Hands-on Learning in Engineering Mechanics using Layered Beam Design

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I. Introduction

A sophomore level Engineering Mechanics project is presented that uses design and construction to reinforce student learning of beam deflection and flexural shear strain. The project requires the student to design, to build, and to test a layered beam that minimizes cost yet provides specific in-plane and out-of-plane stiffness. Each student is presented with an inventory of available layers for their design. Only specific thickness, width, and material combinations are offered. The number of each "type" of layer is also limited to further constrain the design. The students are thus faced with a very real design problem to determine the optimum solution using only available components. After the teams have completed their designs, the beams are built and tested to verify performance. Inconsistencies between theory and reality are routinely found during testing that seed discussion and student learning. Details of the project and advice from two implementations are presented.

II. Project Assignment

The students are given a simple and direct problem statement: "Design a layered beam that satisfies customer specified flexural stiffness criteria and minimizes expense using a finite list of available component layers". The beam may be either simply supported or cantilevered; each student team being assigned different requirements at random (Figure 1).

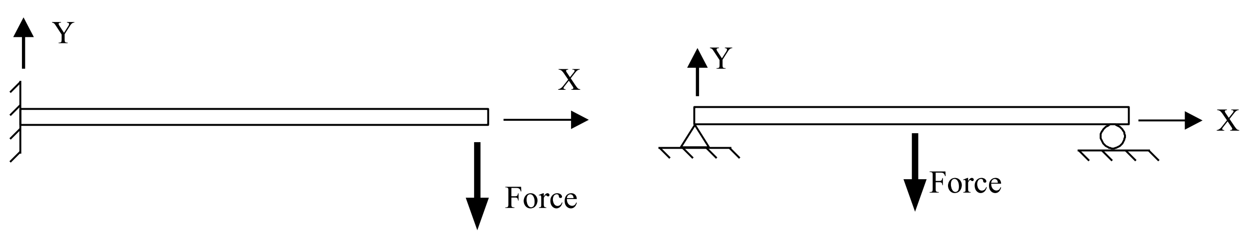

Figure 1: Beam configurations, randomly assigned to each student designer.

The students are next provided a list of layers available for their design. The layers typically consist of 36 inch long strips of material cut to specified widths (1/2", 3/4", 1", 1 1/4", 1 1/2")

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and thicknesses (3/32", 1/8", 3/16"). To limit the design options to a manageable number, each beam is required to have five layers, to have a uniform cross-section over the entire length, and to have a symmetric cross-section with respect to horizontal and vertical bending. Even with these constraints, the students are capable of forming many different cross-sections. Example cross-sections are shown in Figure 2.

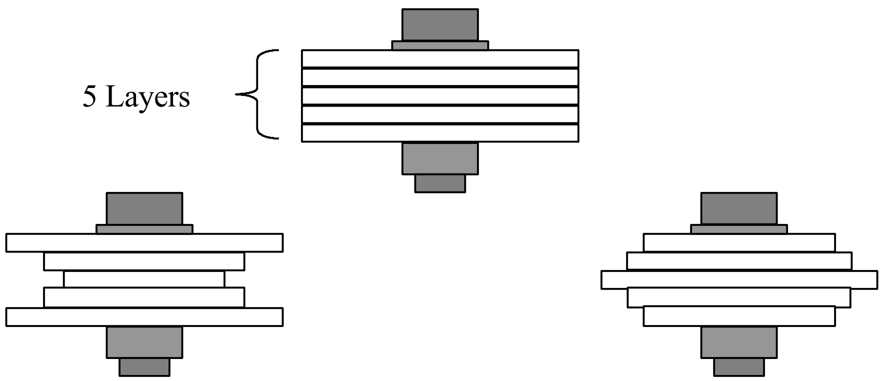

Figure 2: Example cross-sections to satisfy the design objectives. In the diagrams shown, the layers are fastened together at discrete locations using bolts. Adhesives may also be used to fasten the layers together.

