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Heaving Homemade Buoys: A Project Leveraging Smart Phone Movies and MATLAB-Basd Image Processing to Teach Dimensional Analysis in an Undergraduate Fluid Mechanics Course

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Introduction

The COVID-19 pandemic, and the required transition to virtual instruction for many, has taught educators that they must be flexible in their mode of content delivery. While this can be a challenge for many subjects, engineering courses that traditionally benefit from hands-on activities can be especially stressed if the infrastructure to provide those experiences is based on the use of experiments and demonstration apparatus which are often large and expensive pieces of equipment. In these instances, creating hands-on experiences to support virtual learning is a problem. There are reports of some cases during the pandemic where universities were able to ship lab kits to students (e.g., circuit kits for EE classes); however, this approach may be impractical for schools with limited funding and when the nature of the engineering subject matter prohibits the use of a kit. If only we could ship full-scale wind tunnels to students, right?

So what options are there for students in traditional mechanical engineering courses to benefit from meaningful hands-on activities that can be completed at home? How could these experiences make use of materials already found around the home and how could useful data be collected? Can the data be shared, or crowd-sourced, in such a way that the student community benefits from a diversity of experiments? Regarding data collection, given the ubiquity and power of an ordinary smartphone – packed with various sensors and cameras - this seems like a natural choice. The literature contains examples of the use of smart phones and tablets for their sensors and imaging capabilities in activities to support teaching of coupled oscillators [1], projectile motion [2], and rolling motion [3], as examples. The authors of this paper were motivated to explore the use of smartphone-gathered data from at-home experiments in the context of an undergraduate engineering fluid mechanics course. But how to do this and for what topic within that class?

The subject of dimensional analysis and similitude is introduced in undergraduate engineering fluid mechanics courses as a way of dealing with the often-complicated relationships between flow variables and geometric parameters [4]. Dimensional analysis, using what is known as the Buckingham-Pi Theorem (to be explained later), allows a researcher to reduce the number of variables of study in a phenomenon by creating relevant dimensionless groups. These dimensionless groups then lead to a smaller number of experiments which are necessary to understand a flow phenomenon [5]. The power and utility of dimensional analysis is most easily observed when one has data to work with and can view and contrast that data in both a dimensional form and in a dimensionless form. But quite often this data is neither provided in textbook-style problems nor can it be collected readily via experiment in a lecture course. Thus, experiments that can be performed at home and data which can be crowd-sourced to create a large set, are great ways to highlight the utility of dimensional analysis as a tool in the study of fluid dynamics.

Project Description

Given the interest in creating a fluid mechanics problem that could be investigated via experiments performed at home and with minimal materials, more complicated flow phenomena were out of the question. As an example, dimensional analysis is used to help guide aerodynamic studies and results are presented in the form of relationships between dimensionless drag (i.e., a drag coefficient) and a collection of variables that relate fluid properties, geometry, and object speed (aka the Reynolds number). But students at home cannot easily measure drag forces (or infer them for a wide range of objects and velocities). As another example, the results of deformation of droplets as they impact surfaces can be presented in a universal way using dimensionless numbers (dimensionless deformation and Weber number) [6]; however, this phenomenon requires equipment beyond the means of even a typical undergraduate teaching laboratory! A phenomenon which is accessible with limited materials, and which can serve as a dimensional analysis example, is that of the motion of a heaving buoy. In addition, there is a simple analytical solution for the motion that can also be employed to support the experimental findings. This forms the basis of the project presented in this work.

The complete project that is described in this paper comprises a problem statement, instructions for student experiments (including analysis using the `MATLAB` code), and a presentation of the results of those experiments and the results of a dimensional analysis study. The authors feel that the results are best presented to the class in the form of a guided discussion. The portion of the project involving the problem statement and student experiments can be offered as an out-of-class assignment or, with the appropriate resources and time, can be completed in-class. Our overall objective is to provide the details necessary for any instructor to facilitate this project in order to enhance student learning of dimensional analysis in their engineering fluid mechanics course.

The Problem Statement

The inspiration for the problem statement comes from a typical dimensional analysis homework problem that is often posed in undergraduate fluid mechanics textbooks. An example of such a problem statement is as follows [7]:

> *"A spar buoy has a period T of vertical (heave) oscillation that depends on the waterline cross-sectional area A, buoy mass M, and fluid specific weight γ. How does the period change due to doubling of (a) the mass, and (b) the area?"*

While this is a perfectly acceptable problem statement for a homework assignment, we can take something like this and modify it to start as an inquiry statement, driven by a common observation, and ask students to think about the fundamental physics in order to come up with the relevant variables themselves. We suggest starting with the following:

> Most of us have at some time or another likely observed an object bobbing in an otherwise undisturbed body of liquid (e.g., a toy in a bathtub, or an object in a swimming pool). Have we ever considered how certain variables might characterize and determine that motion? Let us think about the frequency of the vertical oscillation of that object, often called the 'heave', and consider what characteristics of the object (referred to here after as the 'buoy') might be related. After discussion, we will create a list.

It takes very little time for students to recognize that the mass and the shape of the buoy play an important role in setting the heave frequency. With a little directed questioning, it is then understood that the cross-sectional area (which is related to the overall shape) of the buoy is important. The original inquiry generally makes the students think only of objects in water, so you might need to get them on track to considering that this question could apply to any liquid and even under circumstances when the gravitation acceleration might be different than that on earth (e.g., "Would the buoy's heave frequency be the same on the moon?"). Now we can set the stage for the experiments by telling students that we want to perform a systematic experimental investigation of this phenomenon with buoys that they can construct with common materials ("around the house" items). So, to simplify matters, we will restrict our study to objects with constant cross-sectional area at the liquid-air interface, although not restricted to circular cross sections so that prismatic containers can be used. This means that the 'shape' of the object can now be thought of as the shape of the bottom end of the buoy. Thus, we conclude our discussion of the problem statement, and transition to experiments, by writing

$$f = \phi(M, A, \rho, \mu, g, S) \quad (1)$$

where f is the heave frequency of the buoy and is a function of buoy mass and cross-sectional area, M and A, the liquid density and viscosity, ρ and μ, gravitational acceleration g, and S captures the general shape of the bottom of the buoy (but without a dimensional definition, so we think of it as a dimensionless qualitative measure at this point). Figure 1 contains a schematic diagram that captures the variables and an example of typical shapes of the bottom of containers (e.g., bottles and cartons) that can generally be found around the home and can be used as buoys for this project.

