Session 1633

GAS TURBINE ENGINE SIMULATION USING MATHCAD: A STUDENT PROJECT

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Abstract

This paper describes an energy system simulation project assigned to mechanical engineering students at the Virginia Military Institute. This project is part of a required, senior level, course in energy conversion design. The class exposes the student to methods of energy system design based on system simulation and optimization. This gas turbine engine simulation is one of several simulation projects assigned in recent years. This project is particularly relevant in that it requires the use of several modeling and simulation techniques. The project statement provides the student with the design point operating characteristics like engine inlet conditions, pressure ratios, and shaft power developed, which are necessary to develop a design point thermodynamic model. Characteristic curves for the various engine components (compressor and turbine) used are also provided. The student must develop the system of equations necessary to model the engine system and use this model to predict the off-design performance (speed, power, efficiency, etc.) of the engine. Off-design conditions may result from changes in throttle setting, changes in applied load, or changes in environmental conditions. The required equations, including the first and second laws of thermodynamics necessary to describe the various components and processes and curve fits of the graphic component characteristics and working fluid properties data, are developed. The graphic data, representing components like compressor ratio, as a function of compressor speed and compressor flow rate, is curve fitted using multiple regression methods. Depending on how the student approaches the problem the system usually involves thirteen to twenty linear/nonlinear equations that must be solved simultaneously. For this project the solution of these equations is accomplished by using Mathcad; although other software capable of solving this system of equations is available. Mathcad was selected because of the ease of programming and the capability to handle systems of nonlinear equations. As it is with the solution of any system involving nonlinear equations, care must be taken in the selection of initial trial values of the unknown variables. For simulations near the engine's design operating point, these initial trial values are simply chosen to be the values at the design point. For simulations that are farther from the design operating conditions, the student must use some judgement in selecting these initial trial values. Physical understanding of the engine's operation is necessary in the selection of these values.

Symbols

C_p ≡ specific heat, Btu/(lb_m R)
m ≡ mass flow rate, lb_m /sec
N ≡ engine speed, rpm
P_o ≡ total pressure, psia
Q ≡ heat rate added to combustor, Btu/sec
T_o ≡ total temperature, R
w ≡ work per unit mass, Btu/lb_m
γ ≡ specific heat ratio
η ≡ isentropic efficiency
Subscript d represents design properties

Introduction

All senior mechanical engineering students at the Virginia Military Institute are required to complete the course, *Energy Conversion Design*. This course includes the applications of thermodynamics, fluid mechanics, and heat transfer in the design of energy conversion systems and system components with emphasis on the use of simulation and optimization in the design process. Typical design projects include engines, heat exchangers, compressors, and power and refrigeration cycles. This paper describes a gas turbine engine simulation assigned as a class project, which is one of several simulation projects used in recent years. The project is particularly relevant because the student is required to use a number of modeling and simulation techniques. Design point operating parameters (temperature, pressures, etc.) and graphs of component (compressor and turbine) operating characteristics are furnished. The student is required to develop the system of equations necessary to model the engine and then use their model to predict the off-design performance (speed, power, efficiency, etc.) of the engine.

Problem Statement

The project statement provides the student with the design point operating characteristics, including engine inlet conditions, pressure ratios, and shaft power, necessary to develop a design point thermodynamic model using the cold air standard assumptions[1]. Additionally, the student is given characteristic curves for the various engine components (i.e., compressor and turbine). These compressor and turbine performance maps were developed based upon typical maps described by Cohen[2], and Wilson[3]. The student is furnished with the following statement of the problem:

Problem Statement

A simple cycle gas turbine engine, shown in Fig. 1, operates with the following parameters when operating at design conditions:
Engine pressure ratio, $P_{o2}/P_{o1} = 4.0$
Engine running speed, $N = 10{,}000$ *rpm*
Total temperature of air entering the engine compressor, $T_{o1} = 530$ *R*
Total pressure of air entering the engine compressor, $P_{o1} = 14.7$ *psia*
Shaft power output of engine = 500 hp
Compressor isentropic efficiency, $\eta_c = 84\%$
Turbine isentropic efficiency, $\eta_t = 90\%$
Turbine inlet temperature, $T_{o3} = 2000$ *R*