In addition to the geometric combinations, the layers are also made available in multiple materials. The author has used two different strategies regarding available materials during the past two years of teaching Engineering Mechanics. The first strategy is to provide the students with reusable layers made of aluminum, steel, and brass. The layers are prefabricated with holes at discrete locations along their lengths. The students simply "stack" their design and fasten the layers together tightly using bolted connections (shown in Figure 2). The second strategy is to provide the students with consumable layers made of wood, foam, and plastic. These layers are sufficiently inexpensive to allow the students to epoxy the layers together for improved load sharing in shear. Other material options are certainly possible.

Based only on these limited supplies, the students are faced with the daunting task of designing the best beam for their assigned specifications. The reader should note that the potential combinations are many, as shown below, even given the fact that symmetry restricts the design. If fewer potential combinations are desired, making certain materials available in fewer sizes effectively limits the options.

$$\text{Combinations} = (3\ \text{Materials x } 5\ \text{Widths x } 3\ \text{Thicknesses})^{3\ \text{Unique Layers}}$$
$$\text{Combinations} = 91{,}125\ \text{cross-sections}$$

Finally, each student team is assigned specific in-plane (X-Y plane in Figure 1) and out-of-plane flexural stiffness values that its beam must satisfy. Some care is taken to select target stiffness

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values that are either impossible to perfectly satisfy using the available beam combinations or are only satisfied by an expensive beam design. As such the students will be required to justify an optimization metric to balance the error in stiffness verses reduced cost.

III. Required Analyses

Each student is individually (not in teams) required to develop a simulation code that will determine the in-plane stiffness, out-of-plane stiffness, and construction cost for a 5-layered, uniform, symmetric beam. Successful development will include the following steps [1,2].

1. Specify the geometry and material of each layer

2. For the in-plane response …

   a. Adjust the layer materials to a single uniform material, making appropriate geometry changes to preserve the flexural stiffness for in-plane bending

   b. Utilize the adjusted geometry to determine the 2nd Area Moment for the cross-section (in-plane bending)

   c. Derive and compute the in-plane flexural stiffness of the beam based on the assigned boundary conditions

3. For the out-of-plane response …

   a. Re-adjust the layer materials to a single uniform material, making appropriate geometry changes to preserve the flexural stiffness for out-of-plane bending

   b. Utilize the adjusted geometry to determine the 2nd Area Moment for the cross-section (out-of-plane bending)

   c. Derive and compute the out-of-plane flexural stiffness of the beam based on the assigned boundary conditions

4. Compute the construction cost of the beam using the original geometry (non-adjusted)

At this initial stage each student's simulation code analyzes a single beam only. Students are next asked to work in teams to extend one of their simulation codes into an iterative optimization routine. This step requires the team to recall its training in structured programming and to develop an optimization metric for the design. Perhaps the simplest metric is a weighted summation of the cost and stiffness errors (in-plane and out-of-plane). Yet even this option requires the team to carefully consider how to weight items with different units and ranges. Figure 3 overviews one such optimization routine, written in Mathcad [3].

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LAYERED BEAM SOLVER
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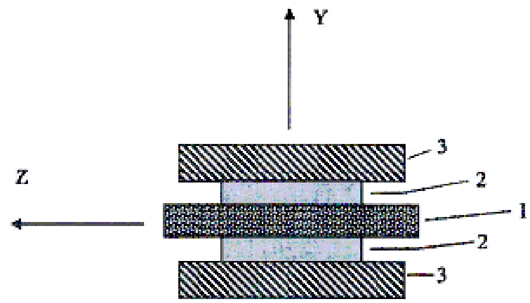
ENGINEERING MECHANICS

This program performs the optimization of a layered beam. The beam is considered to have five (5) layers and to be symetric. Available layer parameters are set by the user. Consistent units are assumed. In the formulation, b denotes the width of the layer, h denotes the thickness, K1 denotes the in-plane stiffness and K2 denotes the out-of-plane stiffness.



