Full Paper: Hands-On Laboratory Exercises for Engineering Applications of Mathematics Course

In Fall 2007, the First-Year Engineering Program (FEP) was started with the intent of increasing student retention and success. One of the main hindrances to retention at a public university engineering program with open enrollment is the unpreparedness of students for rigorous curriculum requirements of the first year. In an effort to help first year engineering students who are one or two semesters behind Calculus I, FEP offers Engineering Applications of Mathematics (E-Math) course, which was inspired by the Wright State model for Engineering Mathematics Education [1]. E-Math aims to teach College Algebra, PreCalculus, and introductory Calculus I concepts using self-paced lectures focused on engineering applications and supported by hands-on laboratory exercises. FEP gained permission from the Department of Mathematical Sciences to have a “C” or better in the E-Math course count as a prerequisite to Calculus I [2].

All math concepts covered in E-Math course are paired up with engineering application problems and in-class activities. Majority of the in-class activities are hands-on labs that support the mathematical concepts being learned in class by relating what they learn to real-life situations. For all hands-on labs, students work in teams of two or three. They are excepted to answer the questions “What math concepts do we know that might be applicable?” and “What materials and information do we need to figure out the answers to these questions and/or tasks?”. At the conclusion of the labs, teams are required to turn in a lab write-up to convey the answer to these questions along with explaining their objective, methods used, calculations and main conclusions. These lab write-ups increase the benefits of teamwork and aim to develop students’ scientific communication skills.

This paper highlights some of our hands-on labs. Due to limited space, we are not able to discuss all lab activities developed for the course or go into all the details of the set-up calculations, but we give an overview of how we pair these activities with mathematical concepts. We then look at the end of semester results to explore if students’ problem-solving skills, communication skills, and ability to work in teams have improved.

Linear Functions: One-Loop Circuits Lab

To explore the concepts of linear functions, students are introduced to basic fundamentals laws of electricity and are asked to make measurements to find the resistance value of a resistor with “unknown” value. In order to accomplish this, students construct a circuit with their unknown resistor using a breadboard (*Figure 1*), then measure voltage and current values across this unknown resistor with a Multimeter for three different sources of voltage: a AA battery (1.5V), a 9V battery and Arduino Uno microcontroller used as a 3.3V and 5V voltage source. Students then utilize Microsoft Excel to create a scatter plot of current vs voltage, add a linear trendline to their plot, and use the equation of the trendline along with the concept of Ohm’s Law to determine the value of their unknown resistor.