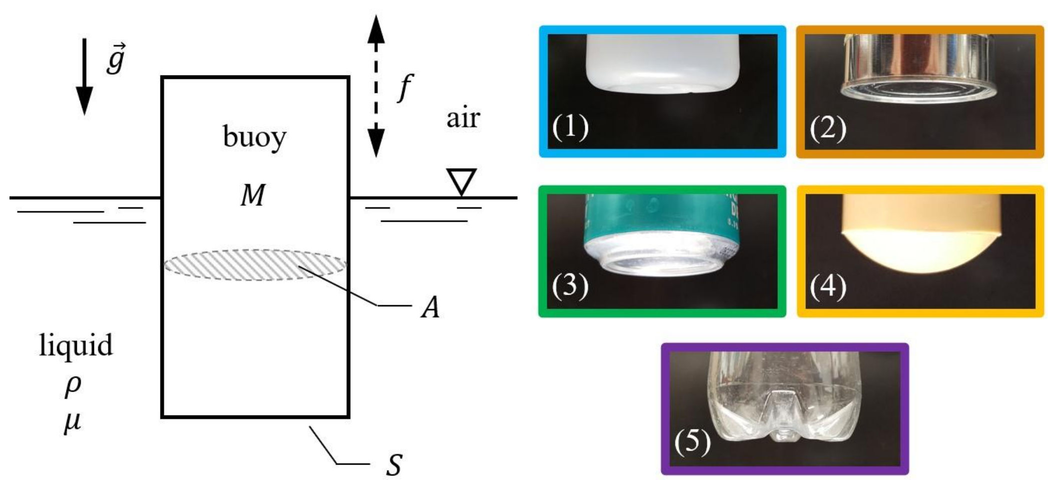

Figure 1. Schematic diagram for problem statement (left). Common buoy shapes (right) obtained from: a plastic bottle, steel can, soda can, cigar tube, and plastic tennis ball canister. Border colors and shape numbers are mapped to content in the results section.

It is at this point in a textbook-style problem we would use the Buckingham-Pi theorem to transform the dimensional variables in Equation (1) into a smaller number of relevant dimensionless groups. This work will be presented later to highlight the difference between a dimensional and dimensionless study of the data. We should also point out that by neglecting any viscous effects (i.e., drag due to shear on the vertical walls of the buoy and form drag from the shape of the buoy bottom), a simple analytical model can be found to describe the heave frequency. This is accomplished by applying the equations of motion to a displaced buoy. This should be made clear to the students, and they should be informed that the rationale of using a heaving buoy as a case study is driven by both the simplicity of the experiments and the accessible analytical solution that can be used for comparison when the results are presented.

The Experiments

As previously stated, the experiments used in this project are designed to be accomplished by students in an at home setting and using a minimum of materials and instruments. Because of this goal, the instructions for building the buoy stress being creative and resourceful. In what follows, we present a synopsis of student instructions for (1) building a buoy, (2) collecting a video of the heave motion, and (3) using the `MATLAB` code to analyze the video and share the results. A more complete version of these instructions, suitable as a document to provide to students, is available from the authors upon request.

Step 1 - Build a Buoy: In order to create a buoy, students will need to select a light-weight container that will float and also accommodate additional mass. The only restrictions imposed on the shape are that it needs to have a constant cross-sectional area around the waterline (liquid-air interface) as it heaves. Beyond that, we encourage students to be creative and to think about getting containers that will span a range of shapes and sizes when all student data is pooled together. All sorts of plastic bottles, cans, containers, and cartons can be used and are available to a college student audience. Since a typical light-weight container does not have sufficient mass to be stable when placed upright into water (which is a great fluid statics / stability of floating bodies question that you can investigate with your students as part of this project!), and since we have speculated that the mass of the buoy plays a role in the heave frequency, additional mass of a known amount needs to be added to the container. A variety of materials found around the house can be used including sand, dirt, rice, nuts-and-bolts, etc. For fill material like sand and dirt, these can be wetted to increase the mass and thereby keep the center of mass as low as possible. A trial-and-error approach might be needed to get a mass that provides stability without capsizing the buoy. The `MATLAB` code that will be used is designed to capture the motion of the buoy by identifying and tracking a specific feature. The code has been designed to find a circular feature and this is why students must attach a spherical marker to the buoy. The color of the marker must contrast with the background behind the buoy. We have obtained the best results when students use ping pong balls or small Styrofoam balls as markers. Students are instructed to attach the markers to a toothpick or straw to extend the marker over the top of the buoy (so that there is no visual interference). Additionally, these types of markers are light so they hardly shift the center of mass of the buoy upward (which affects stability). In Step 3 we provide more details of how the `MATLAB` code processes movies and images, but for now it is important for the students to know that a spherical marker is essential so that if/when the buoy rolls about the vertical axis, the marker will have the same shape as detected by the code. An example of a buoy created using the instructions is shown in Figure 2(a). For this case the buoy is made with a clear plastic bottle, sand is used to add mass, and small foam beads have been added above the sand to help clearly show the level. A ping pong ball is used as a marker and is glued to a darkened toothpick that has been taped to the bottle neck. Note the contrast between marker and background color.

The last part of this step is for the students to measure the relevant properties of the buoy that we have anticipated will drive variations in heave frequency. Students will need to measure the cross-sectional area of the buoy as best they can given their particular set of resources (e.g., using calipers, ruler, etc.) and the mass of the buoy. While some students may have a small scale at home, a reasonable estimate of mass can be made based on the submerged volume. You can encourage your students to ask their friend Archimedes for help if they need to use that approach. Students should also make note of the shape of the bottom of the buoy and attempt to classify it based on general categories (for example those shown/numbered in Figure 1).

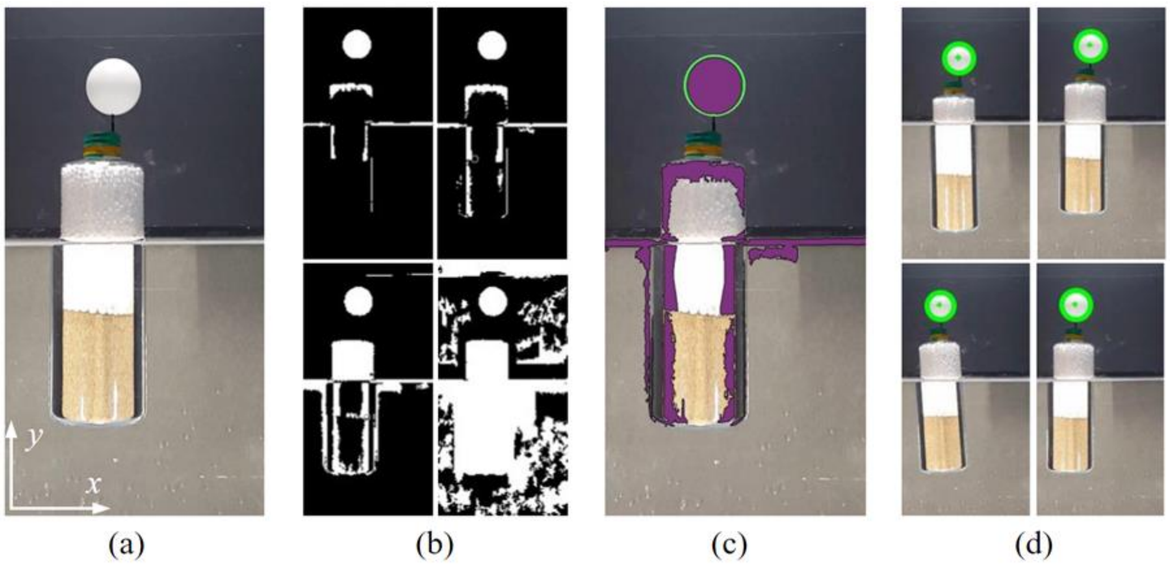

Figure 2. (a) A typical buoy constructed using a plastic bottle with added sand. The xy coordinate system is based on how `MATLAB` interprets position in a movie frame. (b) Examples of the same movie image that has been converted to black and while with various threshold levels. (c) `MATLAB` uses the black and white image to find closed regions and then identifies closed regions which have the property of circles. In this case it has identified only the marker and given it a green border. (d) An output of the `MATLAB` code is a movie where the detected marker is highlighted in each frame. Here we can see the up-down motion of the buoy along with it drifting to the right (time proceeds clockwise beginning in the upper left).

