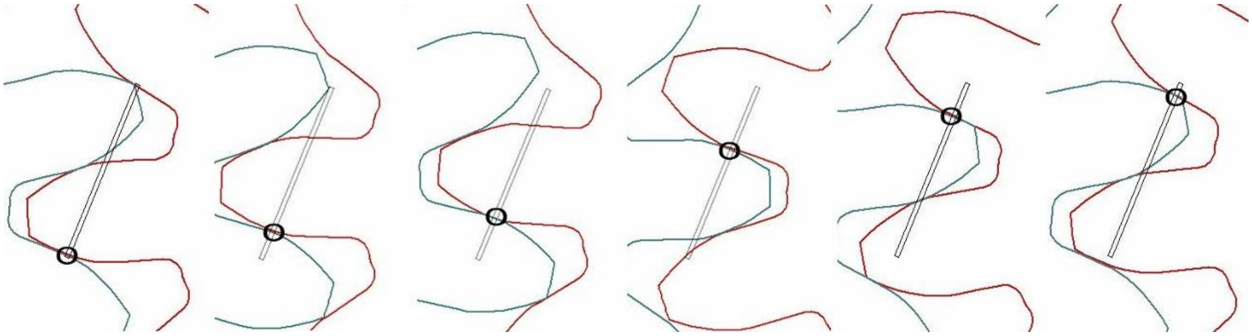

Figure 2 Gear Engagement – contact point is along the pressure line

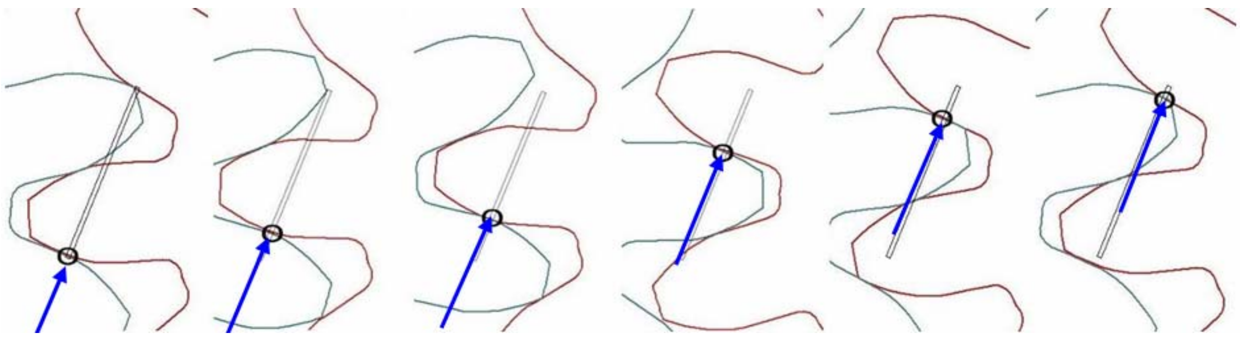

Figure 3 Gear Engagement – the gear force at the contact point moving along the pressure line

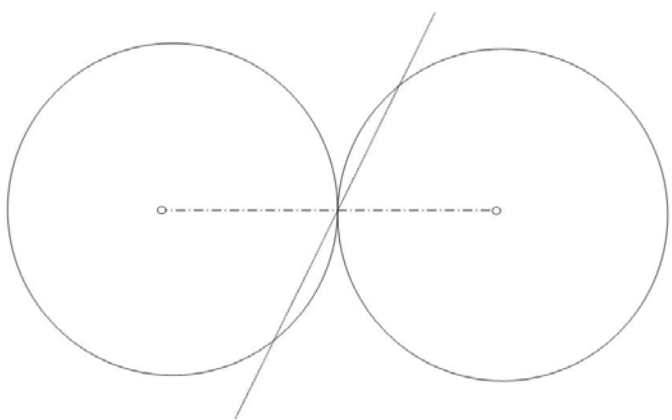

Figure 4 Two meshed gears can be represented by two circles.