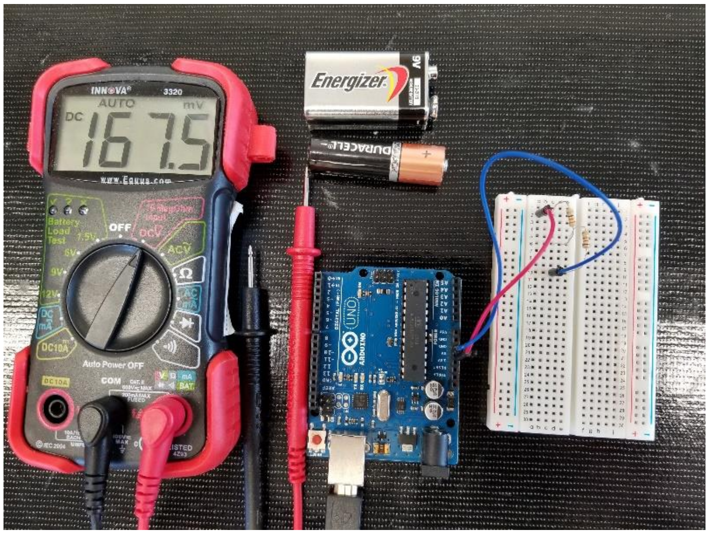

Figure 1: Equipment used and set-up for One-Loop Circuits Lab

Quadratic Functions and Derivatives: Falling Ball Lab

To explore the concepts of quadratic functions, students study one of the most common applications, which is a falling ball. For this lab, students utilize materials from a Lego EV3 kit including an ultrasonic sensor (to measure height at different times) along with a touch sensor and motor to automate the start. The complete apparatus can be seen in *Figure 2*. The most visible part of the apparatus is a six-foot pvc tube with sufficient diameter to allow the sensor to set easily in the bottom and allow the ball (a standard tennis ball) to fall. The box on the side of the tube is the EV3 "Brain", which runs the software including activating the motor to initiate the release, simultaneously starting the ultrasonic reading, and storing the results from the Ultrasonic sensor. The apparatus running up the side to the top is for dropping the ball when a touch switch is activated. The use of EV3 was chosen because the materials were available from other FEP activities.

Figure 2: Falling Ball lab set-up

Before starting the experiment, students measure the distance the ball will fall and use the theoretical equation to predict how long it will take the ball to fall. Students then modify the program given to them in Lego Mindstorm EV3 software to set the length of time the ultrasonic sensor measures to be more than the expected time. To collect data, students place the ball on the moveable arm atop the tube, use the touch sensor to activate the ball drop and start the measurements. The drop takes less than a second as the EV3 program stores the distance readings. Students repeat the drop 3-5 times to be sure they have at least one good curve. Students use EV3 software to analyze the data and use curve fit to approximate acceleration due to gravity and add the theoretical curve to their graph (*Figure 3*). Students also perform percent error calculations using their theoretical and experimental results.

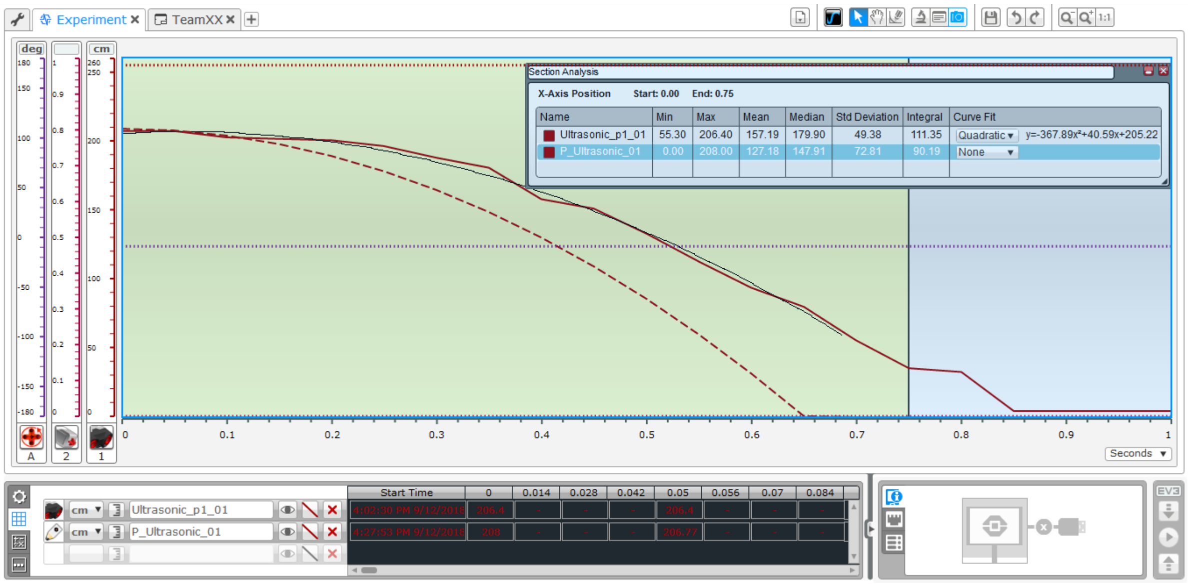

Figure 3:Height vs time graph in Lego Mindstorm EV3 software for falling ball lab

Towards the end of semester, to explore the idea behind finding a derivative as a slope, students again utilize the falling ball apparatus used for the quadratics lab. The same setup is used, but students alter the settings of the program in Lego Mindstorm EV3 software to take a different frequency of measurements. The expectation is that with more precise measurements, students will be able to find a curve that fits closer to the theoretical value.