<u>Step 2 - Collect a Video of the Heave Motion</u>: After the buoy is constructed, the heave motion will be measured as it oscillates vertically in a pool of liquid. For safety reasons, students should be told to use only clean tap water. Recall that in our formulation of the relationship between variables we made no mention of the size of the container that the buoy was floating in. For all intents and purposes, it was treated as infinite in extent (unbounded on all sides of the buoy). Since infinite bodies of water are hard to come by, students should use what they have available. This could be a large kitchen sink, bathtub, bucket, etc. As a general rule, the larger the buoy, the larger the body of water to avoid any influence. Once this is secured, a video recording is necessary. Recording the video is the least time-intensive part of this project. The student need only to use their hand to gently depress the buoy (e.g., tap down on the buoy) and record the motion. The only things to highlight here are that:

- The camera cannot move during the course of filming so that only the buoy motion is captured.
- The background, i.e., the region behind the spherical marker, should be as plain as possible and provide good contrast. A dark background is best to contrast with the light color the marker (see Figure 2(a) as an example). Students have frequently achieved this by using a towel in the background. The main concern are objects in the background, or even reflections, that the code may detect as a circular objects, although there are features in the code to help with this.
- The marker cannot be fully blocked or go out of the field of view of the camera during the recording. If a hand is in the way at the start, the movie must be trimmed in length.
- Only five to ten full heave cycles are required. Anything beyond that is unnecessary and leads to longer run time for the code. It is also not necessary to record the movie in high resolution or at very high speed (e.g., 120+ fps). Good results are achieved with FHD at 30 fps or 60 fps.

Step 3 - Analyze the Video and Share Results: The `MATLAB` code was written by a team of undergraduate mechanical engineering students and the purpose of the code is to track the heave oscillation of the buoy so as to determine the heave frequency. We intended the code to be user-friendly to avoid having students needing to read detailed instructions, and it contains dialog boxes which guide the user through their tasks and the code operation. The code was also written so that the user gains some insight into how the marker is detected and tracked, can see the movie frames analyzed, and then interact with the data before final results are provided. Versions of the code were written for use with the desktop version of `MATLAB` and the online version of `MATLAB`. We chose to use `MATLAB` given its general availability/familiarity to engineering students at the college level, and its robust Image Analysis Toolbox which has pre-written functions for specific tasks (e.g., the `imfindcircles` feature which was used in our code [8]). A flow chart highlighting the main operations in the code is provided in Figure 3, and a brief overview is provided. Once the code is initiated it performs the following tasks, with direction from the user, to analyze to motion of the buoy:

- Code recognizes the movie file and determines movie properties (e.g., frame rate, etc.).
- A single frame in the middle of the movie is isolated, converted from color to black-and-white at nine different threshold levels, and code prompts the user to select a level from those options where the marker is distinguished from the background (an example is show in Figure 2(b)).
- After using threshold level user input, closed regions are identified and analyzed to detect which regions are most likely circles. The circular feature most likely to be the buoy marker is highlighted and the user is prompted to confirm the detection (see Figure 2(c)). If this process fails, the user is instructed to manually identify the marker by drawing a line across it.
- The code now knows the marker size (based on confirmation or manual selection), and it proceeds to track the marker sequentially through each frame of the movie. The (x, y) coordinates of the center of the marker for each frame are recorded along with the frame number. Simultaneously, the code both writes and plays a new movie in which the marker is tracked so that the user can see the results (see Figure 2(d)).

After the frames of the movie have been analyzed, the code prompts the user to enter the measured buoy properties (e.g., mass, cross-sectional area, and fluid specific weight). These are then stored and printed in a final ‘results’ text file. Once this information has been provided by the user, the code plots raw data for the y location of the center of the marker in pixels versus the frame number.

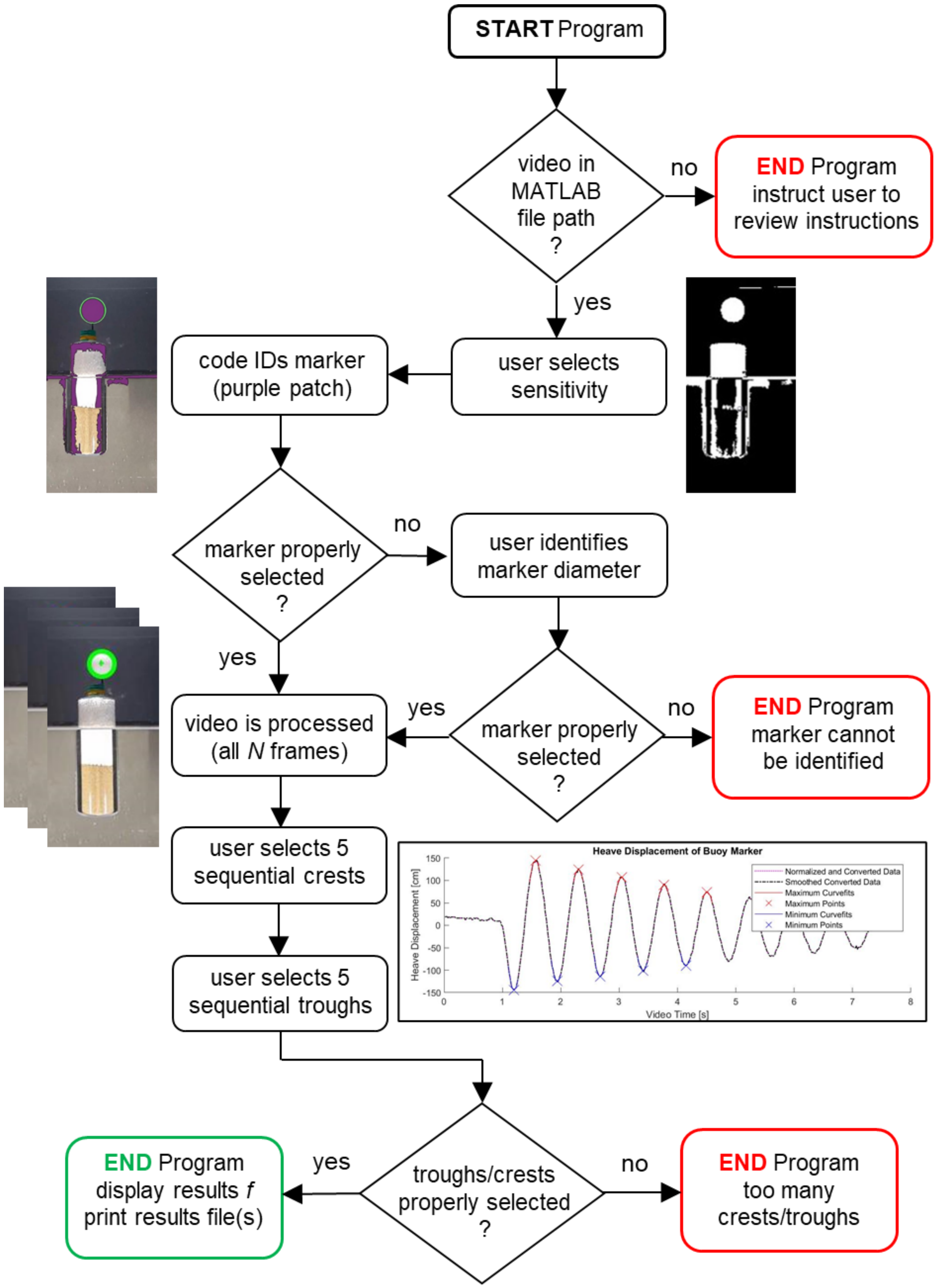

Figure 3. Flow chart showing main operations of the `MATLAB` buoy image processing code.

