AC 2011-1066: HELPING STUDENTS APPROACH FEA SIMULATIONS LIKE EXPERTS

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Helping Students Approach FEA Simulations like Experts

Introduction

Computer simulation has emerged as a fundamentally new approach for solving engineering problems, distinct from theory and experiments. The recent, dramatic reduction in the cost of computing hardware and the maturation of off-the-shelf, commercial software packages has allowed advanced simulation to become an integral part of engineering design, analysis and research. Driven by cost pressures, many industries are vigorously pursuing a zero-prototype future, with simulations minimizing or even replacing expensive physical prototypes. For society to reap the full benefits of simulation, most engineers, not just specialists, need the ability to deploy simulations effectively. There is now widespread agreement that undergraduate engineering students need to be taught the capabilities and limitations of advanced simulation. Simulation also offers the opportunity to enhance learning through an interactive, visual medium and to build excitement among students about the engineering profession. Working with simulations is a visual and interactive experience, something the current generation of students takes to readily. Simulation enables beginners to generate solutions to practical engineering problems without the use of abstract mathematics or expensive equipment. The finite element method (FEM), also referred to as Finite Element Analysis (FEA), is an important numerical technique used to simulate a wide variety of engineering problems. By integrating simulations across several sequential required courses in the mechanical engineering curriculum, we plan to increase students’ ability to use FEA-based simulations effectively and improve their understanding of the concepts developed in these courses.

Cognitive research has shown that people’s understanding lies in a spectrum from “novice” to “expert”. Conventional learning materials tend to relegate beginners to “novice thinking” by presenting simulation exercises as recipes handed down by authorities. Wieman’s group has shown that interactive simulations, when designed using a rigorous scientific approach, are much more effective in helping physics students develop an expert cognitive structure than lectures are. A preliminary survey of best practices guidelines for simulation use, developed by practicing engineers, indicates that the expert approach has an underlying uniformity irrespective of the specific context or discipline. Our project extends this cognitive and simulation research to industrial-standard simulation platforms. We hypothesize that if students, in their formative years, see the same expert approach to simulations being followed repeatedly for a wide variety of problems in different subject areas, they are likely to internalize it and be able to apply it in new situations. Students will thus develop a mental organizational structure similar to those developed by experts with years of experience working with simulations. Students will then be able to work with simulations much more effectively in both academic and industrial settings since they will have a robust scaffold of understanding on which to base new applications.

Learning materials and strategies are being developed to help undergraduate students learn an “expert approach” to FEM so that they can obtain reliable solutions to engineering problems. The materials are being organized into a dynamic, interactive cyberlearning portal for simulation, where faculty, students and practitioners can learn, teach, contribute and interact.
meaningfully. A key insight undergirding this effort is that advanced simulation software takes care of the details of the mathematical models and numerical techniques so that the user (student or non-expert engineer) only needs to be concerned with the essence of relevant concepts to apply the technology intelligently and effectively.

**Methods**

In order to support the goal of guiding undergraduate engineering students towards a more “expert” approach to simulations, this effort seeks to:

1. Identify and formalize an expert approach in simulation that is valid across various applications in finite-element analysis (FEA) and computational fluid dynamics (CFD).
2. Incorporate this expert approach into simulation exercises in three Mechanical Engineering courses at our university and evaluate the resultant impact on student learning.
3. Disseminate the resulting learning materials and strategies so that other instructors can easily incorporate this expert approach into their courses.

The desired student learning outcomes are:

1. Students will be better able to avoid common errors as they apply industry standard FEA and CFD software.
2. Students will be better able to verify and validate FEA and CFD results.
3. Students will be able to apply the expert approach to new problems in FEA and CFD.

Teaching materials have been developed for the first of the three target courses, a required junior-level course in solid mechanics where mechanical engineering students at our institution are first introduced to FEM. Three FEM-based demonstrations have been created where students are presented with the FEM solution to classical problems. Students explore the FEM solution and compare the results with the corresponding analytical solution or empirical data. It is important to note that students do not obtain the FEM solution themselves but instead focus on the exploration and critique of results that have already been obtained. Each of these demonstrations is accompanied by in-class clicker questions and homework problems that ask the student to think more deeply about the simulations. Three longer-term projects ask the students to engage with simulations at a progressively deeper level. Through these tools, students are led through thinking about simulations before they actually start creating their own simulation in the third project. Figure 1 shows how the FEA simulations were integrated into the class. Compared to teaching a “recipe of clicks” to create a simulation, this approach is designed specifically to increase student’s conceptual understanding of both simulations and course content.