Like the quadratics lab, students start by measuring the distance the ball will fall and creating a theoretical equation. Students then use the EV3 falling ball apparatus to collect data for ultrasonic sensor settings of 5 readings per second, 10 readings per second, and 20 readings per second. Students finish analyzing their data in Excel Microsoft Excel template. They use a table to calculate acceleration due to gravity in three ways. The first is to plot height vs. time, use a quadratic curve fit, and multiply the leading coefficient by 2. The second way is to calculate an approximate velocity at each time by using $v = \Delta h/\Delta t$. These velocities are then plotted vs time with acceleration being the slope of this curve. The final way is to use the approximated velocities to approximate the instantaneous acceleration $a = \Delta v/\Delta t$. Acceleration due to gravity is then approximated as an average of these instantaneous accelerations. After calculating the acceleration due to gravity these three ways for each of three settings, the error for each is calculated. These errors are often high, but we expect the accuracy to increase as the frequency of measurements increases.

Systems of Linear Equations: The Penny-Wise Lab

In this lab, student teams utilize systems of linear equations to determine the number of pennies in a sealed container that are minted before and after 1982. At the beginning of the lab, students are provided with the background that the U.S. Mint has changed the compositions of metals that comprise a penny in 1982. Then, students are given a sealed container that contains pennies - some of which were minted before 1982 and some after (*Figure 4*). The task for their team is to develop a method to determine the number of pre-1982 pennies and post-1982 pennies that are in a container without removing them. Students must determine that they need to develop two equations: (1) How many pennies are in the container? and (2) How much do the pennies weigh?

Students can count the number of pennies, N, in the container. They are given a scale to measure the values needed to set up the equations. To develop the equations, students must determine that they need to know the weights of the sealed container (P), weight of an empty container (C), weight of a pre-1982 penny (3.11 g), and weight of a post-1982 penny (2.5 g). If the number of pre-1982 pennies is X and the number of post-1982 pennies is Y the following equations can be used to solve for X and Y.

$$X + Y = N \ (Eq.1)$$

$$3.11X + 2.5Y + C = P \ (Eq.2)$$

Figure 4:Penny-wise lab

Right Triangle Trigonometry: Heights of Trees Lab

In this lab, student teams apply right triangle trigonometry to estimate the height of selected trees on campus. Students are given the following scenario: “The School of Forestry and Natural Resources at University of Arkansas- Monticello needs a device that can be used to measure the height of various trees for their research without the need to climb them. This device must work in all kinds of weather conditions (sunny or cloudy). All the options for sale are too expensive for their budget. They need your team to design a tool for the needs specified above. Please include a manual, with pictures, on how to use the device as well as how to perform any needed calculations. Make sure the manual is clear and concise as well as organized and accurate.”

Students are provided with several materials, which typically includes a protractor, string and measuring tape. Three known heights are marked in the classroom, and students must prove that their “measuring device” is accurate before measuring tree heights. After they check the accuracy, they then spend time outside using their device to measure the height of three pre-selected trees on campus. Students can calculate height by measuring distance between themselves and the tree (D), the angle to the top of the tree (a), the angle to the bottom of the tree (b), and then calculating:

- A = D Tan a
- B = D Tan b
- Height = A + B

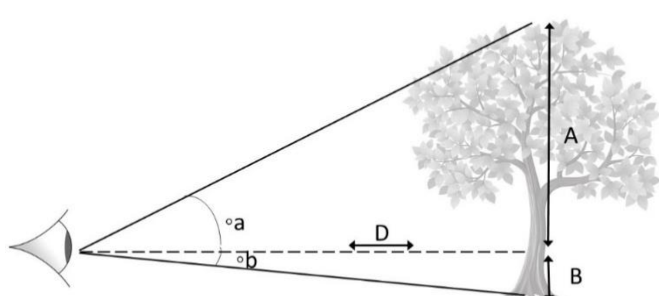

Figure 5: Illustration for calculating the height of a tree

Two Dimensional Vectors: Orienteering Lab

In this lab, we utilize University of Arkansas' long-lasting tradition to etch the names of graduates on the walkways around university campus. More than 170,000 graduates are listed in walkways and sidewalks around the campus, called Senior Walk [3].

