# AC 2011-689: HOME BREW WORT COOLER AS SUBJECT OF PROCESS MODELING AND DESIGN: A COMPELLING EDUCATION MODULE

**Michael A Smith, Villanova University**
**Noelle K Comolli, Villanova University**

Noelle Comolli, PhD, attended University of Delaware for her BS in Chemical Engineering, then Drexel University for her PhD in Chemical Engineering. She is currently an Assistant Professor of Chemical Engineering at Villanova University, specializing in biomaterials and polymeric based drug delivery designs.

# Home Brew Wort Cooler as Subject of Process Modeling and Design: A Compelling Education Module

It is widely accepted that educational outcomes are more successful when students have a keen interest in the subject, and this typically happens when the subject is something near and dear to them. It is also widely acknowledged that most college student show a keen interest in beer. In our experience with students in engineering, this often translates into an interest in the brewing process and at times has culminated in students engaging in home brewing. It is natural then to use this interest to engage students in educational exercises around one or more aspects of brewing technology.[1, 2] In the Department of Chemical Engineering at Villanova University, we have developed an educational module consisting of a demonstration/ laboratory experiment, a transient modeling exercise, and a design exercise of the wort (unfermented beer) cooling step in the overall home brewing process.

In this paper we describe a typical home brewing operation, outline the objectives and constraints of the wort cooling step, and describe the apparatus we use in the demonstration or lab. The governing model equations are given, and we show how these are used as a theoretical modeling exercise for the cooling step, where results can be compared with experimental values using different model assumptions and simplifications. We show how non-dimensional version of the model equations are used to show how the magnitude of different factors can be used to *a-priori* suggest appropriate model simplifications, providing a useful lesson in non-dimensional analysis for this real physical system.

We believe the module provides some very useful lessons regarding design, and provides a clear introduction to the laws of process equipment scaling. An outline of the educational assessment of these lessons and preliminary results are provided as well.

### *Introduction to home brewing technology*

The typical homebrewed batch size is 5 gallons (which is roughly 48 12-oz. bottles). This is the most common batch size since the homebrew supply trade provides kits and equipment consistent with this size. The basic process flow diagram is illustrated in Figure 1. Steps include mashing, lautering, wort boiling, wort cooling, separation of spent hops from wort, pitching the yeast to start fermentation, and bottling or kegging.

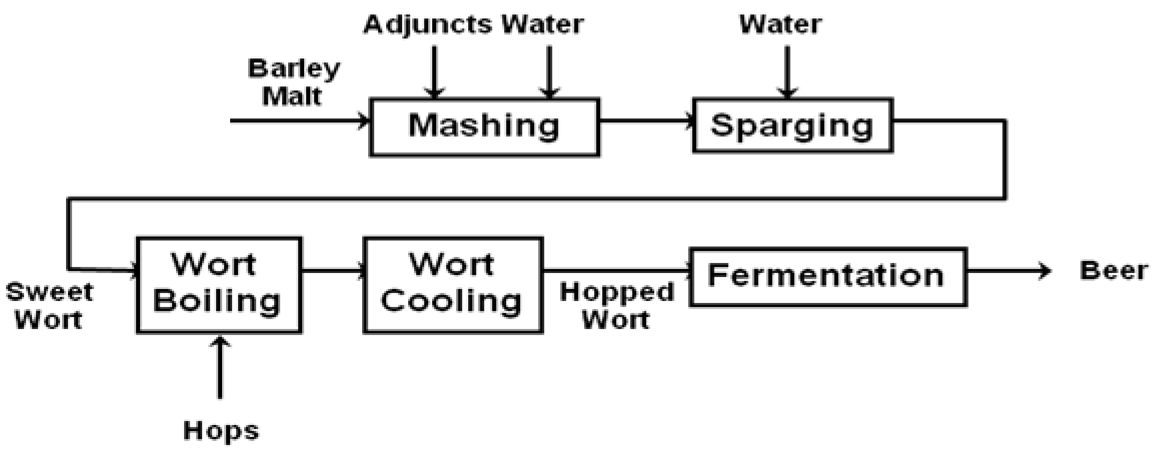


**Figure 1:** The basic home brewing process is shown in the flow diagram, where the home brewer begins with a barley malt adds water to mash, then boils the wort followed by rapid chilling. After chilling, the wort is "pitched" with yeast to start the fermentation and ends with a drinkable beer. The entire process ranges from weeks to months depending on the type of yeast and sugars present.