**Lecture Demonstrations**

The three FEM-based demonstrations were designed and created using ANSYS Workbench 12.1, a leading commercial FEM software. The demo topics -- tensile bar, plate with a hole, and curved beam -- were chosen to readily connect with traditional topics in the textbook. Each demo was presented in class along with clicker questions designed to engage students in discussing the demos. Students could download these demos, easily change parameters, and re-run the demos for further investigations outside the lectures.
The first demonstration showed a tensile bar fixed at one end and loaded with a point force at the other (Fig. 2). It followed the introduction of the Differential Equations of Equilibrium (DOE) to show students the differences between solving 2D Boundary Value Problems (BVPs) analytically and solving them numerically using FEM. Solving a BVP often requires making appropriate assumptions and simplifications. Therefore, the analytical solutions may not be valid everywhere in the domain and may fail when the assumptions are not appropriate. In these demonstrations, students observed that the analytical results they obtained by assuming the bar has a length much larger than the other dimensions and reducing the problem to 1D were not valid at the fixed end or the end where the point force was applied. This can be seen by comparing Figs. 2a and 2b: the axial stress is uniform across the bar in the 1D analytical solution while variations could be observed near both ends in the 2D numerical solution.

Figure 2. Demonstration of tensile bar: (a) Analytical solution: Uniform axial stress = $2 \times 10^8$ Pa (parameters: $P=20kN$, length of the bar=$50mm$, width=$10mm$ thickness=$10mm$), (b) Numerical solution (axial stress), (https://confluence.cornell.edu/x/77dyBw)
In the second demonstration, the stress field of a rectangular plate with a hole in tension (Fig. 3a), was shown. For simplicity, symmetry was considered so only a quarter of the geometry was used for numerical solutions (as shown in Fig. 3b). This demonstration helped students clearly visualize the stress distributions and variations around the hole where stress concentration occurs. Students used the numerical solutions as concrete visual aids to interpret the abstract analytical solutions.

![Plate with hole](image1.png)

Figure 3. Plate with a hole demonstration. (a) geometry (parameters: $\sigma_0 = 1 \times 10^6$ psi, length of the plate=10in, width=5in, thickness=0.2in, diameter of the hole=0.5in) (dashed lines represent symmetry axes), (b) FEM solution - Shear stress in polar coordinate system (upper right quadrant of the plate in Fig. 3a) (https://confluence.cornell.edu/x/orqTBw)

The third demo was used to show the stress field of a curved beam under pure bending (Fig. 4). Analytical solutions of this problem using elasticity theory, Winkler Bach theory and straight beam theory were presented. The numerical solution was compared with analytical solutions from all three theories. (See examples in Fig. 5). This demonstration showed that the Winkler Bach theory gave very good approximations of the stress distributions for curved beams under pure bending. Assuming the beam was straight and then using the straight beam theory can be a quick way to estimate the stress distribution, but how close it is to the real solution is limited by the geometry. This demonstration therefore helped students understand these three analytical methods for solving pure bending of curved beam so that they would have a better basis from which to pick appropriate methods for particular problems. Comparing these theories and the numerical solution helped build students understanding of the strengths and limitations of analytical and FEA solutions.

![Curved beam](image2.png)
Figure 5. Stress Distribution for $\sigma_\theta$ along a curved beam where $a = 10$ in, $b = 12$ in, $t = 0.25$ in and $M = 300$ lb/in$^2$. (a) Comparison of analytical solution from straight beam theory with numerical solution (circumferential stress), (b) Comparison of analytical solution from elasticity theory with numerical solution (circumferential stress).