In first part of this hands-on lab, student teams are asked create a short orienteering course using vectors that guides another team to a particular name on the senior walk. This course must have a well-defined starting point and reference axes. The course also must have at least five locations: a starting point, three intermediate locations that are on sidewalks or easy to spot landmarks, and a final location. Each set of instructions should include a vector in polar form (that is, distance and bearing, $l\measuredangle\theta$, i.e 25ft $\measuredangle$ 42°). Students are also expected to draw their vectors on the map and calculate the resultant of their vectors to check accuracy (*Figure 6*). Students practice their orienteering skills first by understanding a path created in the classroom before going out to senior walk for their measurements and calculations.

In the second part of the lab, index cards from teams are distributed to other teams. Let's assume Team 1 gets the index card for Team 2: Team 1 uses the starting location and vectors given on Team 2's index card to find the particular name on senior walk that was originally assigned to Team 2. Team 1 takes a picture at their final location, and both Team 1 and 2 are given bonus points if the name assigned to Team 2 can be seen in this picture.

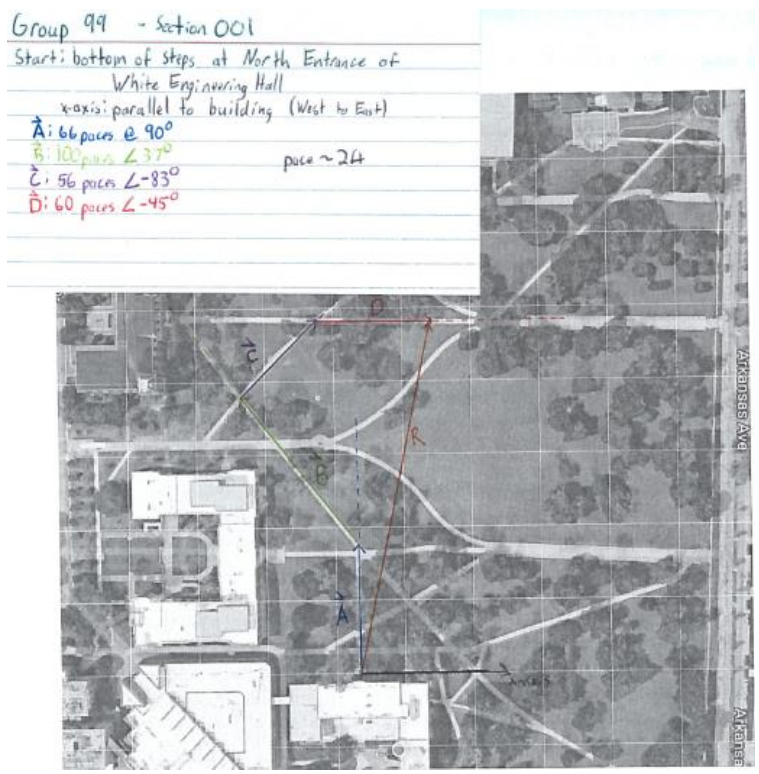

Figure 6: Example of a completed map and index card for Orienteering Lab

Conclusions

In this paper, we summarized the hands-on lab activities that support the mathematical concepts in E-Math course. Most of our labs were developed utilizing the equipment that were readily available to us in our department, but we hope that the ideas would be easy to modify for those who want to adopt them for their classroom. We found that it was very beneficial to incorporate lab write-ups as a requirement. Lab write-ups opened the opportunity for students to have discussions of their results, furthered their understanding of the concept (since they had to explain what they did clearly), and served as a good technical writing practice.