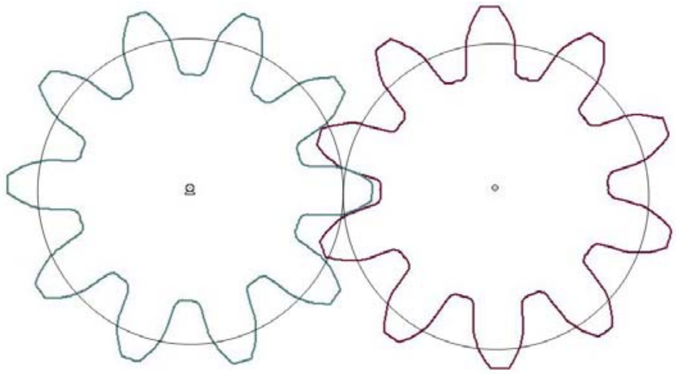

Figure 5 Two meshed gears are represented by two pitch circles

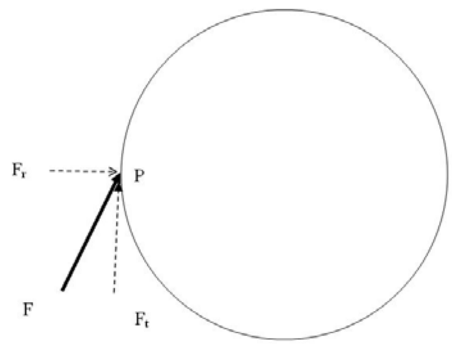

Figure 6 Gear Force at the pitch point

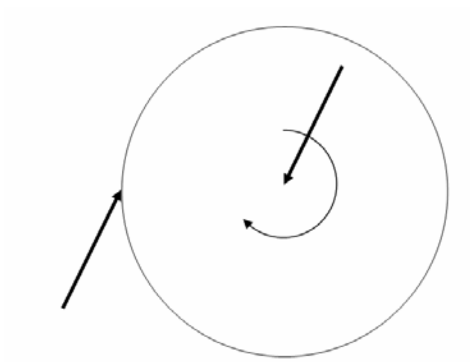

Figure 7 The FBD of the gear

A FBD is a sketch of a mechanical system cut free of its surroundings to shows all the external forces acting on the system. Therefore, the two engaged gears need to be separated to draw a FBD for each gear. For a body to be in equilibrium, the external forces and the moments acting on the body both sum to zero. Therefore, Fig. 6 shows the force on a gear, not a FBD. The FBD will include the reaction force and torque on the gear, as shown in Fig. 7. FBDs of a pair of gears are shown in Fig. 8.

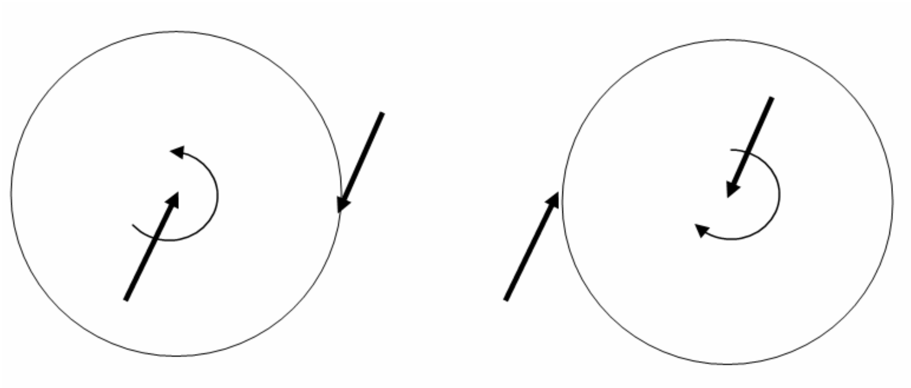

Figure 8 Gear force of two meshed gears

FBD of a helical gear set

Spur and helical gears are used to connect two shafts that are parallel to each other. Spur gears are teeth parallel to the shaft, while helical gears are with teeth at an angle to the axis, as shown in Fig., 9. The force on a helical gear has three components: radial, tangential and axial. The force is still normal to the gear tooth, at an elevated angle of φ. Because the teeth are at a helix angle to the gear axis, the gear force has an axial component.