This is analogous to plotting the heaving motion (vertical motion) versus time. An example of the results of the code for the buoy shown in Figure 2 is provided in Figure 4(a).

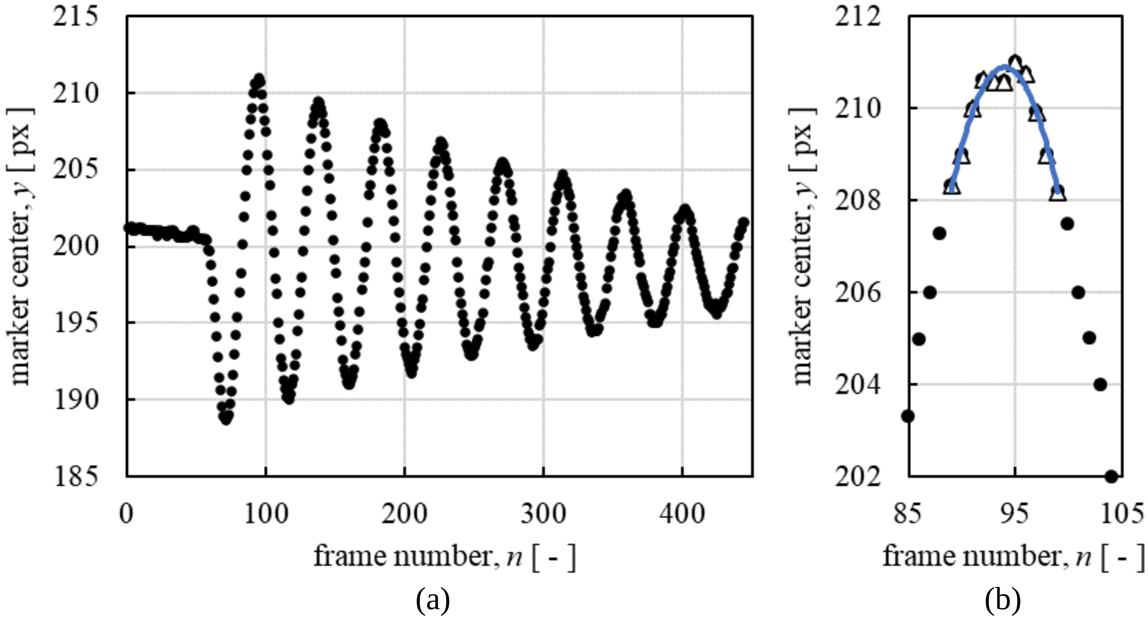

Figure 4. The heaving (vertical) motion of a buoy is captured by tracking the y location of the center of the marker. (a) The first ~50 frames capture a stationary buoy which is then pushed down at frame ~75. The buoy then oscillates for the remainder of the frames. Not surprisingly, the motion of the buoy is damped, and the amplitude of the heave motion decays. (b) A detailed view of one peak showing that, after user input, a curve is fit to identify the frame number location of the crest. This is done for the first five crests and first five troughs.

What remains is for the user to review the plot of heave motion to ensure that it is reasonable (free of any serious flaws) and then select, as prompted by the code, the approximate locations of the first five sequential crests and five sequential troughs corresponding to the heave motion. The user does this by clicking on the plot near the crests and troughs in an open plot window provided by the code. The code uses these approximate locations to locally fit a second order polynomial to the data, thereby identifying the frame number values of the peaks and troughs (interpolating between integers). An example of this is shown in Figure 4(b). The code now has all of the information to compute an average of the heave frequency in addition to an estimate of the statistical uncertainty of the values of heave frequency used to determine the average. This information is provided in the form of summary plots, in addition to a standardized 'results' text file. The code can be modified as needed to save all figures for future use.

Students now have data from their experiments to share. Instructions need to be provided by the instructor to indicate how students can share the data. During past versions of this project, we have found success with using an Excel file in OneDrive that can be linked through the course learning management system. This sharing of data is key to this project – a large number of students each running a small number of experiments can create a large and diverse data set. The project leverages the power of crowd sourcing this data.

Project Results

The data presented in this section were collected from experiments performed by the authors and are indicative of the typical results obtained by students in an undergraduate fluid mechanic course setting (based on assigning this project several times). Experiments performed with 69 unique buoys, which were used to collect 251 values of heave frequency, are provided. A majority of the data was collected with buoys constructed using commercially available bottles and cartons; however, several buoys were fabricated with thin-walled plastic tubes (e.g., shipping tubes). These provided additional buoys with a more slender shape than most bottles.

<u>Data in Dimensional Form</u>

Figures 5(a) and 5(b) capture the results of buoy experiments and are presented in a dimensional form. In Figure 5(a) we see heave frequency f (Hz units) plotted versus waterline cross-sectional area A (mm^2 units). In Figure 5(b) we see heave frequency plotted versus buoy mass M (gm units). In both figures, the data markers have been chosen to distinguish six basic classifications of the buoy shape (i.e., bottom of the buoy). The numbering for these classifications is provided in Table 1. The first observation we can make from these plots is that there is a wide range of buoy area and mass. The values of area span from ~ 120 – 20,000 mm^2 (corresponding to buoy diameters of ~ 12 – 158 mm). The buoy mass range is from ~ 9 – 3900 gm. All but five of the data points use water as the liquid. Overall, the heave frequencies measured for these wide ranges of area and mass fell between ~ 0.7 – 2.2 Hz.