While we have observed that students appreciate being able to see engineering problem solving applications of the mathematical concepts, we did not ask the students to give feedback for each hands-on lab separately. Instead, we analyzed some of the questions from a survey given to all students to get a general idea of students' perception of these labs and other engineering oriented assignments embedded in E-Math. At the end of each semester, students are asked to complete an end-of-semester survey as their last assignment, which counts toward their grade. As a part of this survey, students rate certain aspects of the projects and course using a 5-point Likert scale and are allowed to leave additional comments and suggestions for improvement. We analyzed the end-of-semester survey results for the following five questions:

The assignments associated with this course:

1) improved my engineering problem-solving skills.
2) improved my ability to communicate solutions to engineering problems.
3) provided me with a meaningful experience working on a diverse team.
4) helped me appreciate the multi-disciplinary nature of engineering.
5) helped me appreciate the role of engineering in modern society.

Figure 7 shows the results for the 123 students who responded to the survey over 5 semesters (from Fall 2016 to Fall 2018). Mean scores, shown in parenthesis on x-axis, are calculated assuming a point-scale where strongly disagree = 1, disagree = 2, neither agree or disagree = 3, agree = 4, strongly agree = 5. The mean scores ranged from 3.89 to 4.07 and the total percentages for "agree" and "strongly agree" ranged from 72% to 81%. This is a good indication that the students benefited from the engineering applications embedded with math concepts.

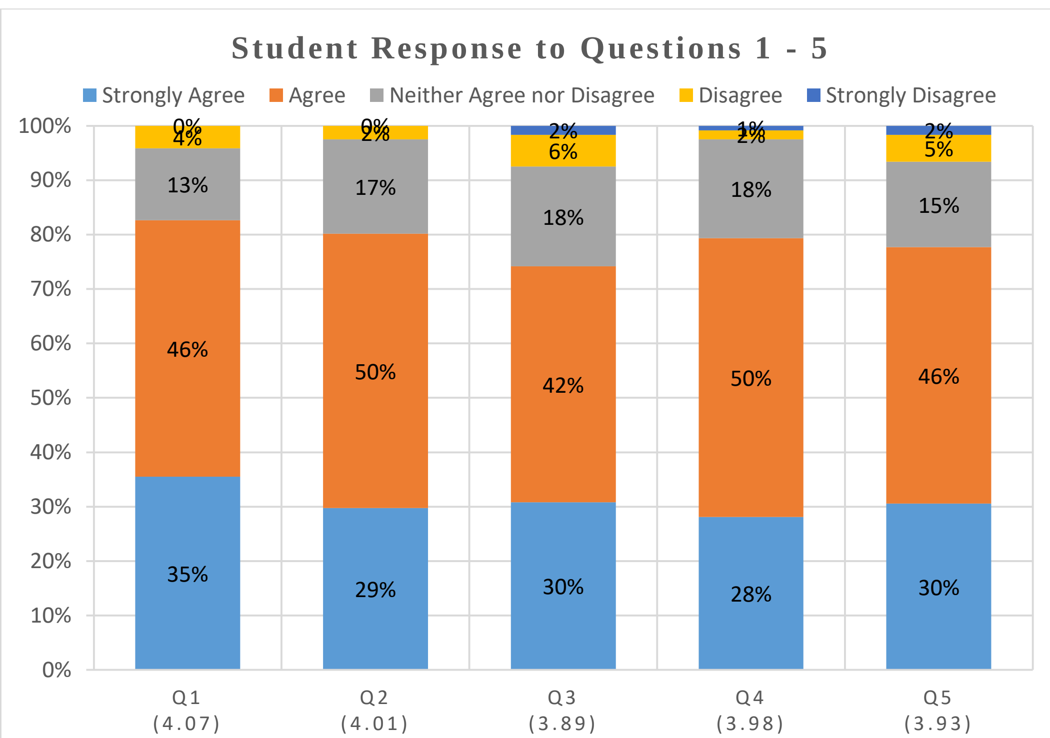

Figure 7: Percentages of responses to End of Semester Survey Questions 1 to 5 in 5-Point Likert scale with the mean for each question given in parenthesis on x-axis under question number. The sample size for all questions is 123 students.

References

1. https://engineering-computer-science.wright.edu/research/the-wright-state-model-for-engineering-mathematics-education
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