Assumptions:
Neglect mass of fuel added
Assume constant specific heat of 0.24 Btu/lb$_m$*R for air and combustion product*
Neglect pressure loss in combustor and connecting ducting
Shaft power required by the load varies directly with the cube of the shaft speed

You are to determine the engine operating parameters as the engine fuel rate is decreased in 10% increments back to 50% fuel flow (i.e., operating parameters for 90, 80, 70, 60, and 50%

fuel flows). Compressor and turbine characteristics are as provided in Figs. 2 and 3. Provide summary of operating parameters, computer program, sample calculation, and discussion of results.

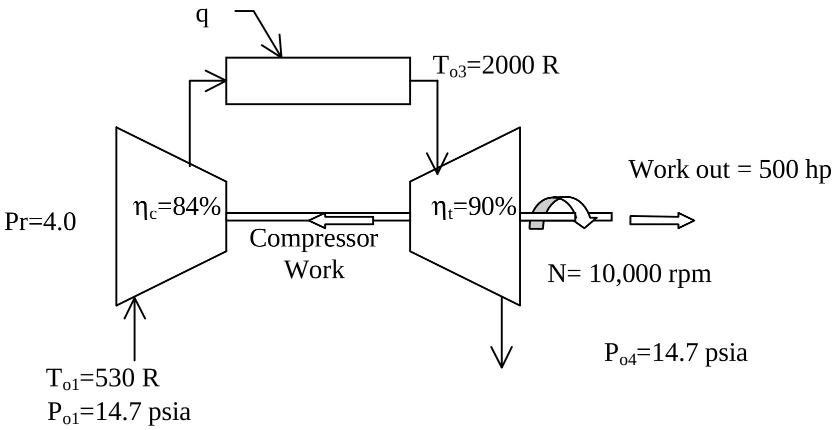

Fig. 1 Gas Turbine Engine at Design Conditions

Engine Model

The engine model is developed by modeling the performance of each component in the engine using energy and mass balances with the performance characteristics provided for each component[4]. The required equations, including the first and second laws of thermodynamics necessary to describe the various components and processes and curve fits of the graphic component characteristics and working fluid property data, are developed. The graphic data, representing components like compressor ratio, as a function of compressor speed and compressor flow rate, is curve fitted using multiple regression methods. The system simulation is accomplished by matching speeds, mass flow rates, pressure ratios, and work for each of the components in the engine, as described by Cohen[2]. The component models include a compressor model, a combustor model, and a turbine model. The development of these models is described in the following sections.

Compressor Model

The compressor model is developed using the thermodynamic relationships for the compressor work, Eq. (1); the isentropic temperature rise change, Eq. (2); and the definition of compressor isentropic efficiency, Eq. (3). The operating characteristics of the compressor are modeled by developing multiple regression curve fits for both the pressure ratio and efficiency as functions of non-dimensional mass flow rate and non-dimensional speed. Equations (4) and (5) define the non-dimensional mass flow and speed parameters[2]. Equation (6) describes the compressor pressure ratio and equation (7) describes the compressor efficiency as multiple regression curve fits. Multiple regression curve fitting for this project was accomplished using FARNSFIT[5].