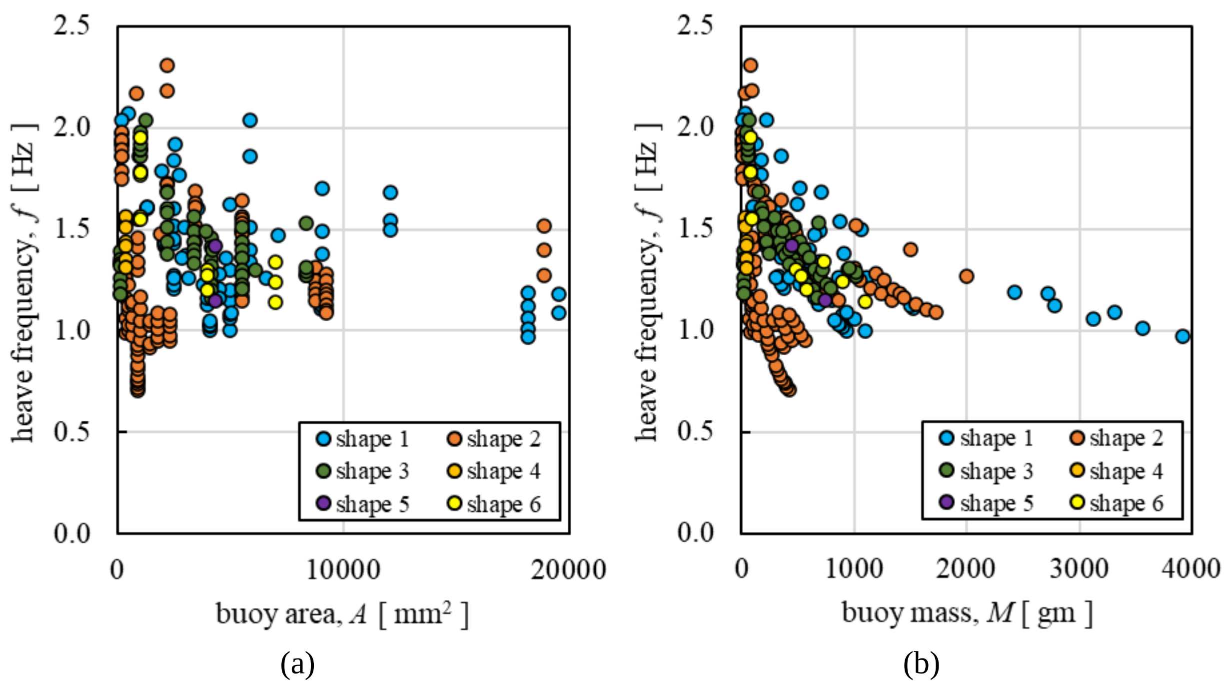

Figure 5. Results from buoy experiments presented in a dimensional form where frequency is shown to depend on both (a) cross-sectional area, and (b) buoy mass. Given the scatter in the data (e.g., the vertical range for any fixed value along the abscissa), it is difficult to determine any precise trend. The average error for values of f are ~ 3.5% and so error bars are not plotted.

The second observation that we can make from Figure 5 is that, in general, both area and mass have a role in setting the buoy heave frequency. It appears from both figures that heave frequency decreases with increasing mass and decreases with increasing area. However, the scatter in the data is significant and we would not expect to achieve an accurate fit with a single trendline. The conclusion that we can draw here is that neither mass nor area are good single variable predictors of heave frequency. There exists a more complicated relationship between the variables that dimensional analysis may help elucidate.

Data in Dimensionless Form

Given that there are several variables which influence heave frequency, we can now turn to dimensional analysis in order to seek relevant dimensionless groups that characterize the heave motion. The goal is a smaller number of dimensionless groups (i.e., π groups) that shed light on the relationships between all of the variables involved. If performed correctly, the scattered data as shown in Figure 5 collapse to show definite relationships if relevant dimensionless groups are found and used. This is the utility of dimensional analysis and using dimensionless groups to find relationships.

One procedure for finding these dimensionless group is called the Buckingham-Pi Theorem and is outlined in most undergraduate fluid mechanics textbooks [4]. We will briefly summarize the process here in order to arrive at groups for our specific problem. We have already started this process through the listing of dimensional variables earlier in the paper – refer to Equation (1). Note that within these variables are contained the primary dimensions of mass, length, and time (an inspection of all of the units of the variables will demonstrate this). The Buckingham-Pi Theorem requires the selection of what are known as 'repeating parameters' from the collection of independent variables. These are variables (i.e., parameters) that we will use to create dimensionless groups. We do this by combining the group of repeaters with individual remaining variables, one at a time, and setting the exponents of all of the variables such that the group contains no net dimensions. In fluid mechanics problems in which the overall group of variables contains primary dimensions of mass, length, and time, the repeating parameters are typically chosen so that one is characteristic of the fluid, one descriptive of the flow field, and one representative of the geometry of the flow field, for a total of three repeating parameters. The independent parameter is never chosen as a repeater as it should only appear once – in the dimensionless independent group (the independent parameter in our case is frequency f). A challenge in a student project like this is to have meaningful variations in fluid properties while still adhering to the at-home materials philosophy. Thus, typical viscous fluids that students might have around the house are cooking oils. Even these, in quantities necessary for experiments with bottle-sized buoys, can be a burden to purchase or work with, so we restrict ourselves to a study using only water. This means that we will eliminate viscosity μ from our list of variables. The product of density and gravity, called specific weight γ, can be used instead of the two separate variables. Recall that we already claimed S is a factor that characterizes shape, but we assign it no dimensions. Thus, we are now left with a list of variables that appears as in Equation (2)

$$f = \phi(M, A, \gamma, S) \tag{2}$$

where there are only three independent variables that have dimensions, and so they must be picked as the repeating parameters. Given that S is already thought of as a dimensionless group, using the Buckingham-Pi Theorem we now seek to find a dimensionless group by combining f, M, A, and γ. We do this by setting exponents on the variables, $f^1 M^B A^C \gamma^D$, and solving for the exponents to yield zero dimension of mass, length, and time for the group. This is accomplished with three algebraic equations to set the dimensions of mass, length, and time to zero in the group (hence solving for the three unknowns B, C, and D). When this is completed, we now find that we anticipate describing heave frequency with the following relationship,

$$\frac{f M^{\frac{1}{2}}}{A^{\frac{1}{2}} \gamma^{\frac{1}{2}}} = \phi(S) \quad \text{or} \quad \frac{f}{\sqrt{\frac{\gamma A}{M}}} = \phi(S) \tag{3}$$

which suggests that dimensionless heave frequency is only a function of shape. If the shape is fixed, say for example by having bottom shapes that are only sharp-edged, then $\phi(S) = C$, where C is a constant for that particular shape. The dimensionless frequency would be equal to a constant value, i.e., $f^1 M^{1/2} A^{-1/2} \gamma^{-1/2} = C$. If this result is true, then the values of the constants for each shape can be found by analyzing the data in a dimensionless form. That is a key idea in this entire process – we find the dimensionless groups that may be relevant, but the relationship between them (i.e., the function ϕ) is found through experiments.

To see if each shape yields a unique value for C, we first plot the data in a dimensionless form as shown in Figure 6, where we have grouped the data by shape (the classifications are admittedly crude as, for example, we do not measure the radius of the rounded edge) and then plotted as dimensionless frequency versus data point number i. For simplicity, we will use f^* to denote the dimensionless heave frequency in figure axis labels.

