Geostatistical analysis of geotechnical parameters

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Abstract

Application of various techniques for statistical analysis to the field of civil engineering is well documented. In this paper a case study is presented, where kriging is applied for producing a map showing the probability that the existence of collapsing soil at certain depth of Tucson, Arizona area exceeds a critical threshold value. The assessment is based on existing criteria with spatial analysis used to build up model for low, medium and high collapse potential.

Key words: Map, variogram, kriging, collapsing, soil.

Introduction

Understanding the spatial distribution of data from phenomena that occur in space constitute a great challenge. Due to availability of high speed computing system such studies are becoming common in almost every field of study, such as health, environment, geology, engineering, and many others. Besides visual perception of the spatial distribution of the phenomenon, the analysis is useful to translate the existing patterns in to objective and measureable quantities by estimating parameters at an unknown location. Since the emphasis of spatial analysis is to measure properties and relationship, taking in to account the spatial localization of the phenomenon under study, such analysis may be used for geotechnical parameters. Spatial analytical technique, known as geostatistics may be useful for this purpose. Geostatistical technique of simple indicator kriging can be used with the probabilistic model to develop the probability contour with the contour of estimation variance.

A soil deposit in a region may be either residual or transported. Also a transported soil may be either *alluvial* (stream borne) or A*eolian* (wind borne) or *colluvial* (gravity transported). When alluvial soils are deposited in an arid or a semi-arid environment, they develop larger voids and undergo a large decrease in bulk volume upon saturation or load application and are known as collapsing soils. However, it is difficult to identify collapse susceptible soils with this definition due to the existence of many different types of clay minerals and many other factors that contribute to the collapse phenomenon. Therefore geostatistical methods in analyzing collapsing soil parameter would provide an optimum solution.

In this study geostatistical techniques of simple kriging were applied to selected collapse criteria and collapse-related soil parameters for soil in Tucson, Arizona. Previous works on this topic was limited only to studies involving either specific areas or specific soil parameters. The purpose of this study was to gather as much information as possible from reliable sources and to use this data with statistical techniques, such as regression and factor analysis, to determine the variation of selected collapse criteria and collapse-related soil parameters in three dimensions.

Mathematical Details:
Kriging is a geostatistical technique to interpolate the value $Z(x_0)$ of a random field $Z(x)$ at an unobserved location x_0 from observations $z_i=Z(x_i)$, $i= 1,\ldots n$ of the random field at nearby locations $x_1,\ldots,x_n$. Kriging computes the best linear un biased estimator $\hat{Z}(x_0)$ based on a stochastic model of the spatial dependence quantified either by the variogram $\gamma(x,y)$ or by the expectation $\mu(x)=E[Z(x)]$ and the covariance function $c(x,y)$ of the random field.

The kriging estimator is given by a linear combination $\hat{Z}(x_0)=\sum_{i=1}^{n} w_i x_0 Z(x_i)$ of the observed value $z_i=Z(x_i)$ with weights $w_i(x_0)$,

$i=1,\ldots,n$ chosen such that the variance, known as kriging variance or kriging error

$$
\begin{aligned}
&\sigma_k^2(x_0):\\
&= Var\left(\hat{Z}(x_0 - Z(x_0)\right)\\
&= \sum_{i=1}^{n}\sum_{i=1}^{n}\ \sum_{j=1}^{n} w_i(x_0)w_j(x_0) +\\
&Var(Z(x_0)) -\\
&2\sum_{i=1}^{n} w_i(x_0)c(x_i, x_0)
\end{aligned}
$$

is minimized subject to the unbiasedness condition:

$$E[\hat{Z}(x) - Z(x) = \sum_{i=1}^{n} w_i(x_0)\mu(x_i) - \mu(x_0) = 0$$

The kriging variance must not be confused with the variance

$$
\begin{aligned}
&Var\left(\hat{Z}(x0)\right)\\
&= Var\left(\sum_{i=1}^{n} w_i Z(x_i)\right)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j c(x_i, x_j)
\end{aligned}
$$