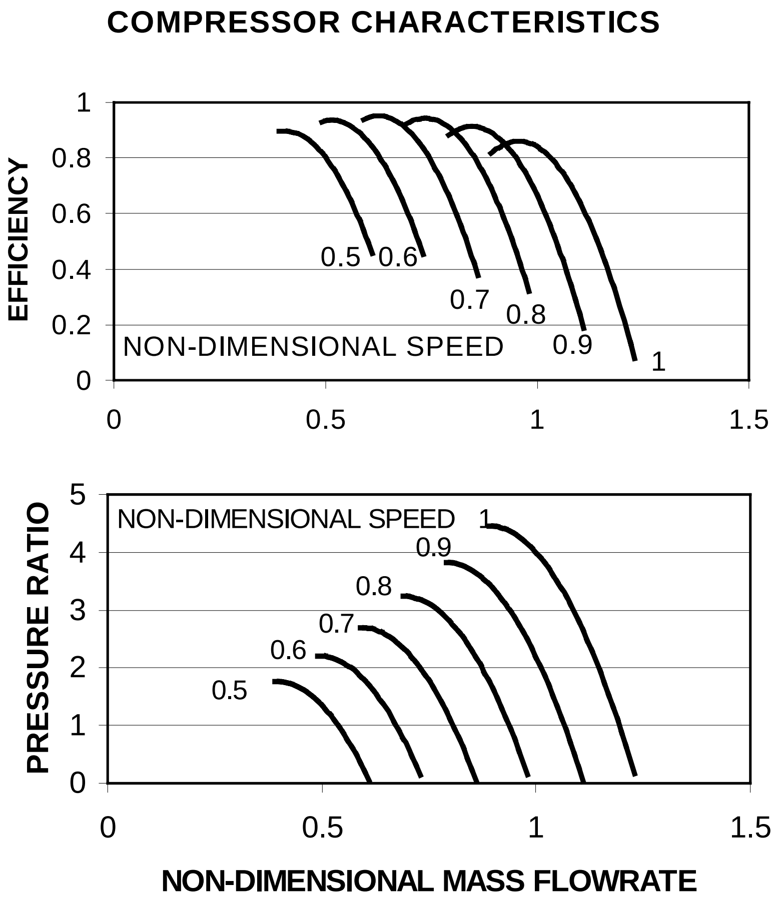

Fig. 2 Compressor Performance Curves

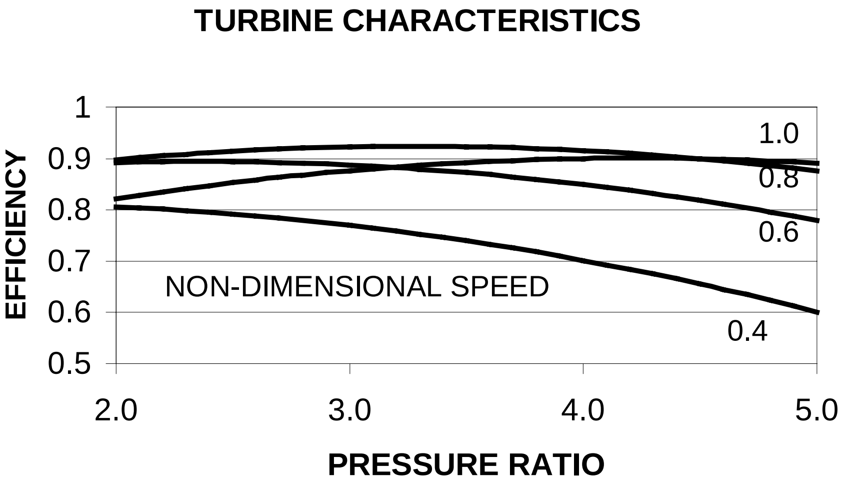

Fig. 3 Turbine Performance Curves

Engine Compressor Model

(1) $$wc = Cp\cdot(To2 - To1)$$ (2) $$To2s = To1\cdot Pr^{\frac{\gamma - 1}{\gamma}}$$ (3) $$\eta c = \frac{To2s - To1}{To2 - To1}$$