Mashing consists of steeping a crushed barley malt, sometimes with other crushed grains (adjuncts used to create different varieties of beer), in hot water at temperatures between 40 -70º C. A Typical homebrew batch would begin with 4-6 kg of malt plus adjuncts. At temperatures around 60 - 65º, $\alpha-$ and $\beta-$ amylase enzymes from the barley malt are very activate and serve to hydrolyze starches to disaccharides suitable for yeast fermentation. The resulting sugary liquid is extracted from the spent malt in a process called lautering (essentially a filtration step). A common apparatus might consist of a 5 gallon insulated beverage cooler fitted with a false bottom and extracted sugars are drained from the mash above the false bottom. Additional hot water is washed through spent mash to collect 7 – 8 gallons of sweet wort. Many home brewers begin their process here with malt extracts they purchase from a supply store. These malt extracts are concentrated of the sugary liquids resulting from the mashing step and are commonly referred to as "sweet wort".

Beer made from sweet wort would be insipidly sweet, cloudy and highly subject to contamination from micro-organisms due to its high sugar content. In order to obtain a more flavorful beer, this sweet wort is vigorously boiled with hops and sometimes other spices for up to two hours. The primary objective of the long boil is to extract hop resins (mostly $\alpha$-acids and $\beta$-acids, consisting of humulones, lupulones and related compounds), these resins provide a necessary bitter component to beer flavor, as well as impart some antibacterial activity to prevent spoiling. Hops added late in the boil provide volatile essential oil that contributes to beer flavor and aroma. The long boil is also used to sterilize the wort prior to fermentation. A second objective of the boil is to coagulate various proteins fragments from the malt with tannins from the malt husks and hops to form a "hot break"; if left in the beer the tannins would impart an unwelcome bitter and astringent taste, and the proteins would contribute to shelf life instability. The wort boiling operation of a typical home brewing apparatus consists of a 10-gal boiling vessel containing about 6 gallons wort at the end of the boil (about 1-2 gallons of water evaporate during the boil). Heat is provided by a propane burner or electric heating element (such as the kitchen stove-top).

After the boil, the boiled wort must be cooled quickly and without contamination. Contamination occurs easily in this warm sugary liquid, therefore the faster it can be chilled the better. A common solution is to employ an immersion cooling coil (typically consisting of 25-ft of 3/8- in copper tubing, coiled into a cylinder 1-ft in diameter ) which suspended with the top of the coil at the liquid surface. The coil is hooked to a cold water source (usually the kitchen sink for a home brewer). The coil is simply placed in the boiling wort about 10 minutes before the end of the boil, which ensures the outside of the coil is also sterilized. As soon as the boil is stopped, cooling water is allowed to flow through the coil. It typically takes 45 minutes to 1 hour to cool 6 gallons of wort to 15º C (the typical desired temperature to "pitch" yeast for an "ale" style beer). During the cooling process additional protein fragments may coagulate into loose flocs, this is called the "cold break". The wort is covered and the cold break allowed to settle for an additional 15 minutes or so, after which the clear, cooled wort is siphoned off into a fermentation vessel. Approximately 1 gallon of spent hops and coagulated precipitate, collectively called "trub", is left at the bottom of the boiling vessel.

As the wort enters the fermenter (a sealed vessel around 5-6 gal in size), it is agitated vigorously to ensure the cooled wort is saturated with oxygen, and the yeast is pitched (added to the liquid). The oxygen is necessary to ensure appropriate yeast growth and reproduction. Fermentation takes place at about 15 º C for ales, and between 4 and 10 º C for lagers. Fermentation typically takes as little as 2 weeks for light ales, longer for strong beers and lagers. The beer is then bottled with a little extra sugar to induce fermentation for carbonation, or kegged. Bottle conditioning takes a couple weeks to a month, again depending upon the strength and style of the beer.

***Wort Cooling Demonstration***

The wort cooling demonstration consists of a 10 gallon stainless steel pot filled with about 7 gallons of water as a wort surrogate (shown in Figure 2).. The cooling coil is 25 ft of 3/8" copper tubing, loosely coiled into 8-10 loops about 10 inches in diameter, with the ends bent to protrude above the level of the pot. The coil is connected to a water supply, and the outlet to a floor drain. The coil is held in place by one or two clamps in such a way that the top of the coil is just covered by water in the pot. This is an important point: if the top of the cooling coil is more than a couple tube diameters below the liquid surface, during cooling a layer of hot liquid remains above, and the liquid in the pot is stratified.