Table 1. Curved Beam Demonstration – Comparisons of analytical solutions from three theories with numerical solution

<table>
<thead>
<tr>
<th></th>
<th>Elasticity Theory</th>
<th>Winkler Bach Theory</th>
<th>Straight beam theory</th>
<th>Numerical solutions</th>
<th>Percent difference: FEM and Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Elasticity Theory</td>
</tr>
<tr>
<td>Maximum radial stress</td>
<td>0.00</td>
<td>N/A</td>
<td>0</td>
<td>-0.11</td>
<td>N/A</td>
</tr>
<tr>
<td>Minimum radial stress</td>
<td>-82.21</td>
<td>N/A</td>
<td>0</td>
<td>-82.30</td>
<td>0.11</td>
</tr>
<tr>
<td>Maximum circumferential stress</td>
<td>1697.00</td>
<td>1696.40</td>
<td>1800</td>
<td>1697.63</td>
<td>0.04</td>
</tr>
<tr>
<td>Minimum circumferential stress</td>
<td>-1917.00</td>
<td>-1915.60</td>
<td>-1800</td>
<td>-1916.20</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Homework Assignments
Following each FEM demonstration, homework was assigned in which an FEM solution in ANSYS Workbench was provided and students were asked to determine if it was correct. To make that determination, students had to obtain the analytical solution and use it to check the FEM solution. The assignments were very similar to the corresponding in-class demonstrations but with a small modifications. Each homework assignment was designed to: (1) reinforce what was taught in the in-class demo, (2) guide students in becoming more familiar with reviewing FEM results, (3) aid students in developing a deeper understanding of important concepts through interactive, visual exploration of FEM results, (4) help students learn how to use analytical solutions to check FEM results, and (5) build students’ understanding of the relationships between the analytical and numerical solutions. Thus, students go through the process of interpreting FEM results with a critical eye, as an expert would, rather than accepting them blindly. This is possible since students were provided with the FEM solution, enabling the focus to be shifted to interpreting and critiquing results rather than obtaining a solution. The latter skill is taught later in the semester in separate exercises.

The first FEM homework assignment required solving the stress distribution for a bar hanging from a ceiling and acted on by a downward force. The FEM solution provided was deliberately incorrect – gravity was omitted. The learning objectives for this homework were to help students (1) understand the difference between body force and surface force, (2) learn to use the Differential Equations of Equilibrium to solve for the stress distribution, (3) practice checking an FEM solution, and (4) realize that an FEM solution could look realistic but be wrong if there was an error in defining the problem.

The second FEM homework required solving for the stress concentration in a plate with two grooves and a hole. The FEM solution was provided for comparison with calculated results. The learning objectives for this homework assignment were to help students: (1) understand the concept of stress concentration, (2) visualize the stress distribution in the structure, and (3) learn to check FEM stress concentration factors using empirical correlations.

The third FEM homework required solving for the bending stress in a curved beam using an elasticity solution and again using Winkler-Bach Theory and then comparing each result with the provided FEM solution. The learning objectives for this homework were to: (1) help students understand the difference between the elasticity solution and Winkler-Bach theory, and (2) realize that for this problem the FEM solution is more accurate than the Winkler-Bach theory since the FEM solution does not assume the radial stress is zero. In summary, the FEM demonstrations and homework problems build a set of skills – visualizing the deformation and stress fields described by equations, critiquing and evaluating results, comprehending the different assumptions in analytical and numerical solutions etc. – that help students emulate the behavior of experts.

Projects
Three individual projects were assigned across the semester to further strengthen students understanding of FEA. The objectives of these projects were to help students (1) understand the post-processing step in FEA; (2) understand the concept of convergence, (3) learn to verify and
validate numerical solutions, and (4) use FEA to solve real engineering problems. In contrast to the FEA demonstrations, the projects required students to obtain the FEA solutions themselves, either by modifying an in-house MATLAB-based FEA code called redAnTS or by using ANSYS Workbench. The skills of reviewing, checking and interpreting results learned through the FEA demonstrations were necessary to successfully complete the projects. Thus, the demonstrations and projects complemented each other, enabling students to move along the spectrum from novice to expert.

The first project required students to implement the post-processing procedure of FEA in MATLAB, i.e., obtaining stresses and strains from the nodal displacements for a single triangular element. The nodal displacements of a constant strain triangular element were provided to students for five simple cases: uni-axial tension, bi-axial tension, simple shear, pure shear, and pure rotation. Students needed to develop a Matlab code to output the displacement field, calculate the strain field using the displacement-strain relationship, and then calculate the stress field using Hooke’s Law. Deformed shapes for a square material element were plotted for each case, helping students to identify important deformation modes in an FEA solution.