(4) $$mc = \frac{\frac{m\cdot\sqrt{To1}}{Po1}}{\frac{md\cdot\sqrt{To1d}}{Po1d}}$$ (5) $$Nc = \frac{\frac{N}{\sqrt{To1}}}{\left(\frac{Nd}{\sqrt{To1d}}\right)}$$

(6) $$Pr = -0.090124 - 7.32282881 mc + 8.99802494 Nc - 36.8122668 mc^2 - 33.4803159 Nc^2 + 72.7075081 mc\cdot Nc$$

(7) $$\eta c = 0.139072009 - 3.05135506 mc + 5.00200918 Nc - 10.3933081 mc^2 - 13.7719419 Nc^2 + 22.9155228 mc\cdot Nc$$

Combustor Model

The combustor model is developed using the first law of thermodynamics to calculate the heat added in the combustor, equation (8). The combustor model could be expanded to include other operating characteristics; such as, pressure loss as a function of flow rate, or mass flow rate of fuel added in the combustor. For this project, the problem statement said to neglect the mass of fuel added and to neglect the combustor pressure drop, that is, let Po3 equal Po2.

Model Engine Combustor

(8) $$Q = m\cdot Cp\cdot(To3 - To2)$$

Turbine Model

The turbine model, similar to the compressor model, is developed using the thermodynamic relationships for the turbine work, Eq. (9), the isentropic temperature change, Eq. (10), and the definition of turbine isentropic efficiency, Eq. (11). The operating characteristics of the turbine are modeled by developing a multiple regression curve fit for the turbine efficiency as function of non-dimensional speed and pressure ratio. Equation (12) defines the non-dimensional speed parameter and Eq. (13) describes the turbine efficiency as a multiple regression curve fit. For this problem the turbine flow is assumed choked, as is common with gas turbines[2], and the non-dimensional mass flow rate is held constant as shown in Eq. (14).

Engine Turbine Model

(9) $$wt = Cp\cdot(To3 - To4)$$ (10) $$To3 = To4s\cdot Pr^{\frac{\gamma - 1}{\gamma}}$$ (11) $$\eta t = \frac{To3 - To4}{To3 - To4s}$$

(12) $$Nt = \frac{\frac{N}{\sqrt{To3}}}{\left(\frac{Nd}{\sqrt{To3d}}\right)}$$

(13) $$\eta t = (0.4828332 - 0.0155\,Pr + 1.15\cdot Nt) - 0.01625\,Pr^2 - 1.0208333\,Nt^2 + 0.1525\,Pr\cdot Nt$$

(14) $$\frac{m\cdot\sqrt{To3}}{Pr\cdot Po1} = \frac{md\cdot\sqrt{To3d}}{Po3d}$$

External shaft load model

The external shaft load is modeled using the cubic shaft power relationship, Eq. (15), as stated in the problem statement. This cubic relationship might represent a pump, a compressor, or a propeller load.

External Shaft Load Model

(15) $$SHP = SHPd\cdot\left(\frac{N}{Nd}\right)^3$$

Engine energy balance

The final relationship necessary to complete the engine model is an energy balance on the engine cycle; that is, the net heat must equal the net work, Eq. (16).

Engine Energy Balance

(16) $$Q - m\cdot Cp\cdot(To4 - To1) = SHP$$

Solution of system of equations

The system of equations involves thirteen to twenty linear/nonlinear equations that must be solved simultaneously. The number of equations depends on how the student has approached the problem. For this project, the solution of the system of equations is accomplished using Mathcad 6.0. Although other software capable of solving this system of equations is available, Mathcad was selected because of its ease of programming. Mathcad allows the student to enter the equations in a form similar to that which they would use if they were solving the equations by hand and is insensitive to the order in which the equations are entered. As with the solution of a system involving nonlinear equations (e.g., solving by Newton-Raphson) care must be taken in the selection of initial trial values of the unknown variables (engine rpm, pressure ratios, temperatures, etc.). For operating points near the engine's design operating point, these initial trial values are simply chosen to be the values at the design point. For operating points that are further from the design conditions the student must use some judgement in selecting these trial values. Physical understanding of the engine's operation aids in the selection of these values.