Of the kriging predictor $\hat{Z}(x_0)$ itself

The kriging weights of simple kriging have no unbiasedness condition and are given by simple kriging equation system:

$$w = cc(0)$$

Where

$$w = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$

$$c = \begin{pmatrix} c(x_1, x_1) & \cdots & c(x_1, x_0) \\ \vdots & \ddots & \vdots \\ c(x_n, x_1) & \cdots & c(x_n, x_n \end{pmatrix}^{-1}$$

$$c(0) = \begin{pmatrix} c(x_1, x_0) \\ \vdots \\ c(x_n, x_0) \end{pmatrix}$$

Simple kriging interpolation:
The interpolation by simple kriging is given by

$$\hat{Z}(x_0) = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}' \begin{pmatrix} c(x_1, x_1) & \cdots & c(x_1, x_n \\ \vdots & \ddots & \vdots \\ c(x_n, x_1) & \cdots & c(x_n, x_n) \end{pmatrix}^{-1} \begin{pmatrix} c(x_1, x_0 \\ \vdots \\ c(x_n, x_n) \end{pmatrix}$$

The kriging error is given by:

$$Var\left(\hat{Z}(x_0) - Z(x_0)\right)$$
$$= \underbrace{c(x_0, x_0)}_{Var(Z(x_0))} -$$
$$\underbrace{\begin{pmatrix} c(x_1, x_0) \\ \vdots \\ c(x_n, x_1) \end{pmatrix}' \begin{pmatrix} c(x_1, x_1) & \cdots & c(x_1, x_n) \\ \vdots & \ddots & \vdots \\ c(x_n, x_1) & \cdots & c(x_n, x_n) \end{pmatrix}^{-1} \begin{pmatrix} c(x_1, x_0) \\ \vdots \\ c(x_n, x_0) \end{pmatrix}}_{\mathrm{var}(Z'(x))}$$

Which leads to the generalized least squares version of the Gauss-Marcov theorem[1].
$$Var(Z(x_0)) = Var\left(\hat{Z}(x_0)\right) + Var\left(\hat{Z}(x_0) - Z(x_0)\right)$$

ORDINARY KRIGING EQUATION

The kriging weights of ordinary kriging fulfill the unbiasedness condition
$$\sum_{i=1}^{n} \lambda_i = 1$$
and are given by the ordinary kriging equation system:

$$\lambda = \gamma\gamma^{-1}\lambda^*$$

where

$$\lambda = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \mu \end{pmatrix}$$
$$\lambda^{-1} = \begin{pmatrix} \gamma(x_1, x_1) & \cdots & \lambda(x_1, x_n) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \lambda(x_n, x_1) & \cdots & \lambda(x_n, x_n) & 1 \\ 1 & \cdots & 1 & 0 \end{pmatrix}^{-1}$$
$$\lambda^* = \begin{pmatrix} \lambda(x_1, x* \\ \vdots \\ \lambda(x_n, x*) \\ 1 \end{pmatrix}$$

The additional parameter μ is a Lagrange multiplier used in the minimization of the kriging error $\sigma^2{}_k(x)$ to honor the unbiasedness condition.

The interpolation by ordinary kriging is given by

$$\hat{Z}(x^*) = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{pmatrix}' \begin{pmatrix} Z(x_1) \\ \vdots \\ Z(x_n) \end{pmatrix}$$

The kriging error is given by

$$\operatorname{var}\left(\hat{Z}(x^*) - Z(x^*)\right)$$

$$= \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \mu \end{pmatrix}' \begin{pmatrix} \lambda(x_1, x^*) \\ \vdots \\ \lambda(x_n, x^*) \\ 1 \end{pmatrix}$$

COLLAPSE CRITERIA AND RELATED PARAMETERS

In arid regions soil deposits become partially saturated with large voids due to high evaporation rate. Application of loads on such soils causes small deformation at low degree of saturation. However as soon as the soil becomes saturated, large deformations take place due to the collapse of the intergranular structure and the phenomenon is referred to as collapse. In general, if the dry density of the soil is sufficiently low to give a void space larger than that required to hold the liquid limit water content, then collapse upon saturation is likely. Otherwise collapse generally occurs only when the soil is loaded.