The directions of $\mathbf{F_t}$ and $\mathbf{F_r}$ are determined the same way as in the spur gear. The direction of $\mathbf{F_a}$ will be decided by that of $\mathbf{F_t}$. This is because $\mathbf{F_a}$ and $\mathbf{F_t}$ are on the same side of the gear tooth since they are components of the gear force **F,** which is a compressive force, pushing the tooth in the normal direction.

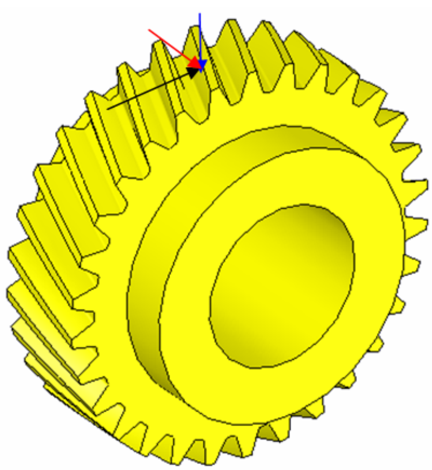

Figure 9 The force on a helical gear

2. FBD of a bevel gear set

Bevel gears are used to connect two shafts, at right angles to each other that would intersect if extended beyond the gears, as shown if Fig. 10. The force on a bevel gear has three orthogonal components (radial, tangential and axial) as the teeth is not on a cylinder as that in a spur gear, but on the side of a truncated cone.

The directions of $\mathbf{F_t}$ and $\mathbf{F_r}$ are determined the same way as in the spur gear. $\mathbf{F_a}$ is pushing into the tooth, as shown if Fig. 11. Note that the axial component of the force on the bevel gear is equal and opposite to the radial gear and are always pointing to the gear.

Figure 10 photos of bevel gears

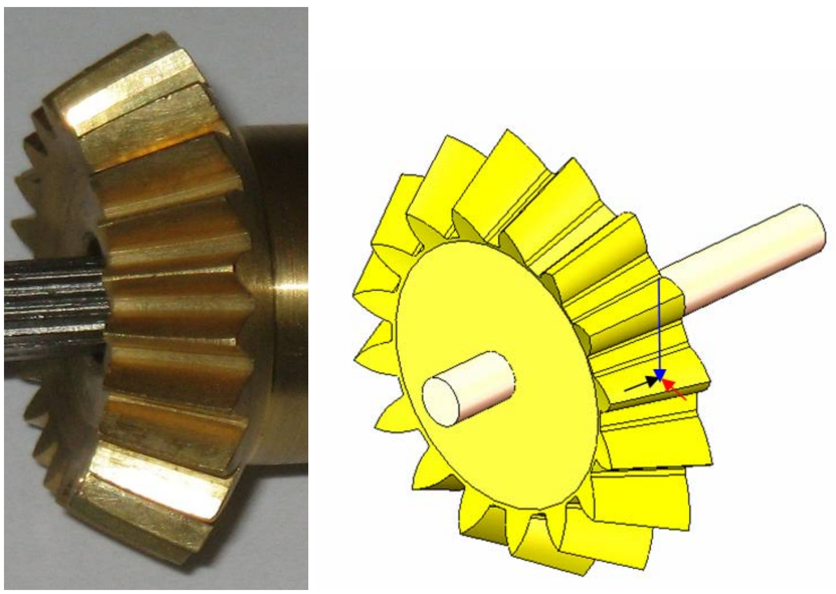

Figure 11 Tooth forces on a bevel gear

FBD of a worm gear set

Worm gears are used to provide right-angle connection of two non-intersecting shafts, as shown if Fig. 12. The worm resembles a screw, and the worm gear is a helical gear with the top of the teeth concaved to provide maximum contact between the worm and worm gear. High gear ratios can be achieved with worm gear sets, but at the expense of efficiency. The directions of the gear forces can then be determined with the following rules.

The direction of $\mathbf{F_r}$ is determined the same way as in the spur gear. The direction of $\mathbf{F_{wt}}$ is determined the same way as in the spur gear. That of $\mathbf{F_{Gt}}$ is not as straight-forward, as $\mathbf{F_{Gt}}$ and $\mathbf{F_{wt}}$ are in different directions and of different magnitudes, unlike other types of gears discussed so far. Because the two gears are at a right angle, the direction of $\mathbf{F_{Ga}}$ and $\mathbf{F_{wt}}$ are equal and opposite. The direction of $\mathbf{F_{wt}}$ can be decided, in the method similar to that for a helical gear –$\mathbf{F_{wt}}$

and $\mathbf{F}_{wa}$ are on the same side of the gear tooth, as shown in Fig. 13, as they are components of the gear force $\mathbf{F}_W$. A FBD of the worm is shown in Fig. 14.