In the second project, students used redAnTS, our in-house FEA software, to find the stress field of a cantilever beam under bending. The software provided the nodal displacements after students created the geometry, specified parameters, generated the mesh and specified boundary conditions. Students needed to augment redAnTS to provide the post-processing capabilities: calculation of strain and stress fields from the nodal displacements. While they implemented this for one element in stand alone code in the first project, here they were required to implement this for an arbitrary number of elements within redAnTS. Students compared their numerical results with the analytical solution and assessed mesh convergence to validate their FEA solution.

The third project required students to design a bicycle crank using both redAnTS and ANSYS 12.1 Workbench. A baseline design was provided to students and they were asked to analyze this design for given static and cyclic loading conditions. Students then selected materials and optimized the geometry with the objective of minimizing the mass. Several constraints were prescribed, for example, that the new design should maintain certain safety factors for both static and cyclic loading conditions and the deflection of the crank should be within certain limits.

**Assessment**

The FEM homework assignments developed students’ critical thinking skills pertaining to FEM solutions. The first assignment required a comparison of the student’s analytical solution with an FEM solution (provided to students) that deliberately contained an error. After the assignment was graded, the types of mistakes made by the 81 students who turned in the assignment were categorized. The results are shown in Fig. 6. Only a quarter of the students completed the comparison correctly. A fifth of the students didn’t spot the error in the FEM solution. The rest of the students realized there was a problem but had varying levels of problems or incompleteness in their calculations and explanations.
The mistakes students made on homework 2 are summarized in Table 2. This information is given in a table rather than a pie chart as some students made multiple mistakes so the percents total more than 100. One of the most important aspects of an FEM solution is the set of boundary conditions applied. Students were asked to deduce the boundary conditions applied in the FEM solution provided to them by looking at the deformed and undeformed shapes (this is an essential step in evaluating an FEM solution critically). Table 2 shows that only 29% of students were able to deduce all the boundary conditions correctly with 64% getting the displacement boundary condition wrong. In contrast, 97% of students were able to correctly determine the empirical stress concentration factors from the associated chart in the book and compare it to the corresponding values from the FEM solution. This calculation is mostly plug-and-chug which students are able to handle competently. They did much more poorly in the boundary conditions part of the homework which involved more critical thinking and no plug-and-chug. In the future, more homework questions will be developed to promote critical thinking faculties.

Table 2. Assessment of 89 Student Papers for Homework 2. Percent of students in each category. Totals will exceed 100% as some students made multiple kinds of errors.

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>Specified Incorrectly</th>
<th>Not Specified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td>Displacement</td>
<td>Traction</td>
</tr>
<tr>
<td></td>
<td>29%</td>
<td>64%</td>
</tr>
<tr>
<td></td>
<td>Displacement</td>
<td>Traction</td>
</tr>
<tr>
<td></td>
<td>8%</td>
<td>12%</td>
</tr>
<tr>
<td></td>
<td>Traction free</td>
<td>Symmetry</td>
</tr>
<tr>
<td></td>
<td>9%</td>
<td></td>
</tr>
<tr>
<td>Stress Away From Hole and Grooves</td>
<td>Correct</td>
<td>Incorrect analytical solution</td>
</tr>
<tr>
<td></td>
<td>97%</td>
<td>1%</td>
</tr>
<tr>
<td>Stress concentration adjacent to grooves</td>
<td>Correct</td>
<td>Incorrect analytical solution</td>
</tr>
<tr>
<td></td>
<td>87%</td>
<td>11%</td>
</tr>
<tr>
<td>Stress concentration adjacent to hole</td>
<td>Correct</td>
<td>Incorrect analytical solution</td>
</tr>
<tr>
<td></td>
<td>87%</td>
<td>12%</td>
</tr>
</tbody>
</table>
By the time the students did the third FEM homework assignment, three quarters of them were able to correctly compare an analytical and FEM solution as shown in Fig. 7.

For each of the three projects, an assessment was carried out to determine whether the students met specific technical goals. The expectations increased for each project. Figures 8-10 show what percent of students correctly accomplished each goal. Figure 8 is compressed relative to Figure 9 and both are compressed relative to Figure 10 to visually show how projects goals expanded across the projects.