Collapsing soils has been recognized in Africa, part of Asia, Europe as well as in the United States. In the United States the severity of the problem has been observed in the Midwestern and Western United States, where soil deposits are generally either *aeolian* or *alluvial.*

Many criteria for predicting the collapsing potential of a soil are available in the literature (Ref from ASCE). Some of the criteria are derived theoretically from consolidation test results and some are empirical. The methods for evaluating collapse susceptibility vary from simple to very complex. Considerable effort has been given to establish criteria for predicting the collapse potential and the critical values for severity of a soil. Some of the more commonly used criteria are described below.

The parameter C_p is known as collapse parameter and is obtained from consolidation test as shown in Fig. 1. (Jennings and Knight, 1957)

$$C_p(\%) = \frac{\Delta e_c}{1+e_0} \times 100 = \frac{\Delta H_c}{H_0} \times 100 \qquad (1)$$

Where Δe_c and ΔH_c are changes in void ratio and sample height after saturation under a pressure of 200 kPa, and H_0 is the initial height of sample

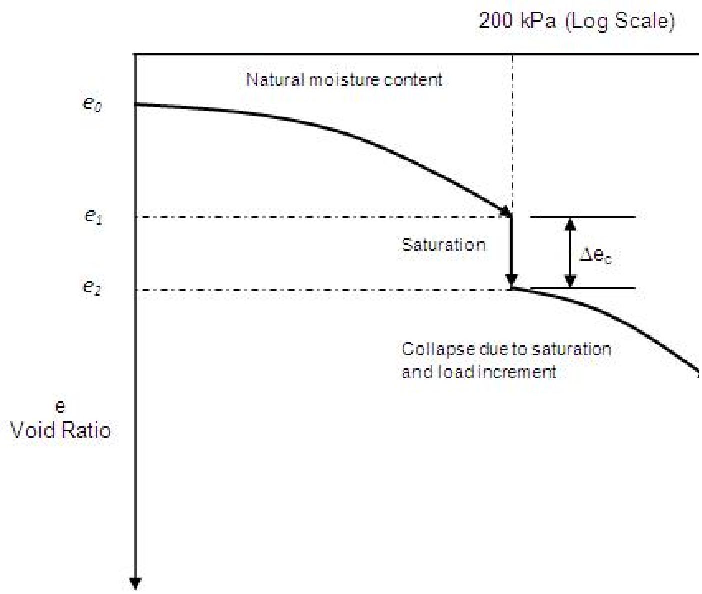

Fig. 1. Typical collapse potential in one dimensional consolidation test

Sabbagh(1982).
R, known as Gibb's parameter is obtained from the following relation:

$$R = \frac{\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}}{w_l} \qquad (2)$$

Where γ_d is the dry unit weight, w_l is the liquid limit moisture content, γ_w is the unit weight of water, and Gs is the specific gravity of soil solids

Denisov's (1964) criterion of collapse susceptibility is expressed as the ratio e/e_{LL}. If the ratio is greater than 1 then the soil is collapse susceptible.

e_{LL} is the void ratio at liquid limit and e is the void ratio at natural moisture content.

Beside the established criteria for soil collapse, there are other parameters that contribute to collapse phenomenon. These related parameters are: initial dry unit weight (γ_d), initial moisture content (w_0), initial void ratio (e_0), initial porosity (n_0), initial degree of saturation (s_0) and plastic limit (*PL*). Specific cut-off values for selected parameters are given below in Table 1

Table 1.Critical Values for and High Collapse (HC) Medium Collapse (MC) and Non-Collapsing(NC), Soil Parameters.