First, general functions that determine the stiffness and cost of a beam based on the individual layers are defined. Note that the beam is assumed to have 5 layers and to be symmetric. I1 defines the inplane area moment, I2 defines the out-of-plane area moment, K1 denotes the inplane stiffness, and K2 the out-of-plane stiffness. Note also that the layered beam has been converted to a single material with unity elastic modulus within the I1 and I2 equations.

$$I1(b,h,E) := \frac{1}{12}\,b_1\cdot E_1\cdot\left(h_1\right)^3 + \frac{2}{12}\cdot\left[b_2\cdot E_2\cdot\left(h_2\right)^3 + b_3\cdot E_3\cdot\left(h_3\right)^3\right] + 2\cdot\left[b_2\cdot E_2\cdot h_2\cdot\left(\frac{h_1+h_2}{2}\right)^2 + b_3\cdot E_3\cdot h_3\cdot\left(\frac{h_1+2\cdot h_2+h_3}{2}\right)^2\right]$$

$$I2(b,h,E) := \frac{1}{12}\left(b_1\right)^3\cdot E_1\cdot\left(h_1\right) + \frac{2}{12}\left[\left(b_2\right)^3\cdot E_2\cdot\left(h_2\right) + \left(b_3\right)^3\cdot E_3\cdot\left(h_3\right)\right] \qquad K1(b,h,E,L) := \frac{48\cdot I1(b,h,E)}{L^3}$$

$$Cost(b,h,Val,L) := L\cdot\left[\left(b_1\cdot h_1\cdot Val_1\right) + 2\cdot\left(b_2\cdot h_2\cdot Val_2\right) + 2\cdot\left(b_3\cdot h_3\cdot Val_3\right)\right] \qquad K2(b,h,E,L) := \frac{48\cdot I2(b,h,E)}{L^3}$$

Next, the available layer widths (bv), thicknesses (hv), material elastic moduli (Ev), and material cost/volume values are set. The materials declared below are representative of balsa wood and foam.

$$bv := \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{5}{4} \end{pmatrix}^T \qquad hv := \begin{pmatrix} \frac{1}{16} & \frac{3}{32} & \frac{1}{8} & \frac{3}{16} \end{pmatrix}^T \qquad Ev := (1000000 \;\; 150000)^T \qquad Cv := (0.16 \;\; 0.02)^T$$

The optimization is performed by evaluating all the available beam combinations (the optimization code which implements this looping, called "Opt" below, is not shown due to space limitations). The "Opt" function searches for the "best" beam to satisfy the K1 goal (K1g) and the K2 goal (K2g) given wieghting factors for the in-plane stiffness accuracy, the out-of-plane stiffness accuracy, and the cost reduction goal. The first example run below demonstrate the process for a beam with desired inplane stiffness of 14 lbs/in and out-of-plane stiffness of 20 lbs/in given 40% weighting on each stiffness goal and 20% weighting on cost minimization. The second example is for a beam with desired inplane stiffness of 15 lbs/in and out-of-plane stiffness of 18 lbs/in given 30% weighting on each stiffness goal and 40% weighting on cost minimization.

Solution Key

$$Opt(14,20,0.4,0.4,0.3) = \begin{pmatrix} 0.25 & 1.25 & 0.75 \\ 0.125 & 0.188 & 0.063 \\ 1000000 & 150000 & 1000000 \\ 13.784 & 20.087 & 0.94 \end{pmatrix} \quad \Longleftarrow \quad \begin{pmatrix} Width1(in) & Width2(in) & Width3(in) \\ Thick1(in) & Thick2(in) & Thick3(in) \\ E1(psi) & E2(psi) & E3(psi) \\ K1\left(\frac{lb}{in}\right) & K2\left(\frac{lb}{in}\right) & Cost \end{pmatrix}$$