**Figure 2.** The wort chilling demonstration set-up uses a 10 gal stainless steel pot and a copper cooling coil 25-ft in length and 3/8-in in diameter. The coil is looped about 8-10 times and is set to be level with the top of the liquid in the pit. Temperature sensors can be attached at various points in the pot or along the coil for measurements.

This apparatus is placed on a propane burner, and water brought to a boil. At such time the heat is turned off and the cooling water flow turned on (we set the cooling water flow to ~ ½ gal/min). For the demonstration, students are assigned to record the temperature in the pot and outlet cooling water temperature at regular intervals. The gentle convective cell created by sinking cooled water near the coil and warmer, rising water in the center of the pot is sufficient to ensure the water in the pot is essentially well-mixed, and that only one record of water (wort) temperature is needed for model calculations. The wort volume, cooling water flow and inlet temperature are recorded once. The class is given the opportunity to observe the cooled water shedding off the tube coils, a vivid demonstration of the boundary layer concept.

After the demonstration the students are asked to use this knowledge in several different ways. First, the students are given a survey to gauge their understanding of the concepts illustrated in the demonstration (questions given in Table 1). The students are then asked to used their design knowledge to scale up to a microbrewery size (100 gal batch) wort chiller design in their Heat Transfer Operations Course. They are also asked to expand their modeling knowledge with this example in their Process Modeling and Analysis course. Both courses use of the demonstration are given in this paper as well as a brief over view of the necessary heat transfer concepts used.

| # | Question |
|---|---|
| 1 | If you were to place the coils at the bottom of the pot, what would happen? Would you expect the same results as the demo? |
| 2 | What would happen if you decreased the coil spacing? |
| 3 | What kinds of heat transfer are occurring? In the coil/ in the pot? |
| 4 | Why can you see the boundary layer in this situation? How wide is it approximately? |
| 5 | What do you think would happen if you floated ice packs on the surface instead of using the cooling coil? |

**Table 1.** This table provides the list of questions given to the students to evaluate during the in class demonstration of the wort chiller immersion coil.

### *Experimental and theoretical heat transfer coefficients*

In order for the authors to explain their use of this demonstration within two different chemical engineering courses, the reader should know the level at which the students are familiar with experimental and theoretical heat transfer coefficicents. What follows is what the authors would expect the students to know from their courses here at Villanova University.

Experimental values of overall heat transfer coefficient (U) are calculated in the usual manner set forth in any standard heat transfer text, see for example Holman[3]. In addition, the technical evaluation of an immersion chiller in this application has been covered by Joye and Smith.[4]

$$Q = m_{cool} \cdot Cp \cdot \left(T_{cw,out} - T_{cw,in}\right) = U \cdot A_{ex} \Delta T_{LM} \qquad [1]$$

where $Q$ is the heat transfer rate, $m_{cool}$ is the cooling water flow rate, $C_p$ is water heat capacity, $T_{cw,out}$ is the cooling water outlet temperature, $T_{cw,in}$ is the cooling water inlet temperature, $U$ is the overall heat transfer coefficient based on $A_{ex}$ , the outside surface area of the cooling coils, and $\Delta T_{LM}$ is the log-mean average temperature driving force. Students are asked to calculate $U$ for each data set and plot these experimental results as functions of time.

The definition of the overall heat transfer coefficient is well known [3].

$$UA_{ex} = \left( \frac{1}{h_o \cdot A_o} + \frac{\Delta r}{k \cdot A_{lm}} + \frac{1}{h_i \cdot A_i} \right)^{-1} \qquad [2]$$

where $h_o$ is the outside heat transfer coefficient due to natural convection, $\Delta r$ is the wall thickness of the cooling coil tube, $D_o$ is the outside diameter of the cooling coil tube, $D_i$ is the inside diameter of the cooling coil tube, $A_i$ is the area based on the inside of coiling coil, $A_o$ is the area based on the outside of the coiling coil, $A_{lm}$ is the logarithmic average area of the cooling coil, $k$ is the thermal conductivity of the cooling coil tube, and $h_i$ is the inside heat transfer coefficient due to turbulent flow of cooling water. The inside heat transfer coefficient can be calculated from the well known Sieder-Tate correlation for turbulent forced internal convection [3] as given in equation 3.

$$h_i = 0.027\left(\frac{k}{D_i}\right)\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3} \qquad [3]$$

In equation 3, Re is Reynolds number and Pr is the Prandlt number. A correction for curvature of the coil is not necessary, because the bend diameter is rather large compared to tube diameter. Reynolds numbers were all in the fully developed turbulent region, therefore mixing due to turbulence is much stronger than currents induced by curvature of the tube. Cooling water properties were evaluated at the bulk average temperature. The viscosity correction was not used here, because we could not accurately measure the wall temperature of the tube. The resistance from the varying viscosity is not the major one, therefore neglecting the viscosity correction term will not have much effect on the final outcome.