Param.	(HC)	(MC)	(NC)
C_p (%)	> 5	$2 < C_p \leq 5$	≤ 2
R	≥ 1.4	$1.0 \leq R < 1.4$	< 1.0
e_0	≥ 0.82	$0.67 \leq e_0 < 0.82$	< 0.67
γ_d,(kN/m^3)	≤ 14.3	$14.3 < \gamma_d \leq 15.6$	> 15.6

For this study field and laboratory test data were collected from local consulting engineers' offices and from the reports of previous researchers (e.g. Sabbagh, 1982). The raw data were reduced to obtain parameters in two categories: established criteria, such *as* C_p, R, and collapse-related soil parameters, such as, γ_d, w_0, e_0, n_0, s_0 and *PL*. Analysis performed on selected parameters are presented in this paper.

Modeling of variogram:

Modelling of variogram is the first and most important step in applying the technique of kriging, which is the method used here for obtaining unbiased estimate of parameters in un sampled location. A considerable amount of computation is necessary to obtain an adequate estimate of the variogram because of the empirical and subjective nature of the estimation process[3]. Parameters of interest with critical values are listed in Table 1. The various data sets containing number of data points of the parameters are listed in Table 2.

Table 2. Data sets used in the Analysis

Data set Numer	Range of depth, m	Number of Data
1	0.0-0.3	125
2	0.3-0.6	286
3	0.6-0.9	254
4	0.9-1.2	100
5	1.2-1.8	104
6	1.8-12.2	123
7	0.0-12.2	219

Representative variograms were obtained for each of the parameters in each of the seven data sets, but only few are presented here. Since modeling of a variogram is, in part, an art requiring some subjective judgment, multiple trials are usually necessary in order to obtain a satisfactory variogram. The important parameters for a variogram are the range of influence, a, and the sill C in a nested model as given below.

Nested Model:

$$\gamma(h) = C\left[\frac{3h}{2a} - \frac{1h^3}{2a^3}\right] + C_0 \quad for\ h < a \qquad (3)$$

$$= C + C_0 \qquad \text{for} \qquad h \geq a$$

and $$\gamma(0) = C_0 \qquad \text{for} \qquad h = 0$$

In geostatistical modeling, the most commonly used model is the nested models as shown in Figure 2. This model bears the same significance as the Normal distribution bears to statistics.