The student will observe the change in operating parameters as the engine simulation is moved away from design conditions and then trial values can be selected.

Prediction of off-design operating conditions

Off-design operation can be simulated after the student has developed a model that predicts design point operating conditions. Off-design conditions may result from changes in throttle setting, changes in applied load, or changes in environmental conditions. For the problem described here, the student is asked to predict the operation of the engine as the throttle setting is reduced from full throttle (design operating point) to fifty-percent throttle in increments of ten percent. The student must predict the engine power, speed, pressure ratio, cycle efficiency, and other engine parameters as the fuel (heat added) is reduced. Table 1 shows selected engine-operating parameters as a function of throttle setting. Additional studies could be assigned; such as, predicting the effects on engine operation due to changing environmental conditions (inlet temperature, pressure, etc.).

TABLE 1

Engine Operating Conditions as a Function of Throttle Setting

Fraction of Fuel Flow	1.0	0.9	0.8	0.7	0.6	0.5
Pressure Ratio	4.0	3.735	3.452	3.148	2.815	2.44
Speed, RPM	10,000	9,574	9,094	8,546	7,906	7,120
Cycle Efficiency, %	24.2	23.6	22.8	21.6	20.0	17.5
Power, hp	500	439	376	312	247	180

At this point, the student might be asked to examine the change in engine performance as engine components are changed. This would demonstrate the effect on engine performance due to changing components and the value of system simulation to the design process.

Possible project enhancements

The project as presented here limits the complexity of the engine simulation by having the student perform a cold air standard analysis. The model could be enhanced to include the temperature dependence of air properties. This could be accomplished by replacing the constant values of air properties, Cp and γ, with the equations[1] that describe the temperature dependence of these properties. The isentropic relationships based on constant specific heats would have to be replaced with the Gibbs Equations to allow for temperature specific heats. Additionally, the air properties could be replaced with combustion gas properties through the turbine to more accurately model that flow.

The combustor model could be further enhanced by modeling the pressure drop, through the combustor, as a function of the square of the volume flow rate of air. The mass flow could be increased through the combustor to account for the fuel added. Combustion calculations could be added to the combustor model to allow the use of different fuels and to calculate the combustor outlet temperature. In general, the complexity of the problem can be raised to a higher level.

Summary

The project described requires a relatively complex simulation of an energy system. The student develops the equations necessary to describe the engine mathematically. These equations include thermodynamic relationships for the components and for the system, as well as developing multiple regression relationships for the graphical component performance data provided. The resulting non-linear system of equations is solved using Mathcad. Mathcad was found to be a convenient and useful tool for the student for the solution of the resulting system of equations. Mathcad's format, which allows the equations to be entered in a form that resembles the mathematical expressions developed by the student, simplifies programming and is insensitive to the order in which the simultaneous equations are entered. These Mathcad attributes allow the student to concentrate on the engineering involved in the project without getting themselves overwhelmed in the details of the mathematics involved in the solution of the resulting system of nonlinear equations.

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APPENDIX

MATHCAD MODEL OF GAS TURBINE ENGINE

Define Design Conditions

$\mathrm{To1d} := 530 \cdot \mathrm{R}$ $\mathrm{Po1d} := 14.7 \cdot \mathrm{psi}$ $\mathrm{Po4d} := 14.7 \cdot \mathrm{psi}$ $\mathrm{SHPd} := 500 \cdot \mathrm{hp}$ $\eta\mathrm{cd} := 0.84$ $\mathrm{Qd} := 1458 \cdot \frac{\mathrm{BTU}}{\mathrm{sec}}$

$\mathrm{To3d} := 2000 \cdot \mathrm{R}$ $\mathrm{Po2d} := 4 \cdot \mathrm{Po1d}$ $\mathrm{Po3d} := \mathrm{Po2d}$ $\mathrm{Nd} := 10000$ $\eta\mathrm{td} := 0.9$