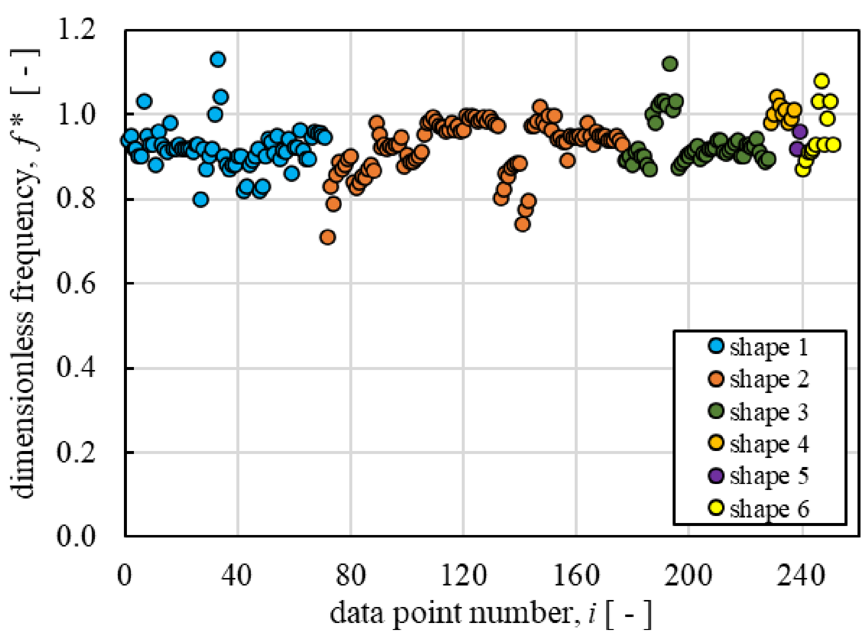

Figure 6. Dimensionless heave frequency plotted to elucidate whether the shape of the bottom of the buoy plays a strong role in determining the frequency.

No definitive trends emerge between dimensionless frequency and buoy shape from an inspection of Figure 6. To further strengthen the claim that there are no significant differences in $f^*$for the shapes studied, we report basic statistics associated with values of f^* in Table 1. Although the rounded bottom buoys have a higher $\bar{f}^*$, this is explained more by the slenderness of the buoys than by the bottom shape as will be explained later. There are some minor differences in the averages of the other shapes (e.g., rounded edge versus concave bottom), but there is overlap of the values when considering the uncertainty. Note that uncertainty is computed using $u = \pm ts/\sqrt{n}$, where t is the Student-t value chosen for 95% confidence (for a two-sided distribution). This leads us to conclude that at least for the buoys tested as a part of these experiments, shape does not play a significant role in setting the dimensionless buoy frequency.

Table 1. Summary of basic statistics for the average dimensionless heave frequency for each buoy shape.

shape description S [-]		sample size n [-]	average $\bar{f}^*$ [-]	std. dev. s [-]	uncertainty u [-]
rounded edge	(1)	71	0.92	0.05	0.012
sharp edge	(2)	105	0.93	0.06	0.012
concave bottom	(3)	52	0.93	0.05	0.014
rounded bottom	(4)	9	1.00	0.02	0.015
undulated bottom	(5)	2	0.94	0.03	0.254
unclassified	(6)	12	0.95	0.07	0.041

But given the scatter in the values of f^*, is there some hidden variable that we have not accounted for? The dimensionless groups that we find from use of the Buckingham-Pi Theorem are only as good as the variables that we identify at the start of the process. Looking back at Equation (2), is there anything else that we can think of that might play a role? One thing to consider is that perhaps the 'shape' of the buoy has less to do with the shape of the bottom, or cross-section shape, and has more to do with the aspect ratio, i.e., the slenderness, of the buoy. We can investigate this by returning to Equation (2) and modifying it to eliminate S and instead replace it with H. We treat H as the submerged depth of the buoy (the portion of the buoy above the waterline does not play a role). We now find that the Buckingham-Pi Theorem leads us to the following,

$$f = \phi(M, A, \gamma, H) \qquad \rightarrow \qquad \frac{f}{\sqrt{\frac{\gamma A}{M}}} = \phi\left(\frac{H}{\sqrt{A}}\right). \tag{4}$$

Since most of our buoys have a circular cross section, we can replace $H/\sqrt{A}$ with H/D, where D is the buoy diameter. We can now return to the data with this new view at finding a predictive relationship between the variables. Note that although students were not asked in the original instructions to measure the submerged depth of their buoys, this value can be computed by equating the mass of the buoy to the equivalent mass of liquid displaced and using the reported cross-sectional area. When this project was presented in class, this re-thinking of the important variables was presented as part of a discussion of the data; however, an instructor can choose to assign H to be computed or measured by the students. With Equation (4) as our guide, we can generate the results show in Figure 7.

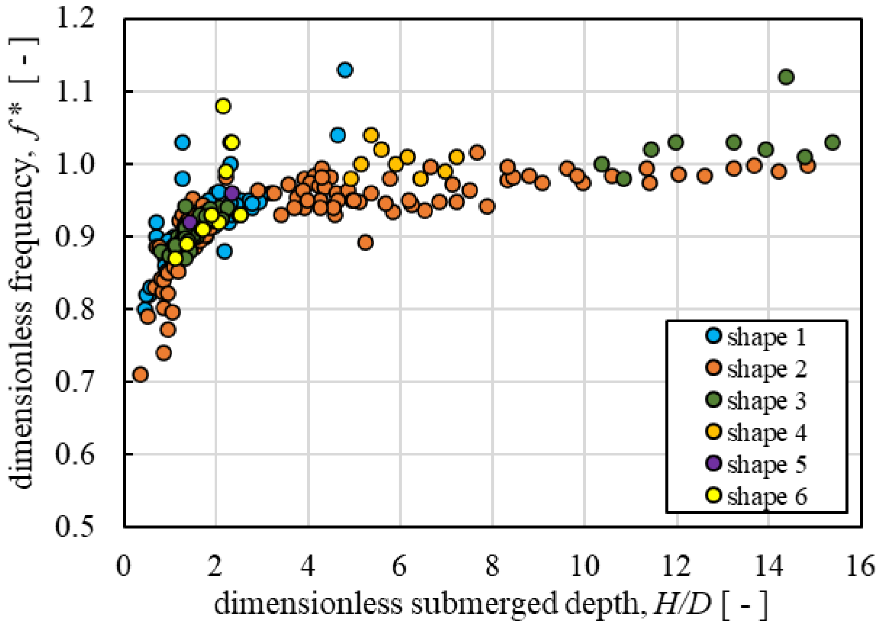

Figure 7. Dimensionless heave frequency plotted against a dimensionless submerged depth. The term H/D indicates the slenderness of the buoy. Large values, i.e., $H/D \geq 5$, were achieved with buoys fabricated from thin-walled plastic tubes.

It is now apparent that there is a relationship between dimensionless heave frequency and the slenderness of the buoy. If we were so inclined, we could attempt a curve fit to yield a predictive relationship; however, this is not necessary for this project. What is worth a reminder is to contrast the results from Figure 5 with those of Figure 7. When viewed through dimensional results only, there is a difficulty in establishing a relationship between heave frequency and buoy mass and area, other than a crude increase/decrease observation. However, when guided by the results of dimensional analysis, and using dimensionless variables which collapse the data onto what seems like a single trend, we can begin to make sense of what could be a complicated fluid dynamics phenomenon.