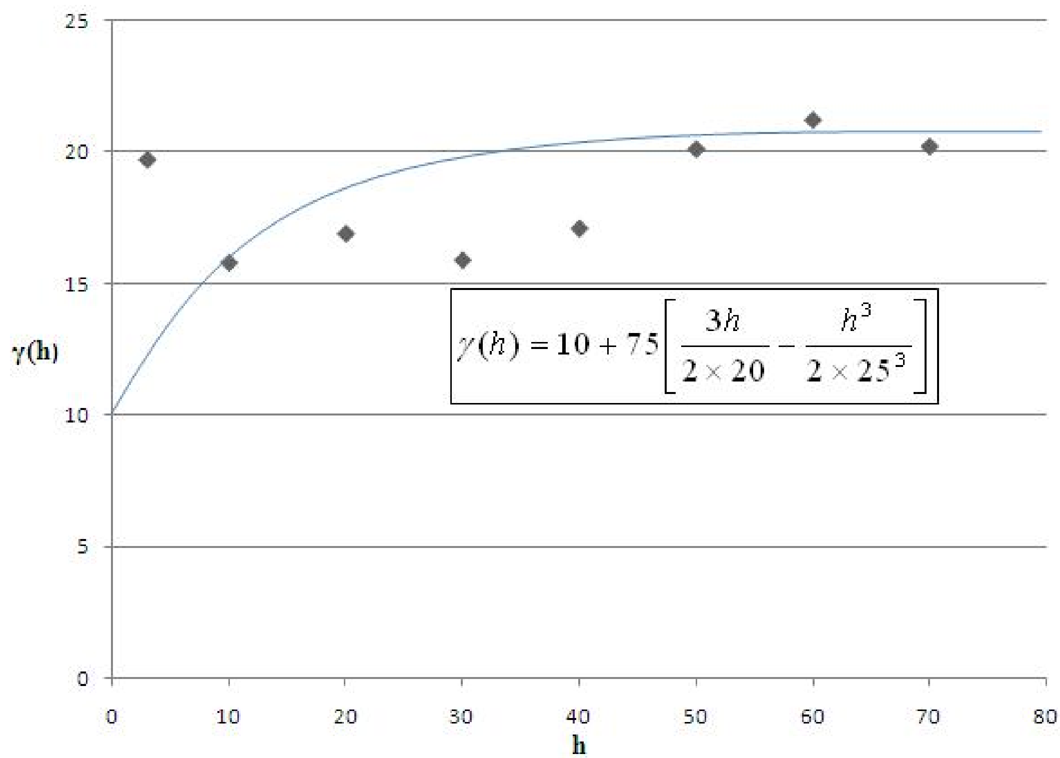

Fig.5. Semi-variogram and fitted equations for Cp of Data Set 5

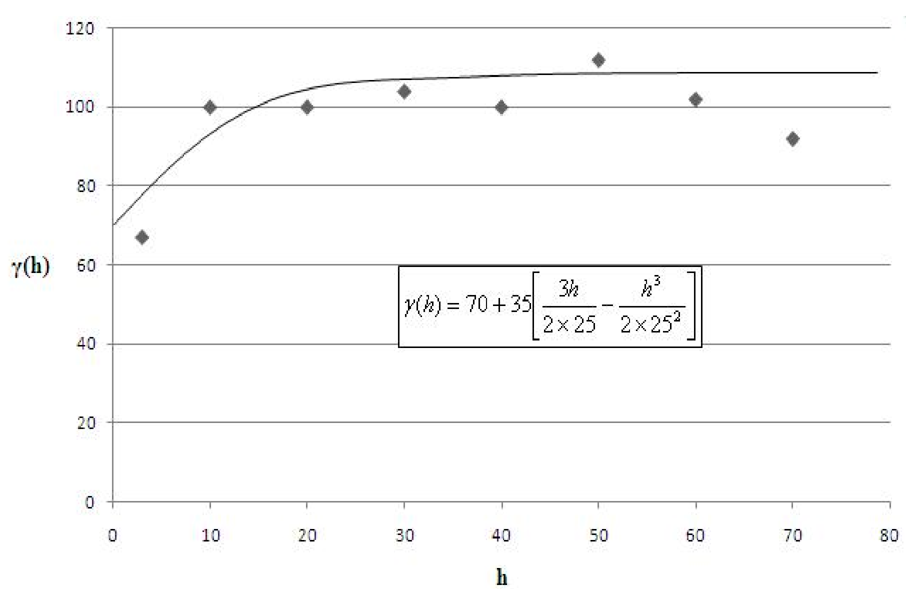

Fig.6. Semi-variogram and fitted equation for dry density of Data set 5.

