Gauge R&R and Troubleshooting

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Abstract

Teaching Gauge Repeatability & Reproducibility (GR&R) to engineering and engineering technology students enables them to possess a practical skill that is popular in industry. It is especially important for engineering technology students, since many of them will conduct testing, take measurements, and analyze data. The ability to analyze data is an important aspect of engineering technology students pursuing B. S. degrees, since this differentiates them from low level technicians who are only responsible for data collection but not necessarily for data analysis. As a part of the curriculum enhancement effort, GR&R was taught to students in a Six Sigma and Applied Statistics course in the Electronic Systems Engineering Technology program at Texas A&M University. A laboratory was developed for the course to provide students with the opportunity to learn how to conduct Gauge R&R analysis. During the laboratory in the first semester, it was discovered that Gauge R&R could also be used as a troubleshooting tool. This paper discusses the details of how Gauge R&R was introduced in the class, implemented in the laboratory, and used as a troubleshooting tool in the laboratory. The students self-evaluated their knowledge of GR&R before and after the course; the results show that on a scale of 10, they improved by more than 4, with a confidence level of 95%. It is proposed that GR&R be used in other courses to reinforce the knowledge students learned.
Introduction

The importance of statistics to engineering majors has been raised since the early 1980s. It has been widely accepted that the use of statistics in engineering design, testing, and troubleshooting is becoming increasingly critical for companies to stay competitive in the global market. Driven by the need to use statistics in industry, enhancing statistics education in engineering programs has become a subject extensively studied in the literatures. There have been many different approaches proposed to improve statistics curriculum for engineering students. Barton et al developed a laboratory-based statistics curriculum. Standridge et al did similar work. Bryce used data collected by students in his introductory engineering statistics course. Levine et al used Microsoft Excel and MINITAB in their book to teach applied statistics to engineers and scientists. Zhan et al proposed to apply statistics in several courses in the curriculum instead of having a separate applied statistics course within the curriculum. They found that applying specific statistical analysis methods in appropriate courses was an effective way for students to learn to use statistics.

Based on these findings, several laboratory exercises were developed for a course in the electronic system engineering technology program at Texas A&M University for the purpose of teaching applied statistics. The laboratory based approach used by Barton et al and Standridge et al works well for engineering technology (ET) students since hands-on learning is the focus for most ET students. The use of Microsoft Excel and MINITAB for statistical analysis was also adopted to help students understand the statistical analysis method they use. One of the laboratory exercises was Gauge Repeatability & Reproducibility (Gauge R&R).

As a measurement system analysis (MSA) tool, GR&R is typically used to analyze the accuracy of a measurement system, part-to-part variation, and operator error. Gauge R&R is a straightforward and useful tool for engineers and technicians who deal with measurement system and data collection in their job functionalities. Research in this area is quite active. It is also commonly used in Lean Six Sigma projects as a part of the MSA. Rosenkrantz conducted a survey to executives in the American automotive industry to assess the values of several quality tools and statistical methodologies. Among the 306 executives, more than 70% responses indicated that GR&R is the methodology most often used by their organization. This percentage was on the top of the list of 17 quality tools and statistical methodologies commonly used in industry.

Teaching GR&R to engineering and engineering technology students enables these students to possess a practical skill that is popular in industry. It is especially important for ET students, since many of them will conduct testing, take measurements, and analyze data. The ability to analyze data is an important aspect of ET students pursuing B. S. degrees, since this differentiates them from low level technicians who are only responsible for data collection but not for data analysis.
Typically, GR&R is taught in industrial/mechanical engineering/ET majors\textsuperscript{14,18}, for instance, as a part of a metrology course\textsuperscript{2,13}. Most of the time, the measurements are dimensions, weights, or other physical quantities\textsuperscript{26}. It is convenient to measure dimensions and weights; however, GR&R is not limited to such measurement systems. Korestky\textsuperscript{12} introduced GR&R to chemical engineering students with encouraging results. Standridge and Marvel\textsuperscript{25} successfully developed a laboratory-based course to teach all engineering freshmen the concept and use of GR&R. As a matter of fact, one of the important aspects of GR&R is that it highlights the concept of variations in measurements and what the sources for the variations are. The concept of variation in measurement can be a paradigm shift for engineering students; for this reason, GR&R can be just as valuable for students in other engineering majors. In this paper, the detailed implementation of a laboratory exercise with the focus in GR&R is discussed.