$$Opt(15,18,0.3,0.3,0.4) = \begin{pmatrix} 1 & 1 & 1 \\ 0.188 & 0.188 & 0.188 \\ 150000 & 150000 & 150000 \\ 15.087 & 17.166 & 0.6 \end{pmatrix} \quad \Longleftarrow \quad \begin{pmatrix} Width1(in) & Width2(in) & Width3(in) \\ Thick1(in) & Thick2(in) & Thick3(in) \\ E1(psi) & E2(psi) & E3(psi) \\ K1\left(\frac{lb}{in}\right) & K2\left(\frac{lb}{in}\right) & Cost \end{pmatrix}$$

Figure 3: Mathcad implementation of the layered beam optimization routine. The optimization function "OPT" is not shown due to space constraints.

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IV. Prototype Testing

Once each team has successfully optimized its design, the beams are fabricated. The nature of the fabrication will vary with the materials used, but is ordinarily simple for the students to perform without the assistance of shop facilities. The students are required to perform all fabrication work outside of class. This requirement limits the in-class testing to a single period.

The beams are tested during a selected class period to determine performance relative to the assigned specifications. The testing is performed using known weights to load the beam. By applying incremental loading and manually measuring the beam deflection using a dial micrometer, a load vs. deflection plot is generated for the in-plane beam response. After the in-plane testing is complete, the beam is rotated 90° and the out-of-plane deflection is tested in the exact same fashion. The slope of each force vs. displacement curve provides the required experimental stiffness values. Routinely, the beams are found to not perform as predicted, providing the students valuable insight into the limitations of their analyses.

Insufficient load sharing between the layers of the beam is one common source of error. The linear beam theory used during design assumes complete shear bonding between the layers. This condition is very difficult to accomplish during fabrication. As such, the students are made powerfully aware that models do not always represent reality. The author has found that this simple demonstration impresses this truth on the students far more than lectures.

Inaccuracies in the assumed material property values are a second common source of error. The properties of consumable supplies (such as wood, plastic, and foam) are difficult to accurately define using textbooks alone. The students are therefore encouraged to experimentally determine the required material properties using individual layers of each available material. Furthermore the teams are asked to consult amongst themselves to workout discrepancies due to different, and often non-ideal, testing methods.

V. Educational Strengths of the Project

Having performed this design project with two Engineering Mechanics classes over the last two years, the author proposes the following advantages associated with this education tool.

1. Asking the students to individually implement the analysis within a computational environment requires they first solidify their understanding of the analysis process. Routinely students find programming the calculations far more difficult than working a simplified problem by hand.

2. The design specifications, combined with the large number of potential configurations, force the students to think about what it means to “optimize”. For many students, this is their first confrontation with a problem that does not have a unique answer. The students are graded based on the merits of their optimization method, not upon their ability to match the instructor’s answer.

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3. The students find the project far more interesting when they recognize that they must demonstrate their design by building it and testing it. The work becomes real; not just another homework exercise.

4. Testing the beam clearly reveals the limitations of simulations and analyses. Students are often too willing to accept simple, linear theory as the absolute truth. Any opportunity to teach a healthy measure of respect for the complexity of real structural response is invaluable.

VI. Conclusions

A novel, layered beam design project, suitable for implementation at the sophomore level in Engineering Mechanics, has been presented. The project offers an integrated opportunity for the students to combine their knowledge of linear beam theory, structured programming, optimization, and experimental testing to reinforce learning. Past executions of this project have demonstrated its ability to communicate the strengths and weaknesses of analytical techniques relative to the actual performance of a structure. Details for implementation are presented.

VII. Bibliography

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VIII. Biographical Information

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Dr. Byron L. Newberry is an Assistant Professor of Mechanical Engineering at Oklahoma Christian University. He holds B.S., M.S., and Ph.D. degrees in Mechanical Engineering (advanced degrees from the University of Michigan). His areas of interest include structural analysis, thermal stress, linear and nonlinear oscillations, and engineering design.

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