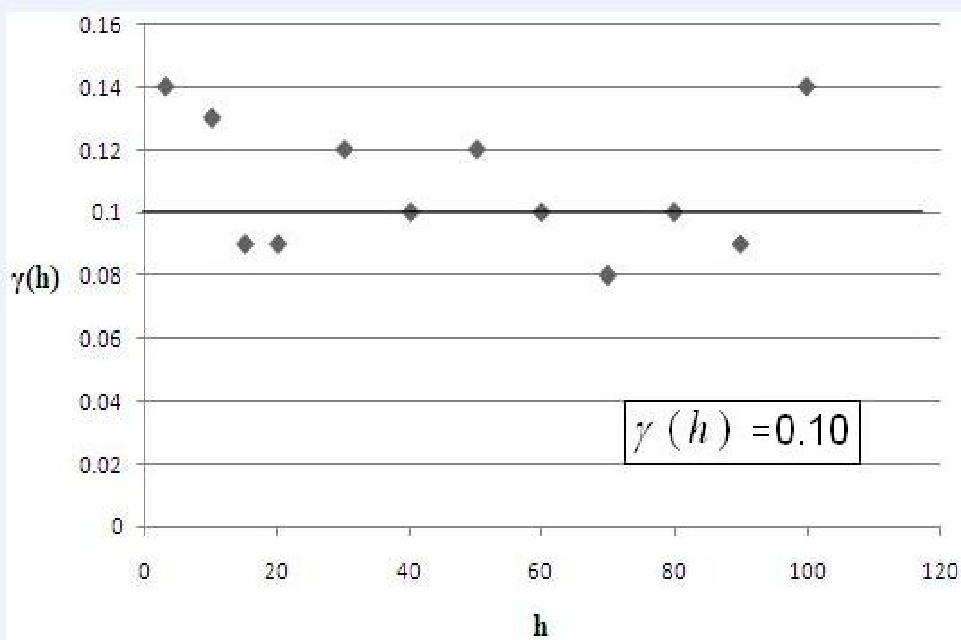

Fig.7. Semi-variogram and fitted pure nugget model for initial void ratio of Data set 6

Variograms were obtained for the analysis for all the parameters, however few of them are presented here. Figures 3 and 4 show the variograms for C_p and γ_d of data set 5, respectively. In several cases a pure "nugget effect" model was obtained, indicating a complete lack of geological structure, Figure 5, shows such a model for the parameter e_0 of data set 6.

Fitting a Model

Constructing a variogram is to find theoretical model that best fits the experimental variogram. The choice is often limited to linear or spherical models, with a spherical model being the most common as the parameters are estimated subjectively. However, it is important that the chosen theoretical model y(h) fits well to the experimental semi-variogram within the model's limits of reliability[5]. The choice of a theoretical model is generally made by examining the experimental variogram and taking in to account the fact that variograms are subject to significant fluctuations at large distances. Since most of the experimental variograms could be approximated by a spherical model, such a model was fitted to all the computed variograms in this study.

The key parameters of the selected spherical model, after cross-validation with different trial models for Data set 1 are presented in Table 3. The nugget value C_0 which is the estimate of y at h=0, provide an indication of short distance variation. The greater the value of C_0, the greater is the variance of the data set. Some of the model, e_0, for example shows a "pure nugget" effect indicating lack of spatial correlation.

Table 3. Parameters of Spherical Models fitted to Data Set 2.

Data Set	Parameter	Nugget C_0	Range, a	Sill C
2	C_p	18.5	-	-
	γ_d	12.5	35.0	100.0
	n_0	32.4	35.0	45.0
	s_0	102.0	30.0	148.0
	w_0	0.002	25.0	0.0025
	e_0	0.042	30	0.053

The range, a, of the variogram can be interpreted as the diameter of the zone of influence which represents the average maximum distance over which a soil property is spatially related. In our study this distance was found to be 5.5 to 8 miles which is large relative to the distance over which soils are usually sampled for laboratory tests for a particular project, This suggest that , geostatistical concept can be applied successfully to the study of geotechnical problems.

There are three methods of kriging: punctual, block, and universal. Puntual kriging, which provides estimates for values of a random variable at points where there is no drift, has two forms: simple kriging if the mean value of the variable is known, and ordinary kriging, if the mean value is not known. Drift is defined as a non-stationary expectation of a random function. Block kriging is used when an estimation of the spatial average is required over a volume or an area. Universal kriging is an optimal method of interpolation that applies in all cases where drift must be taken in to account because of lack of data to make stationary or quasi-stationary estimates.