Students can calculate a theoretical external heat transfer coefficient, $h_{o,}$ based off Nusselt number correlations for external natural convection off a cylindrical geometry as shown in equation 4.

$$\mathrm{Nu} = \frac{h}{k\ D} = 0.53\ \mathrm{Gr}^{1/4}\ \mathrm{Pr}^{1/4} \qquad [4]$$

In equation 4, Gr is the Grashof number, where the distance measure is "D", the outside diameter of the tube. The Grashof and Prandtl numbers are evaluated at the film temperature, which is an arithmetic average of bulk fluid temperature and wall temperature. Wall temperature can be estimated by subtracting the ΔT due to the outside film resistance (equation 5).

$$T_{wall} = T_{wort} - (h_o/U_o)\Delta T_{av} \qquad [5]$$

Thus it is possible to obtain as estimate of the overall heat transfer coefficient entirely from correlations and compare this directly with experimental values. These calculations are consistent with the typical approach used in heat transfer analysis and design, and would typically be done in a course in unit operations or heat transfer during the junior year on a more traditional double pipe heat exchanger.

### *Heat exchanger design – applying the demonstration to a Heat Transfer course*

In the Heat Transfer Operations course (junior year fall semester), the wort chiller demonstration was used to help introduce the concepts of natural convection and heat exchanger design. The demonstration was also as a springboard for a heat exchanger design for a microbrewery (a larger scale~100gal batch). Additional design criteria are discussed at length, including the need for scrupulous cleanliness. During this discussion we try hard to describe the alternatives: immersion coils, shell and tube, and plate and frame exchangers without clearly establishing which is best. We do not mention that the overwhelmingly favored brew-house design is plate and frame, primarily for their compact footprint and ease with which they can be dismantled and completely inspected for cleanliness. We would perhaps note that there are a number of shell-and-tube heat-exchangers expressly marketed to the home brewing hobbyist;

however, the simple coil design has significant advantages in cost, ease of operation, reduced possibility for contamination, ease of cleaning. Since shell and tube exchangers are not easily dismantled, and inspected for cleanliness, shell-and tube heat exchangers are not widely used by microbrewers and industrial-scale brewers. Students tend to struggle with this decision, as most textbook examples use a shell and tube heat exchanger, so they feel most 'comfortable' with this design. They realize, however, as they attempt to justify their decision by cost and sterility issues it is not the best choice.

The relationship between fermentation batch volume and copper heating coil area provides a clear introduction to the laws of process equipment scaling. In the Heat Transfer project, students who try to make an oversized version of the immersion chiller for the microbrewery may intuitively realize that designs that place adequate are of copper coil into the boiling vessel are cumbersome at best, or more likely simply unrealistic. However, the Process Modeling and Analysis course, this conclusion is explored more fully (described in the next section).

The brewing process is also an excellent way to introduce some other design considerations besides efficacy and cost to students. Sterilization, already discussed, is key in this design as well as the consideration of the proper coolant since the process is food grade and therefore regulated heavily. The students are also urged to be "green" in their designs and make efforts to reduce their energy consumption. The authors have found that students are very creating in their attempts to "green" their process and have proposed ideas such as recycling the coolant (which exits as hot water) to either the mash, lautering or boiling steps in the brewing process. Students have also proposed using the "trub" (left over fermentables) as a biomass source that can be further processed into an energy source.

### *Investigations Using Model Equations in Process Modeling and Analysis*

In our curriculum, there is a required, senior level Process Modeling and Analysis course. Here our objective is to introduce relatively challenging mass and energy balance systems and explore the underlying mathematics and how the solutions (both analytical and numerical) to the models equations are affected by simplifying model assumptions. The course is a prerequisite for Process Control, and so an underlying course theme is the distinction between linear and non-linear systems of model equations. Model equations are built from first principles using a shell balance approach, so the simplifying construct of a log-mean $\Delta T$ is not introduced *a-priori*. In this context, the governing overall model equations are energy balances around the wort and cooling water are provided in equations 6 and 7, respectively.