Figure 8. Student achievement of components of the first FEM project
Figure 9. Student achievement of components of the second FEM project

Figure 10. Assessment of the components of the third (and final) FEM project. The first three components were done in RedAnTS. Components 4-6 were done in ANSYS. The 7th component was a comparison of the two solutions and the last five components were related to the optimization of the solutions. In each component the blue (lowest part of bar) represents success; the shades of blue for success were varied to visually group the types of tasks.
Additional details on the students final projects is given in Fig. 11.

![Evaluation of Final Projects](image)

**Figure 11.** Details on what percent of students met the design objectives and constraints on the third (final) project.

A mid-semeser survey was used to gauge student response to the materials. Answers were given on a 1-5 Likert scale with 5 being strongly positive. Average student responses are shown in Fig. 12. All but one of the responses averaged above 4.

![Usefulness of FEM solutions vs Analytical Solutions](image)

**Figure 12.** Usefulness in understanding FEM solutions and in relating FEM solutions and analytical solutions.
The mid-semester evaluation also asked students how helpful they felt the demonstrations were in helping them visualize solutions and in helping them understand the solutions. The mid-semester evaluation was handed out the same day the third, and final, project was assigned. Students were asked how confident they felt about their ability to succeed on the final project. Sixty percent of the students felt confident and ready to start. About thirty percent weren’t sure or didn’t want to say until they had more time to think about the project. Only about 10% didn’t feel ready to start the project. This is a strong indication that the earlier projects, demonstrations and homework were successful in preparing students for the more difficult final project. Fig. 13 summarizes the student responses.

Figure 13. Evaluation of in-class FEA demonstrations. This survey was conducted in the same lecture in which the project 3 was assigned; students had just read the project and some students therefore indicated that they had just barely read the project so they didn’t yet know if they were adequately prepared for it or were only somewhat sure of their preparation.

Conclusions and Future Work
This paper describes ongoing efforts to facilitate students' progression from novices to engineers who are able to emulate the behavior of experts in the application of FEM, an important technique to solve a variety of engineering problems. The current effort focuses on a large, required, solid mechanics course. Students are taught analytical and FEM solutions in parallel across the semester to facilitate a deeper understanding of both. Students learn to use analytical solutions to validate FEM results as well as to use FEM-based results to understand the information contained in equations obtained analytically. The latter is enabled by the rich, visual data provided by FEM software that students can interact with.

In FEM-based demonstrations, students explore FEM solutions presented to them and compare the numerical results with the corresponding analytical solutions or empirical data. These demonstrations focus on exploration and critique of FEM results rather than on obtaining the numerical solution, guiding students towards becoming critical users of computer simulation. The demonstrations are accompanied by in-class clicker questions to improve student
engagement in lectures as well as to connect the FEM results with corresponding topics from the textbook. Homework problems follow each demonstration. Our assessment indicates that students fared poorly on questions that involved critical thinking such as determining the boundary conditions applied from the FEM solution. In contrast, they did very well on questions that involved plug-and-chug. In the future, we intend to create more homework questions that promote critical thinking in order to help students develop a more expert-like cognitive structure.

In addition to the FEM-based demonstrations, three longer-term projects engage students with simulations at a progressively deeper level. In the projects, students obtain the FEM solution either by developing MATLAB code or using ANSYS. Through this multi-pronged strategy, students are led through thinking about simulations at a deeper level. The mid-semester and final surveys indicated that integration of FEM into this course was effective in helping students’ understanding of numerical as well as theoretical approaches. Students were also confident of their ability to accomplish the final FEM-based design project.

Future work will involve building on current findings to provide a robust framework for students to progress from novice to expert not only in this course but also two following required courses. In Fall 2011, we will continue to integrate FEM simulations into lectures with further improvement. Based on our experience and the final course evaluations, we will introduce ANSYS earlier in the semester and provide more opportunities for interactive exploration of ANSYS results by changing parameters etc. This will serve to improve understanding of results, and better connect analytical and numerical solutions. More clicker questions will be used to increase student engagement during the lecture. We have developed a wiki (https://confluence.cornell.edu/display/simulation/ansys) where the material is posted and available for use by other faculty. The wiki encourages the sharing of content and offers the opportunity for others to add to the content and discussion so as to promote more effective use of simulation in engineering education.

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Bibliography