Additional Explorations

Recall that in our development of a problem statement we asked students to consider the influence of various liquids on the heave frequency. Neglecting any variations in viscosity, we can see from Equations (3) and (4), that density (within specific weight) can play a role. In other words, the same buoy (mass and area) in liquids with different density ought to have different dimensional heave frequencies, but that dimensionless frequencies should be the same as they are set only by the ratio H/D. To demonstrate this, the authors performed a limited set of experiments to highlight the motion of a buoy in both water and acetone. Acetone was chosen as it has a low viscosity, similar to the value for water, but a noticeably lower density of 0.79 gm/cm^3 [9]. Because acetone is a strong solvent, can dissolve many common plastics, and should be handled in small quantities, we chose to use a small buoy made from a thin metal tube. The buoy was floated in approximately 1 liter of acetone stored in a glass container. Five different buoy masses were tested for both the experiments with water and those with acetone. The results are presented in Figure 8. Note the buoy masses for the two liquids are different and the purpose here was to test a similar range of H/D for each buoy. The results presented in Figure 8 show that when plotted as f versus M, the two data sets are noticeably separated. Error bars indicate no overlap of the trends. However, when the frequency is plotted using f^*, we see the two sets collapse onto a single trend.

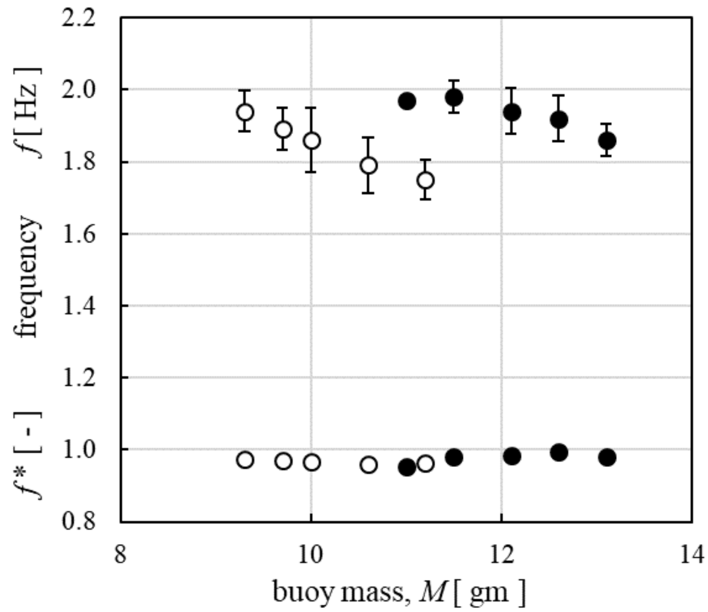

Figure 8. Results of experiments using a small metal tube buoy tested in water (black markers) and acetone (white markers). Both the dimensional frequency f data (in Hz units ranging from ~1.7 – 2.0 Hz) and dimensionless frequency f^* (unitless and ~1) data are shown on the same plot. Error bars represent the magnitude of the standard deviation from each value. It is obvious that buoy frequency changes with fluid density, but that dimensionless frequency is insensitive to it.

Because we chose to create buoys with slender plastic tubes with larger values of H/D for which $f^* \sim 1$ (versus using only bottles – typical of student experiments), we have the ability to look back at those particular experiments and investigate changes in heave frequency by isolating changes in buoy mass and buoy area. A type of investigation such as this further strengthens the results of the dimensional analysis study, i.e., it can be used to confirm the exponents predicted for each variable (cf. Equation (3)), and it can be used to investigate the more typical textbook dimensional analysis question (cf. Page 2). We start by taking the data from seven buoys made from clear plastic tubes with circular cross section. The waterline cross-sectional areas, A, of these buoys are: 349, 471, 665, 985, 1432, 1754, and 2291 mm^2, and all are classified as having a square edge. The results of the experiments with just these buoys are shown in Figure 9(a). The abscissa of this plot is a log-scale, and although the ordinate is a linear scale, because of the narrow variation of ordinate values, it would appear the same on a log-scale (we chose the linear scale to show the major divisions). Each data set is connected with a power-law curve fit, produced using the trendline feature in Excel®, and buoy area A increases from left to right. The curve fits appear linear on this plot because of the actual power-law trend of the data. Dimensional analysis predicted that $f \propto M^{-1/2}$, and the average of the seven exponents from the trendlines is 0.49 with a standard deviation of ±0.05. In other words, the buoy data predicts that $f \propto M^{-0.49}$. Figure 9(a) contains seven data sets with regular cross-sectional areas because we used tubes of set diameter. The masses used in these experiments were chosen to create a spread in the submerged depths. Thus, regular repeated buoy masses were not used. Because of this we cannot isolate the trend in heave frequency with area alone. To remedy this, and because of the result found from Figure 9(a), we can investigate the effect of area on the product $fM^{1/2}$. Dimensional analysis predicts that $fM^{1/2} \propto A^{1/2}$ according to Equation (3). Results from the buoy data are shown in Figure 9(b).

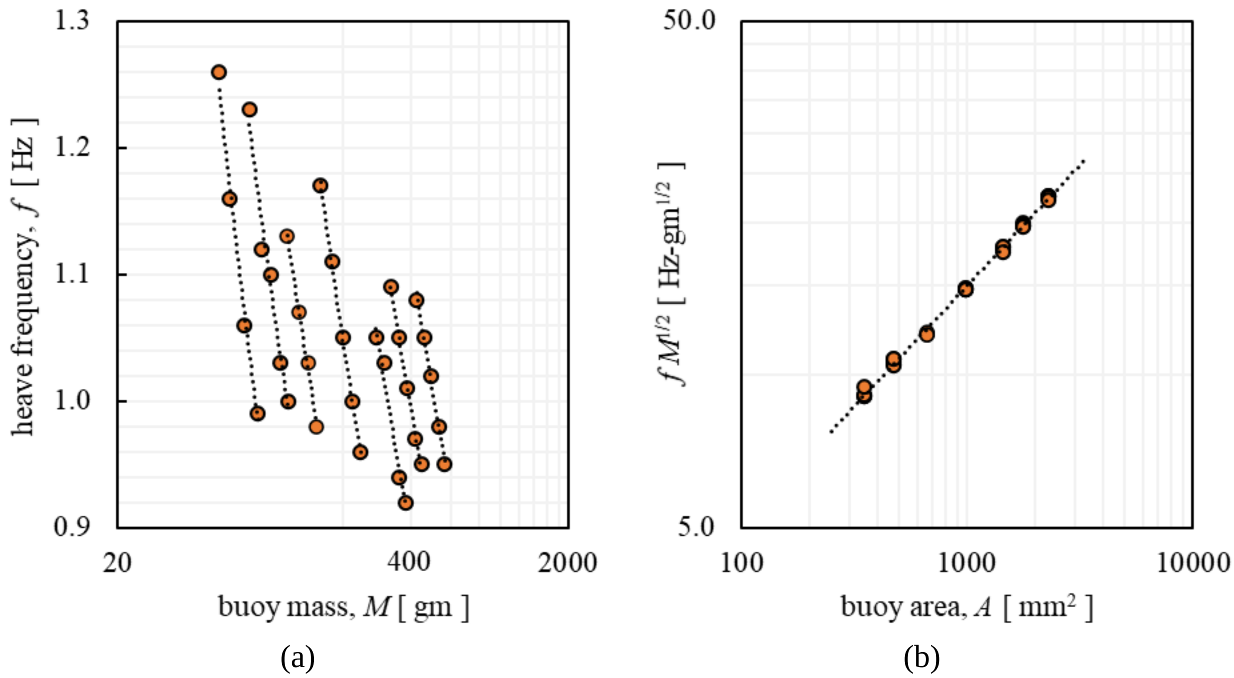

Figure 9. Results from buoy experiments using slender plastic tubes of constant cross-sectional area. These results confirm the dependence of heave frequency on (a) buoy mass M and (b) area A. In both cases, the experimental results are consistent with the predicted exponents of 1/2 on both area and mass as given in Equation (3).