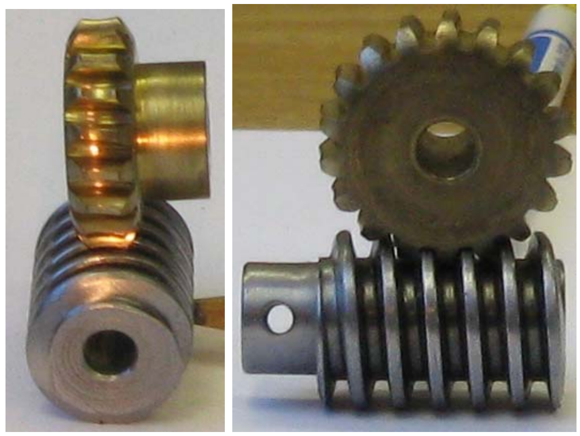

Figure 12 Photos Worm Gears

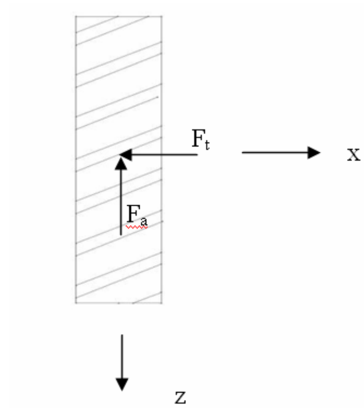

Figure 13 Force component ($\mathbf{F}_{wt}$ and $\mathbf{F}_{wa}$) on the worm – the bottom view

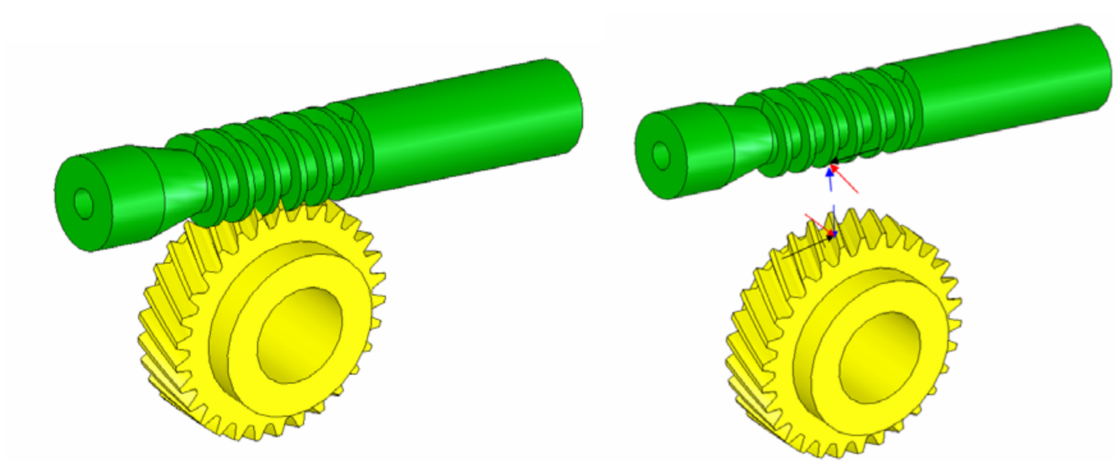

Figure 14 FBD of the Worm

2. FBD of a gear train

In a double reduction gear train shown in Fig. 15, the counter shaft has the function of an idle gear, as there is no input nor output torque on this shaft. To draw a FBD of gear train, the first step is to isolate the system (a shaft with gears mounted) from its surroundings (meshing gears and bearings), as shown in Fig. 16. The second step is to identify the pitch point – the contact point of the pitch circles. The gear force, with its components in radial, tangential (and axial) directions, is applied at the pitch point. The direction of each component will then be decided based on the rules introduced earlier, as shown in Figures 17-19. These three figures are then combined to have the Figure 20.