$\mathrm{Prd} := 4$ $\mathrm{md} := 5.222 \cdot \frac{\mathrm{lb}}{\mathrm{sec}}$

Set Percent Fuel Flow

$\mathrm{Fuel} := 1.0$

$\mathrm{Q} := \mathrm{Fuel} \cdot \mathrm{Qd}$

Define Gas Properties

$\gamma := 1.4$ $\mathrm{Cp} := 0.24 \cdot \frac{\mathrm{BTU}}{\mathrm{lb} \cdot \mathrm{R}}$

Assume starting values of variables (initially assume design point values)

$\mathrm{To1} := 530 \cdot \mathrm{R}$ $\mathrm{Po1} := 14.7 \cdot \mathrm{psi}$ $\mathrm{SHP} := 500 \cdot \mathrm{hp}$ $\mathrm{Po4} := 14.7 \cdot \mathrm{psi}$

$\mathrm{To3} := 2000 \cdot \mathrm{R}$ $\mathrm{Pr} := 4.0$ $\mathrm{N} := 10000$ $\eta\mathrm{t} := 0.9$

Calculate additional starting values of engine parameters based on above assumed values

$$\mathrm{To4s} := \mathrm{To3} \cdot \left(\frac{1}{\mathrm{Pr}}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \mathrm{To2s} := \mathrm{To1} \cdot \mathrm{Pr}^{\frac{\gamma - 1}{\gamma}}$$

$$\mathrm{To4s} = 1.346 \cdot 10^{3} \cdot \mathrm{R} \qquad \mathrm{To2s} = 787.577 \cdot \mathrm{R}$$

$$\mathrm{To2} := \frac{\mathrm{To2s} - \mathrm{To1}}{\eta\mathrm{c}} + \mathrm{To1} \qquad \mathrm{To4} := \mathrm{To3} - \eta\mathrm{t} \cdot (\mathrm{To3} - \mathrm{To4s})$$

$$\mathrm{To2} = 836.639 \cdot \mathrm{R} \qquad \mathrm{To4} = 1.411 \cdot 10^{3} \cdot \mathrm{R}$$

$$\mathrm{wt} := \mathrm{Cp} \cdot (\mathrm{To3} - \mathrm{To4}) \qquad \mathrm{wc} := \mathrm{Cp} \cdot (\mathrm{To2} - \mathrm{To1})$$

$$\mathrm{wt} = 141.286 \cdot \frac{\mathrm{BTU}}{\mathrm{lb}} \qquad \mathrm{wc} = 73.593 \cdot \frac{\mathrm{BTU}}{\mathrm{lb}}$$

$$\mathrm{m} := \frac{\mathrm{SHP}}{(\mathrm{wt} - \mathrm{wc})} \qquad \mathrm{ws} := \mathrm{wt} - \mathrm{wc}$$

$$\mathrm{m} = 5.22 \cdot \frac{\mathrm{lb}}{\mathrm{sec}} \qquad \mathrm{ws} = 67.692 \cdot \frac{\mathrm{BTU}}{\mathrm{lb}}$$

$$\mathrm{Nc} := \frac{\frac{\mathrm{N}}{\sqrt{\mathrm{To1}}}}{\left(\frac{\mathrm{Nd}}{\sqrt{\mathrm{To1d}}}\right)} \qquad \mathrm{Nt} := \frac{\frac{\mathrm{N}}{\sqrt{\mathrm{To3}}}}{\left(\frac{\mathrm{Nd}}{\sqrt{\mathrm{To3d}}}\right)} \qquad \mathrm{mc} := \frac{\frac{\mathrm{m} \cdot \sqrt{\mathrm{To1}}}{\mathrm{Po1}}}{\frac{\mathrm{md} \cdot \sqrt{\mathrm{To1d}}}{\mathrm{Po1d}}}$$