Teaching of GR&R

It is well-known that ET students learn more effectively when the theory is applied in a practical problem. Based on this, the introduction of GR&R was largely done in a laboratory session. The concept of GR&R being a methodology for analyzing measurement systems was first discussed briefly in a lecture. The laboratory exercise was to use digital multimeters to measure the resistances of ten resistors with the same nominal resistance value. Students were divided into teams to take turns to measure the resistances. After the data were taken, there were several options for analysis of the data:

1. Using Minitab software
2. Using free online program written in Excel, such as the one developed by QIMacros\textsuperscript{22}
3. Developing an Excel program by the students following an example for GR&R implementation written by the instructor

Each of these options has their advantages and disadvantages. The following list summarizes the advantages and disadvantages for each option:

**Advantages for using Minitab:** It is easy to use; the whole process of GR&R just requires a series of selections (click of buttons). The users don’t need to know how the calculations are done; they just need to know how to import the data and how to explain the analysis results. Knowing how to use Minitab is a skill that is useful in industry.

**Disadvantages for using Minitab:** The users do not understand the underlying calculations required by the GR&R analysis. If the software is not available, the users may not know how to do the analysis. The users cannot check for intermediate results if something appears to be wrong. For most students, there will be no high level learning in the process.

**Advantages for using free online programs:** It is easy to use; the whole process of GR&R just requires copy and pasting of the raw data. The users don’t need to know how the calculations are done; they just need to know how to explain the analysis results.
Disadvantages for using free online programs: The users do not understand the underlying calculations required by the GR&R analysis. The details of the calculation are protected and not visible to the user. The users cannot check for intermediate results if something appears to be wrong. For most students, there will be no high level learning in the process.

Advantages for student developed Excel program: It requires the students to understand the steps in GR&R analysis. It involves higher level of learning. It is easier to check for errors and to add more features if necessary.

Disadvantages for student developed Excel program: It takes longer to implement than other options.

These options were tried out in the first two semesters of the offering of ESET 329. Based on the feedback from students and the observation made in the laboratory, the final approach is a combination of using Minitab and developing Excel program by students. The two-method approach also allows for comparison of the results from each method.

In a lecture and the notes handed out, the formulas\(^28\) to be used to create an Excel program for GR&R were introduced to the students.

Suppose there are a total of \(N\) teams, taking measurements of \(L\) parts, with each part measured \(M\) times. Denote the \(k\)-th measurement of part \(j\), taken by team \(i\) as \(X_{i,j,k}\), \(i=1,2,...,N, j=1,2,...,L, k=1,2,...,M\). The sum for the measurement taken by each is team is defined as

\[
TeamSum_i = \sum_{j=1}^{L} \sum_{k=1}^{M} X_{i,j,k}, \quad i = 1,2,...,N
\]

Denote \(n_1\) is the number of measurements each team took and \(n_2\) the number of measurements taken for each part, then

\[
n_1 = ML, \quad n_2 = MN
\]

The team average is calculated as

\[
TeamAvg_i = \frac{TeamSum_i}{n_1}, \quad i = 1,2,...,N
\]

The sum of the average of each team’s total measurements squared is calculates as

\[
ColSq_i = \frac{TeamSum_i^2}{n_1}, \quad i = 1,2,...,N, \quad SumColSq = \sum_{i=1}^{N} ColSq_i
\]

For each part, the sum of all measurements is calculated as
\[ PartSum_j = \sum_{i=1}^{N} \sum_{k=1}^{M} X_{i,j,k}, \quad j = 1,2,\ldots,L \]

The average measurements for each part is calculated as
\[ PartAvg_j = \frac{PartSum_j}{n_2}, \quad j = 1,2,\ldots,L \]

The sum of the average of each part’s total measurements squared is calculated as
\[ RowSq_j = \frac{PartSum_j^2}{n_2}, \quad SumRowSq = \sum_{j=1}^{L} RowSq_j \]

The sum of all measurements taken can be calculated two different ways
\[ SumX = \sum_{i=1}^{N} TeamSum_i = \sum_{j=1}^{L} PartSum_j \]

Students were required to implement a check in their program that the two different calculations gave identical results. If this was not true, then they had to go back and find the mistake in their Excel coding.

The sum of all measurement squared is calculated as
\[ SumXSq = \sum_{i=1}^{N} \sum_{j=1}^{L} \sum_{k=1}^{M} X_{i,j,k}^2 \]

The sum of the average of repeated measurements sum squared is calculated as
\[ InteractSq_{i,j} = \left( \frac{\sum_{k=1}^{M} X_{i,j,k}}{M} \right)^2, \quad SumInteractSq = \sum_{i=1}^{N} \sum_{j=1}^{L} InteractSq_{i,j} \]

The correction factor for mean is calculated as
\[ CM = \frac{SumX^2}{M \times N \times L} \]

The total sum of squares is calculated as
\[ TotSS = SumXSq - CM \]
The sum of squares for teams is calculated as

\[ \text{TeamSS} = \text{SumColSq} - \text{CM} \]

The sum of squares for parts is calculated as

\[ \text{PartSS} = \text{SumRowSq} - \text{CM} \]