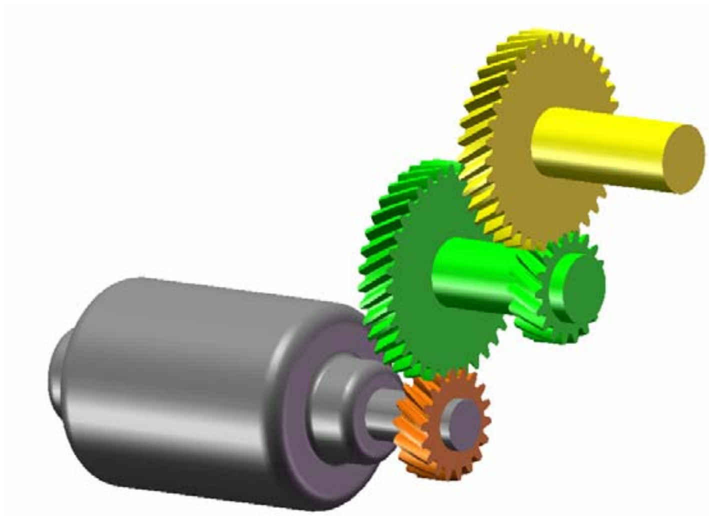

Figure 15 A double reduction gear train

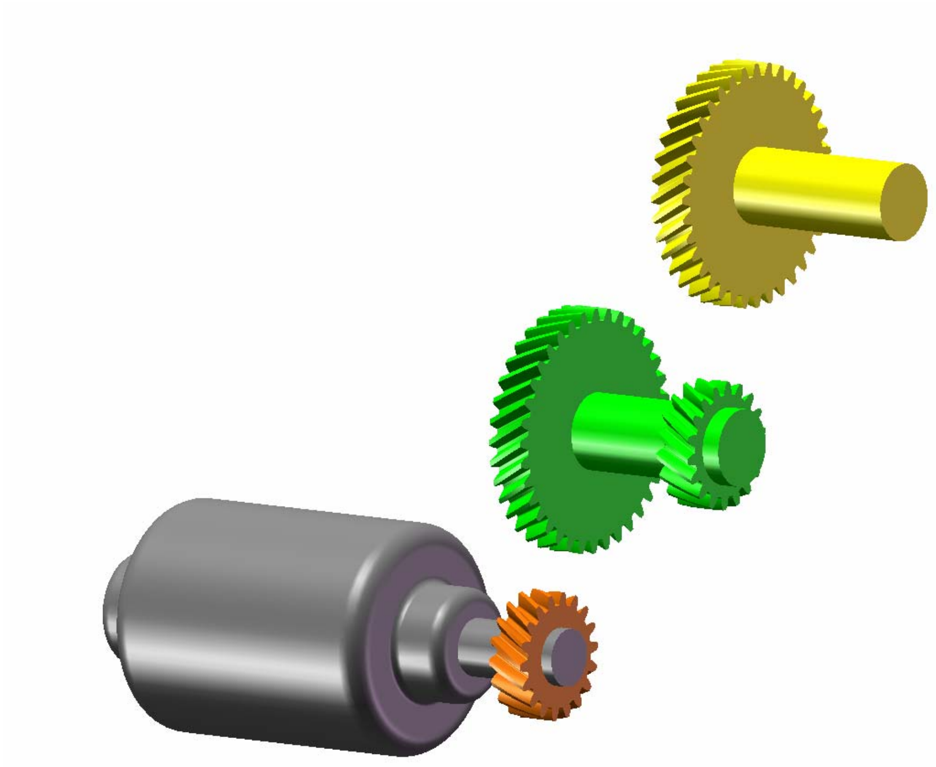

Figure 16 The three shafts with mounted gears are separated to draw FBD

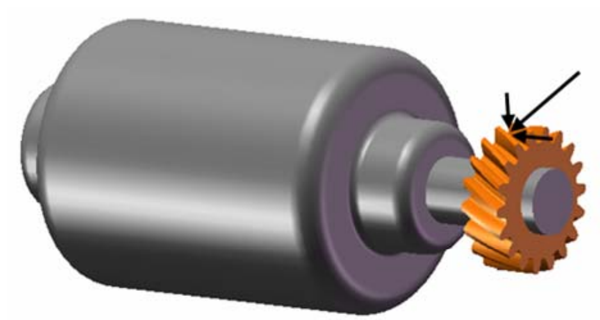

Figure 17 Force on the driving pinion

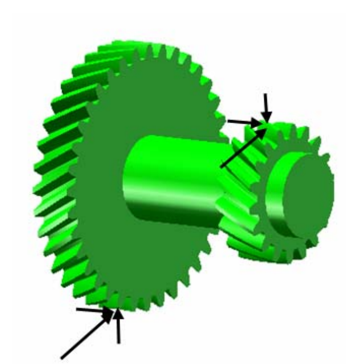

Figure 18 Forces on the gears mounted on the counter shaft

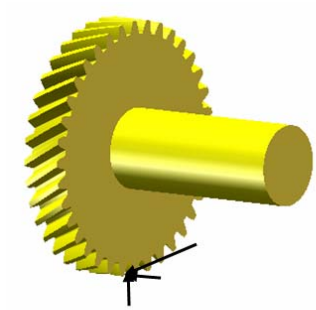

Figure 19 Force on the output gear

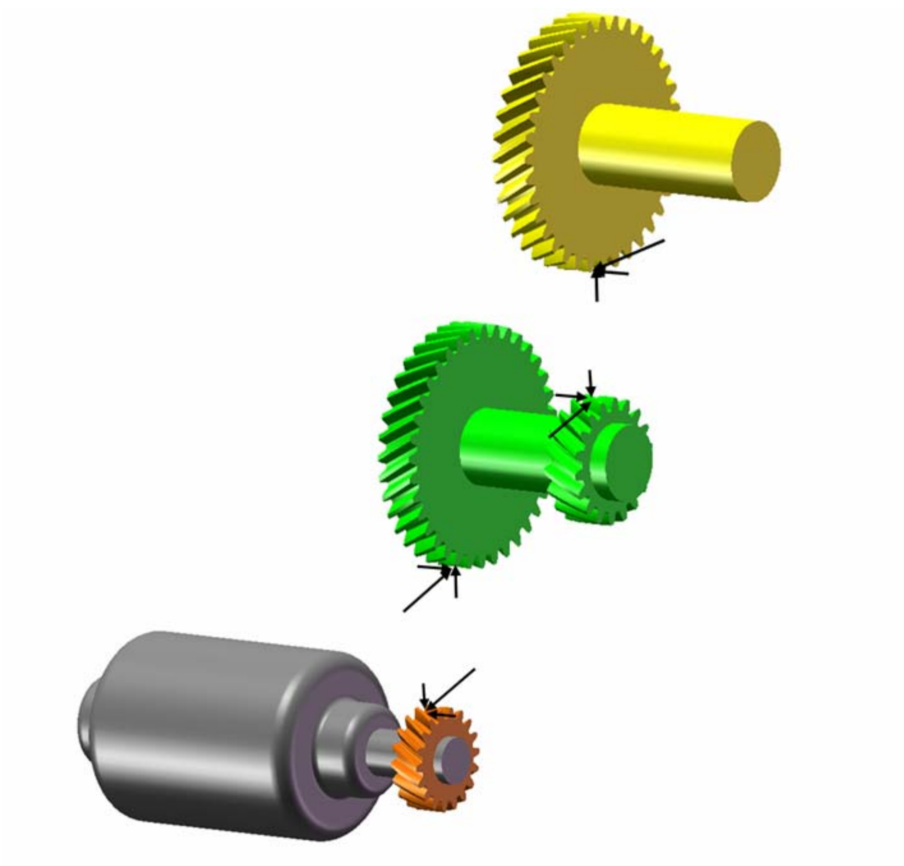

Figure 20 Gears forces are shown with three axes separated

FBD of a planetary gear train

A planetary gear train is a gear train that allows the input and output shaft to be co-axial. The end view and a 3D view are shown in Fig. 21. Figure 22 shows a carrier with planet gears. Planet gears are just idle gears from the force perspective, and the FBD is shown in Fig. 23. The FBD should work progressively from the input gear to the output (carrier), as shown in Fig. 24. The FBD of the carrier is shown in Fig. 25.