$$\mathrm{Nc} = 1 \qquad \mathrm{mc} = 1$$

System of Simultaneous Equations Necessary to Model Gas Turbine Engine

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Model Engine Compressor

$$wc = Cp\cdot(To2 - To1) \qquad \eta c = \frac{To2s - To1}{To2 - To1} \qquad mc = \frac{\dfrac{m\cdot\sqrt{To1}}{Po1}}{\dfrac{md\cdot\sqrt{To1d}}{Po1d}} \qquad Nc = \frac{\dfrac{N}{\sqrt{To1}}}{\left(\dfrac{Nd}{\sqrt{To1d}}\right)} \qquad To2s = To1\cdot Pr^{\frac{\gamma - 1}{\gamma}}$$

$$Pr = -0.090124 - 7.32282881\cdot mc + 8.99802494\cdot Nc - 36.8122668\cdot mc^{2} - 33.4803159\cdot Nc^{2} + 72.7075081\cdot mc\cdot Nc$$

$$\eta c = 0.139072009 - 3.05135506\cdot mc + 5.00200918\cdot Nc - 10.3933081\cdot mc^{2} - 13.7719419\cdot Nc^{2} + 22.9155228\cdot mc\cdot Nc$$

Model Engine Combustor

$$Q = m\cdot Cp\cdot(To3 - To2)$$

Model Engine Turbine

$$wt = Cp\cdot(To3 - To4) \qquad \eta t = \frac{To3 - To4}{To3 - To4s} \qquad \frac{m\cdot\sqrt{To3}}{Pr\cdot Po1} = \frac{md\cdot\sqrt{To3d}}{Po3d} \qquad Nt = \frac{\dfrac{N}{\sqrt{To3}}}{\left(\dfrac{Nd}{\sqrt{To3d}}\right)} \qquad To3 = To4s\cdot Pr^{\frac{\gamma - 1}{\gamma}}$$

$$\eta t = (0.4828332 - 0.0155\cdot Pr + 1.15\cdot Nt) - 0.01625\cdot Pr^{2} - 1.0208333\cdot Nt^{2} + 0.1525\cdot Pr\cdot Nt$$

Model External Shaft Load

$$SHP = SHPd\cdot\left(\frac{N}{Nd}\right)^{3}$$

Engine Energy Balance

$$Q - m\cdot Cp\cdot(To4 - To1) = SHP$$

Solving the System of Simultaneous Equations Yields

$$\begin{bmatrix} mc \\ Nc \\ Pr \\ To2 \\ To2s \\ \eta c \\ wc \\ Nt \\ To3 \\ \eta t \\ wt \\ To4 \\ To4s \\ m \\ N \\ SHP \end{bmatrix} := \text{find}(mc, Nc, Pr, To2, To2s, \eta c, wc, Nt, To3, \eta t, wt, To4, To4s, m, N, SHP)$$

Compressor Results

$Pr = 4$

$To2 = 836.677 \cdot R$

$\eta c = 0.84$

$wc = 73.602 \cdot \frac{BTU}{lb}$

$mc = 1 \quad Nc = 1$

Combustor Results

$To3 = 2 \cdot 10^{3} \cdot R$

$Q = 1.458 \cdot 10^{3} \cdot \frac{BTU}{sec}$

Turbine Results

$\eta t = 0.9$

$wt = 141.282 \cdot \frac{BTU}{lb}$

$To4 = 1.411 \cdot 10^{3} \cdot R$

$Nt = 1$

Engine Parameter Results

$m = 5.222 \cdot \frac{lb}{sec}$

$\eta cycle := \frac{SHP}{Q}$

$\eta cycle = 0.242$

$Fuel = 1$

External Shaft Load Results

$SHP = 500.098 \cdot hp$

$N = 1 \cdot 10^{4}$