The sum of squares for interaction is calculated as

\[ \text{InterSS} = \text{SumInteractSq} - \text{CM} - \text{TeamSS} - \text{PartSS} \]

The error sum of squares is calculated as

\[ \text{ErrorSS} = \text{TotSS} - \text{TeamSS} - \text{PartSS} - \text{InterSS} \]

The degree of freedom for teams is \( N-1 \). The degree of freedom for parts is \( L-1 \). The degree of freedom for interactions is \((N-1)(L-1)\). The total degree of freedom is \( NML-1 \). The degree of freedom for error is calculated as

\[ \text{ErrorDF} = \text{Total DF} - \text{Team DF} - \text{Part DF} - \text{Interaction DF} \]

The error, team, part, and interaction mean squares are calculated as the ratios of sum of squares and the degree of freedoms

\[ \text{MS} = \frac{\text{SS}}{\text{DF}} \]

The F-test statistics is calculated as

\[ F_{\text{cal}} = \frac{\text{MS}}{\text{Error MS}} \]

This test statistics is compared to the F-critical value with certain confidence level. Typically, 95% confidence level is used. If the F-test statistics is greater than the F-critical value, then one can conclude that there is significant variation in the respective category.

The variances for teams, parts, and interactions are calculated as

\[ \text{Var}_{\text{Team}} = \frac{\text{TeamMS} - \text{ErrorMS}}{N \times L} \]
\[ \text{Var}_{\text{Part}} = \frac{\text{PartMS} - \text{ErrorMS}}{N \times M} \]
\[ \text{Var}_{\text{Int}} = \frac{\text{IntMS} - \text{ErrorMS}}{M} \]

The variances are adjusted so that if they are negative, the value will be replaced by 0. The total variance is calculated as
\[ \text{TotVar} = \text{adjVar}_{\text{Team}} + \text{adjVar}_{\text{Part}} + \text{adjVar}_{\text{Int}} \]

The percentage contributions from team, part, interaction and error are calculated as

\[ \%\text{Var}_{\text{Team}} = \frac{\text{adjVar}_{\text{Team}}}{\text{TotVar}} \times 100\% \]

\[ \%\text{Var}_{\text{Part}} = \frac{\text{adjVar}_{\text{Part}}}{\text{TotVar}} \times 100\% \]

\[ \%\text{Var}_{\text{Int}} = \frac{\text{adjVar}_{\text{Int}}}{\text{TotVar}} \times 100\% \]

These formulas are given together in the lab instructions with a simple example of Excel implementation of GR&R analysis. Students were asked to go through the Excel program example to understand how each formula was implemented. The example Excel program was constructed in a way that it would only work for specific values of \( M, N, \) and \( L \). Therefore, students could not simply copy and paste the test data to complete the GR&R analysis. Instead, they had to write their own Excel program for different \( M, N, L \) values. A part of the program is shown in Fig. 1. The last column in Fig 1 has the percent contributions from team, which is the reproducibility, and from measurement error, which is the repeatability. R&R is the sum of these two terms. According to AIAG\(^1\), if the gage R&R contribution is more than 9 percent, the measurement system is not acceptable.

![ANOVA Table](image)

**Figure 1. Code of GR&R in Excel**

In the laboratory, students first worked on their Excel program for GR&R analysis. Using the formulas and the Excel example provided to them, they were asked to create their own program for the case with 10 parts, \( x \) teams, with the measurement of each part repeated by each team 5 time, where \( x \) is the number of teams in the laboratory section. The value of \( x \) may vary from one semester to the next. A set of data with appropriate size was provided to the student teams. The solution to this GR&R analysis problem was worked out by the instructor beforehand. This solution was used in the laboratory to check each student team’s correctness of their Excel program. Only those teams with the same answer as the solution were allowed to move the data collection and final GR&R stage.

Once they had the correct answer for the given test data, student teams took resistance measurements of 10 resistors with the same nominal resistance. For each resistor, the
measurement was repeated 5 times by each team. The values for M, N, and L matched the size of the GR&R program they created in the first stage. Therefore, they could directly record the data in their GR&R analysis program. After recording the test data to the GR&R program, the students first check for calculation errors. If no error was found, they compared the calculated F-test statistics against the F-critical values for team, part, and interaction to find those that had significant variations. The percent contributions from team, part, interaction and error were used as indications for the relative amount of variations from each category. Their GR&R analysis was compared to the results from MINITAB.

**GR&R as a troubleshooting tool**

In the laboratory exercise for GR&R, students were asked to analyze the results from GR&R. For instance, if R&R was greater than 9%, they were supposed to identify whether the variation was mainly from repeatability or reproducibility. If it was mainly from the repeatability, then the equipment was not acceptable. This could be due to a calibration problem or equipment malfunction. If the main variation was from the reproducibility, then there may have been a problem with the operators using the equipment. Training for operators might be required to reduce the variation.