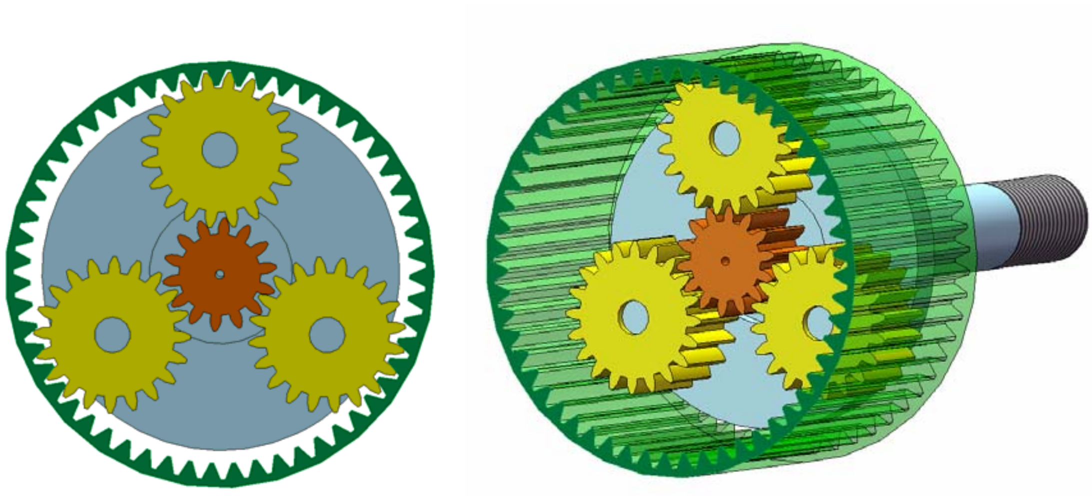

Figure 21 A planetary gear train

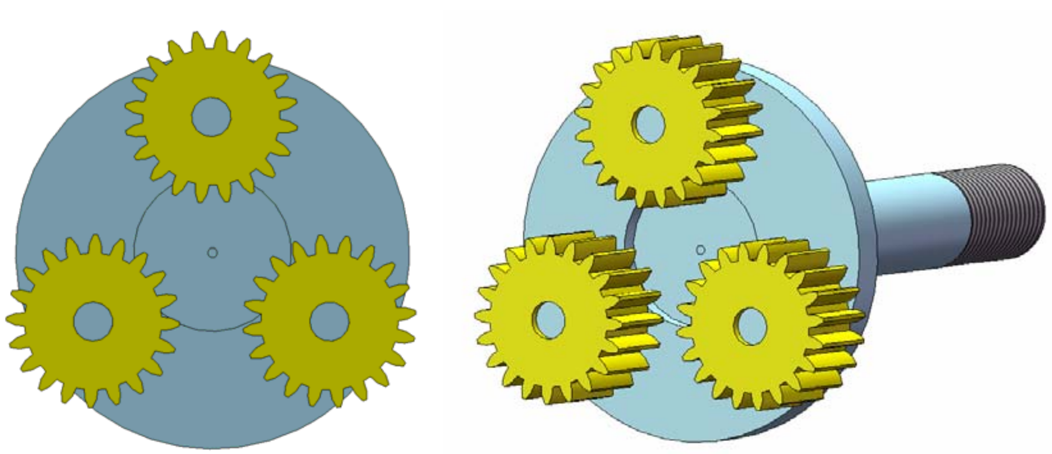

Figure 22 A planetary gear train with the sun gear and ring gear hidden

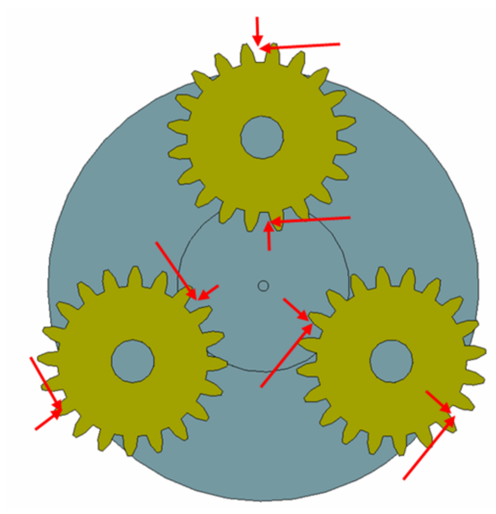

Figure 23 Forces on the planet gears

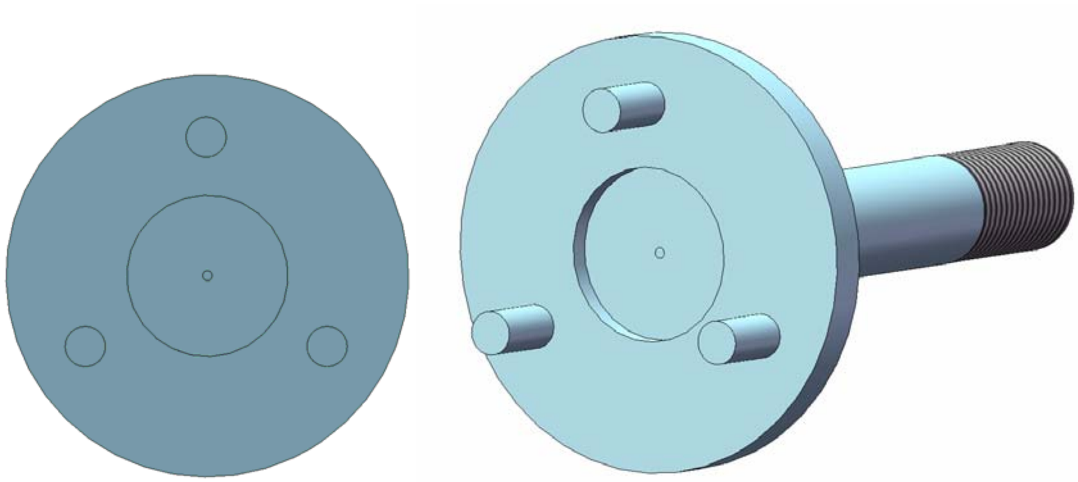

Figure 24 The carrier of a planetary gear train with the sun gear and ring gear hidden

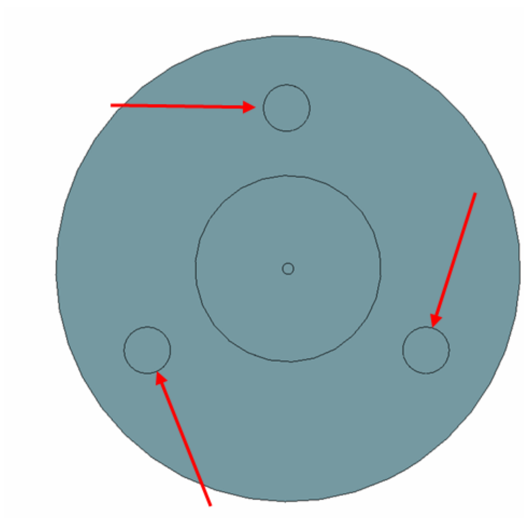

Figure 25 Forces on the carrier

Discussion

Proper gear force directions are necessary to determine bearing reactions on shafts with mounted gears. Gear force components in tangential, radial and axial directions can be determined by based on the rules presented here. The rules are applied to spur, bevel, helical and worm gear sets with illustrations to clarify their use. The rules rely on Newton's laws (1st and 3rd) so that student can relate it to the FBD. The 1st law (equal and opposite) can be used to explain the opposing gear force at the contact point. The 3rd law can be used to explain the direction of tangential component of the gear force. The paper also shows the progression of FBDs of a gear train, solving successfully the unknowns along each step.

The direction of rotation is not factored in determining the direction of force. Rather, it can be determined after the force directions have been determined. The driven gear will be rotating in the direction of the tangential component of the force. This is especially helpful to determine the direction of rotation of worm gear, as most textbooks do not present a concise way in doing so.

References:

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[5] C. R. Thomas and V. S. Hillsman, “Simple Word Rules for the Determination of Gear Force Directions”, Proceedings of 1993 Frontiers in Education Conference pp. 862-5.