Each data set from Figure 9(a) collapses to nearly a single point, with negligible variation, in Figure 9(b). A power-law curve fit through the data of Figure 9(b) yields $fM^{1/2} \propto A^{0.48}$ (the curve fit appearing linear on a log-log plot confirms the power-law nature of the data). This exponent is essentially the value of 0.5 predicted by theory.

What remains is to introduce the simple analytical model for f (in Hz units), mentioned earlier, for the heaving motion of a buoy. The purpose of providing this model for this project is to demonstrate that the data students have collected can be used to assess the validity of a theoretical model (something else that typical homework problems rarely give the student an opportunity to do). We need only to analyze the motion of the displaced buoy to accomplish the task of developing the model. Using the coordinate system show in Figure 4(a), Newton's Second Law yields

$$\frac{d^2y}{dt^2} = \frac{\rho g A}{M} y \tag{5}$$

where the force acting on the buoy is the additional buoyant force due to the displacement y from equilibrium. This equation yields an expression for $y(t)$ that predicts oscillatory motion. Upon solving this equation for the frequency of that oscillation we find

$$f = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{\gamma A}{M}} \qquad \text{and} \qquad \bar{f}^* = 1. \tag{6}$$

where the coefficient $1/\sqrt{2\pi}$ provides units for frequency in Hz. Equation (6) provides us with a means of computing a theoretical (expected) heave frequency for any values of γ, A, and M. We should note that this theoretical frequency does not account for drag forces on the buoy. Figure 10 provides a direct comparison between the experimental heave frequency and the theoretical prediction for heave frequency.

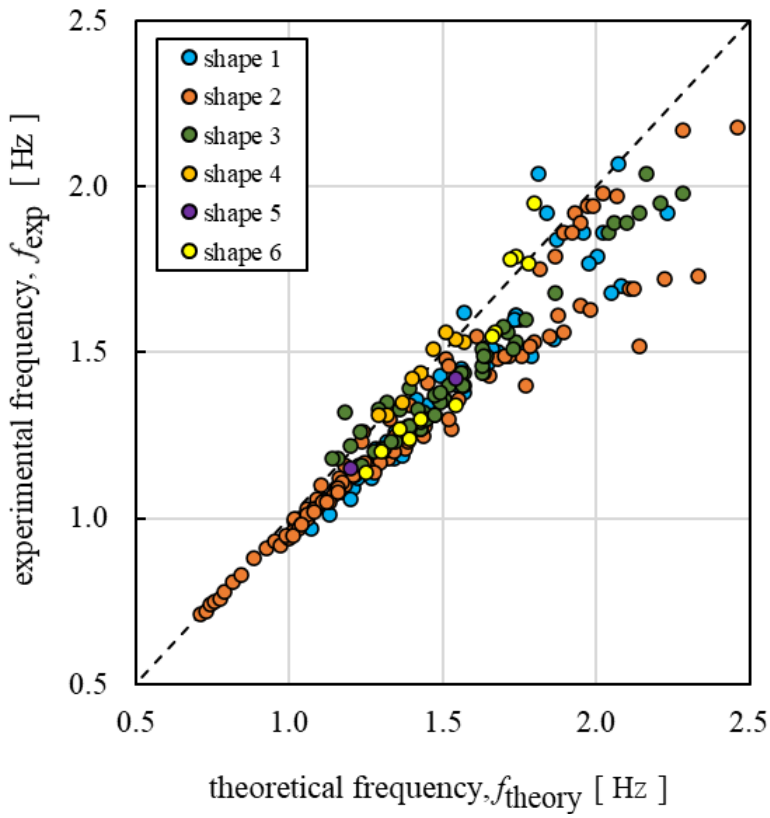

Figure 10. A comparison of experimental heave frequency measurements to theoretical values predicated using a simple analytical model that neglects drag forces and added mass.

What we discover from inspection of Figure 10 is that in general, the experimental data is in excellent agreement with the theoretical prediction at low frequencies (e.g., $f < 1$), but as frequency increases the agreement decreases. The data points for $f < 1$ correspond to slender buoys, which we may classify as spar buoys (vertically elongated) in contrast to other types of buoys such as boat buoys (horizontally elongated), sphere buoys, etc [10]. Nearly all of the experimental data points for $f > 1$ have frequency values below the theoretical. This makes sense if we remind ourselves that the theoretical model does not consider drag forces which we anticipate would increase in magnitude with higher frequencies (and thus higher buoy speeds). What the simple theoretical model also does not account for is an 'added mass' which is often used to more accurately predict buoy heave frequency by accounting for the mass of the liquid that the buoy must accelerate as it moves [11]. If the actual mass is combined with the added mass, we expect a decrease in the theoretical frequency compared with the prediction from Equation (6).

Future Work

The project described in this work has been implemented several times in an undergraduate fluid mechanics course at Cal Poly San Luis Obispo and from the anecdotal feedback we have received, we recognize that there are many opportunities to improve and expand on the project. These improvements and expansions include:

(1) Implementing the project with an eye toward assessment of learning outcomes as compared to a course that does not have a hands-on component for teaching the utility of dimensional analysis;

(2) Develop a version of the code that operates using open-source software to further broaden the pool of potential users and promote inclusion (e.g., students that do not have access to `MATLAB`);

(3) Build a web-based set of experiment movie files, code, and data so that students can supplement their own experiments with more data or share their data to add to the usable collection. Currently, the authors are more than happy to directly share the code and movie files upon request; and

(4) Adapt the code to use artificial intelligence for marker detection. A trained version of the code could then be used to detect markers that are not highly spherical, for example a wadded-up ball of paper, thus making further reductions to the materials needed for the experiments. We see this project as a starting point for future undergraduate engineering studies that could benefit from smartphone-based image analysis.

Conclusions

Not all colleges and universities can provide hands-on experiments in traditional laboratory settings. This could be due to budgetary constraints or, as we have seen more recently, due to circumstances that require a sudden transition to virtual instruction. But this does not mean that students cannot be provided with opportunities to explore concepts and collect data outside of the classroom. As we have demonstrated in this paper, simple at-home experiments using minimal materials, utilizing smart phone video recordings analyzed with image processing software, can be used by students to collect and share data. Crowd sourcing this data can reduce the burden on any one student to complete a multitude of experiments, and by encouraging creativity and resourcefulness a diversity of experimental results can be obtained by the group (i.e., the crowd). In this particular instance, the experiments yield a rich set of data and were used to support the teaching and learning of dimensional analysis in an undergraduate fluid mechanics course – which can be used to elucidate trends in phenomena in which a number of variables play a role.

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