During the first semester of adopting the GR&R laboratory exercise, a significant level of reproducibility was found after the GR&R analysis. All the student teams used the raw data to try to find the cause of the large reproducibility value. The regular GR&R laboratory turned into a troubleshooting exercise.

The student teams calculated the mean and standard deviation in Excel for the measurements by parts and by teams. They quickly identified that the data from a specific team was the cause of the large GR&R value. This team’s data looked very different from other teams’ data. The team went on to repeat their measurements and found that their measurements were very inconsistent. A different digital multimeter was used and the results were still inconsistent. Measuring a short circuit gave a large resistance value. This led to the investigation of the wires used to connect the resistors and the digital multimeter. The resistance for the wire was found to be large and changing when the wire was giggled slightly. Cutting open the wire eventually revealed the cause of the problem: the conduct inside the wire was broken. The two pieces of conducts were still in contact with each other, but the resistance could be a few hundred Ohms.

This troubleshooting success led to the additional laboratory step for the GR&R analysis laboratory in the following semesters. The resistors were wrapped in labels so that students could not see the color codes. A resistor with different nominal value was fixed in the group of resistors that have the same nominal resistance. Using GR&R analysis, students could always trace the problem to the “wrong” resistor.
Through this GR&R laboratory exercise, students learned the concept of variation in measurements, GR&R analysis using Excel and MINITAB, and troubleshooting techniques, all these are useful and practical knowledge in industry.

**Evaluation of student learning**

During the GR&R laboratory exercise, it was observed that students were more actively involved compared to other laboratory work. In the student evaluation conducted at the end of the first semester, many students specifically wrote positive comments about the laboratory experiences during GR&R analysis.

To carry out a quantitative analysis of student learning, students were asked to give themselves two evaluations on the knowledge of GR&R among other things, one in the beginning and one at the end of the semester. Students rank themselves with a score of 1-10, with 1 implying “know nothing about this area” and 10 implying “an expert in the area”. Over three semesters, 83 students submitted their self-evaluation forms. The raw data are plotted in Fig.1.

![Figure 2. Before and after comparison of student self-evaluation](image)

One can see that the values for “After” are significantly higher than those of “Before”. The statistics of the two evaluation results are summarized in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std</th>
<th>min</th>
<th>max</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>1.28</td>
<td>0.72</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>
The paired t-test is conducted using the raw data to check if the mean was improved significantly, let us say, by 4 with 95% confidence level.

The null and alternative hypotheses are defined as

\[ H_0: \mu_1 \leq \mu_2 + 4 \quad \text{(The mean of the self-evaluated student knowledge in GR&R are not improved by more than 4.)} \]

\[ H_1: \mu_1 > \mu_2 + 4 \quad \text{(The average of the self-evaluated student knowledge in GR&R in the end of the semester is higher than that in beginning of the semester by 4.)} \]

The paired t-test is used paired t-test is to compare the following quantity to \( t_{0.05, 82} = 1.664 \)

\[ t = \frac{\bar{d} - 4}{s_d/\sqrt{n}} \]

where \( n \) is the sample size, and equal to 83, \( \bar{d} \) is the difference between the average of the after and before evaluation data, \( s_d \) is the standard deviation of the difference, and \( t_{0.05, 82} \) is the value in the \( t \)-distribution table.\(^{28,29} \) A 95% confidence level is used. The \( t \) value can be calculated to be \( t = 3.837 \), which is greater than \( t_{0.05, 82} \). Therefore, with a confidence level of 95%, the null hypothesis \( H_0 \) is rejected and the alternative hypothesis \( H_1 \) is accepted. In other words, one can conclude that the students thought that their knowledge in GR&R improved after taking the course by at least 4 out of the scale of 10.

**Future work and Conclusions**

This paper discusses the implementation of GR&R in a course “Applied Statistics and Six Sigma” offered to the electronics system engineering technology students at Texas A&M University. An introduction in a lecture was followed by a laboratory exercise to enhance student learning. The focus is on using GR&R as a tool for troubleshooting. The laboratory setting allowed students to use data they collected to conduct statistical analysis. Implementing the analysis in Excel provided students an opportunity to have deeper understanding of the GR&R analysis process. It also enabled students to check intermediate results and conduct troubleshooting. The hands-on experiential learning worked well for engineering technology students.

Future research work will be conducted. More data will be collected to monitor improvement of student learning. Additional surveys will be designed to get more detailed information about the effectiveness of the method being used to teach GR&R. In addition to the “wrong” resistor being mixed up with the “right” resistors, other failure-insertion methods are being considered for the troubleshooting laboratory exercise. It is also proposed that GR&R be used in sequential courses
to reinforce the knowledge students learned. New findings will be submitted to future ASEE conferences.

References


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