In this study, variograms were estimated for each collapse criterion and collapse related soil parameters (Table 1) using a discrete number of values obtained from test data at incremental distances corresponding to sampling locations throughout the area. These variograms (Table 3) are then used in conjunction with ordinary kriging to estimate values of the parameters at un-sampled locations. Indicator kriging[6] was then utilized to produce contour plots of estimated probability and associated kriging variance for each parameter in each data set.

Results and discussion

Results of analyses showing probability contour of high collapse potential with estimation variance are shown are in Fig 6 (a) and 6(b). The shaded zones show areas where there is a 60%-80% probability of encountering collapse susceptible soil. The variance of estimation is seen to lie within a range of 0.5 and 0.6. Similar plots were developed for all of the other collapse criteria and collapse-related soil parameters for each of the seven data set, however, they are not presented here.

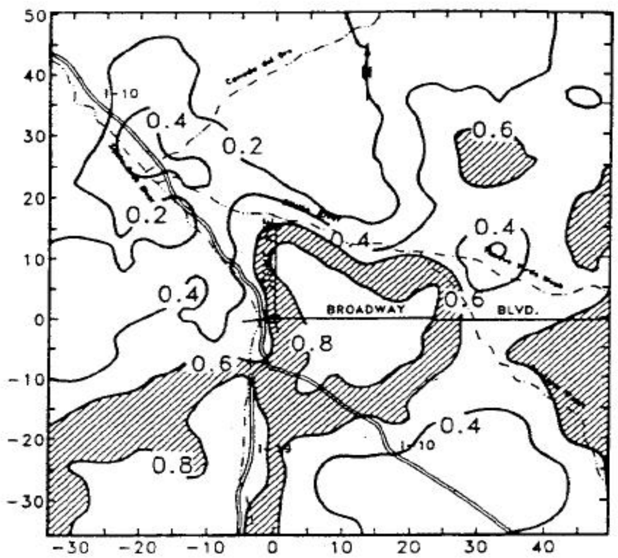

Fig. 6(a) Probability Contour showing areas of high collapse potential

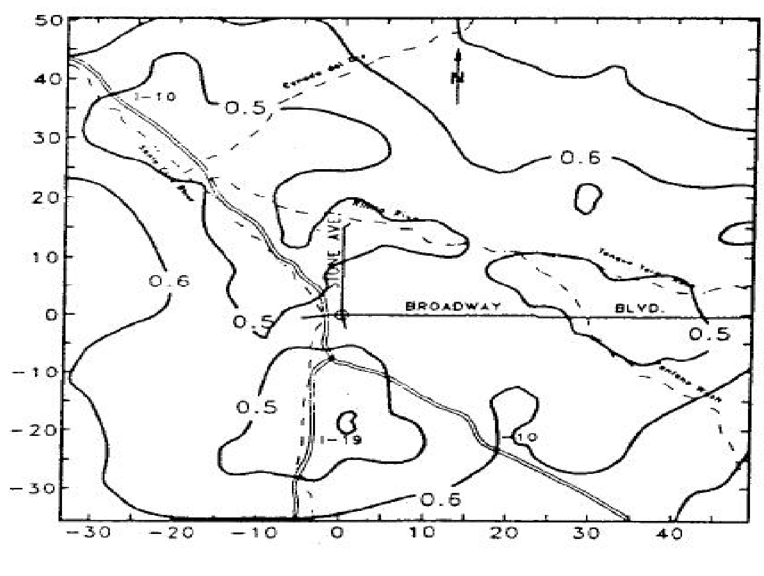

Fig. 6(b) Contour showing estimation variance of estimating probability of high collapse potential

Conclusions

The following conclusions are logically made from the foregoing discussions:

1. Collapsing soil parameters can be considered as regionalized variable and the concept of geostatistics is applicable.

2. Linear estimation method of kiriging was found to be a valuable tool for characterizing and modeling the spatial variability of geotechnical parameters.
3. The parameters under investigation can be best be fitted by a spherical model variogram
4. Simple Kriging methods can be used to develop probability contour with estimation variance for selected soil parameters at. This information is extremely valuable to planners and Government officials.

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