$$\frac{dT_w}{dt} = \frac{Q}{V_w \rho_w Cp_w} = \frac{-U(T_w)\cdot A_{ex}\cdot\left(T_w(t)-T_c(t,z)\right)}{V_w \rho_w Cp_w} \qquad [6]$$

$$\frac{dT_c}{dt} = \frac{F_c}{\pi\, r_{ex}^{\;2}}\frac{dT_c}{dz} + \frac{U(T_w)\cdot A_{ex}\cdot\left(T_w(t)-T_c(t,z)\right)}{V_{ex} \rho_c Cp_c} \qquad [7]$$

In developing these we have made the assumption that the wort is a well-mixed system, and the overall heat transfer coefficient is not a function of coolant temperature. They function as a starting point for simplifying instructions depending upon the specific objectives or instructional point to make. For example, we show that if we assume *U* is constant, then the result is a linear system of equations, and we can discretize the cooling coil (equations 8 and 9).

$$\frac{dT_w}{dt} = -\sum_{i=1}^{n} \frac{U \cdot A_{ex}}{nV_w \rho_w Cp_w} \cdot \left( T_w(t) - \frac{(T_{c,i-1} + T_{c,i})}{2} \right) \quad [8]$$

$$\frac{dT_c}{dt} = \frac{F_c}{V_{ex}} T_{c,i-1} - \frac{F_c}{V_{ex}} T_{c,i} + \frac{U \cdot A_{ex}}{n\, V_{ex} \rho_c Cp_c} \cdot \left( T_w(t) - \frac{(T_{c,i-1} + T_{c,i})}{2} \right) \quad [9]$$

For example, using n=8 and the values for α, β, and ω given in equations 10a, 10b, and 10c, respectively, we can derive the A, $TT_0$ and B matrices (equations 11a, 11b and 11c).

$$\alpha = \frac{-U \cdot A_{ex}}{V_w \rho_w Cp_w} \quad [10a]$$

$$\beta = \frac{U \cdot A_{ex}}{n\, V_{ex} \rho_c Cp_c} \quad [10b]$$

$$\omega = \frac{F_c}{V_{ex}} \quad [10c]$$

$$A = \begin{bmatrix} n\alpha & -\alpha & -\alpha & -\alpha & -\alpha & -\alpha & -\alpha & -\alpha & \frac{-\alpha}{2} \\ 2\beta & -\omega-\beta & & & & & & & \\ 2\beta & \omega-\beta & -\omega-\beta & & & & & & \\ 2\beta & & \omega-\beta & -\omega-\beta & & & & & \\ 2\beta & & & \omega-\beta & -\omega-\beta & & & & \\ 2\beta & & & & \omega-\beta & -\omega-\beta & & & \\ 2\beta & & & & & \omega-\beta & -\omega-\beta & & \\ 2\beta & & & & & & \omega-\beta & -\omega-\beta & \\ 2\beta & & & & & & & \omega-\beta & -\omega-\beta \end{bmatrix} \quad [11a]$$

$$TT_0 = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix} K \qquad B = \begin{bmatrix} \alpha \frac{T_{c,in}}{2} \\ (\omega - \beta) \cdot T_{c,in} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad [11b, 11c]$$

The numerical solution to this example can then be found from using the A, B, and TT matrices in equation 12.

$$TT_j = TT_{j-1} + \left(A \cdot TT_{j-1} + B\right) \cdot \Delta t \qquad [12]$$

In equation 12 the index $j$ is over the time domain. Both the temperature profile in the coil and temperature profile of wort temperature versus time can be extracted from the resulting matrix of temperature values.

There are number of directions that this exercise can take depending upon the instructor's interests and objectives of the course. We briefly describe two here. First, in order to illustrate the value in non-dimensionalizing model equations for use in numerical packages, we can use the reference values given in equations 13a, 13b and 13c.

$$t_{ref} = \frac{V_{ex}}{F_c} \qquad [13a]$$

$$z_{ref} = L \qquad [13b]$$

$$T_{ref} = T_{w,init} - T_{c,in} \qquad [13c]$$

Using the reference values, equations 14 and 15 can be derived.

$$\frac{dT_{NDw}}{dt_{ND}} = \frac{-U(T_{NDw}) \cdot A_{ex} \cdot V_{ex}}{F_c V_w \rho_w Cp_w} \left(T_{NDw} - T_{NDc}\right) \qquad [14]$$

$$\frac{dT_{NDw}}{dt_{ND}} = \frac{-U(T_{NDw}) \cdot A_{ex} \cdot V_{ex}}{F_c\, V_w \rho_w Cp_w} \left(T_{NDw} - T_{NDc}\right) \qquad [15]$$

If we again assume that $U$ is constant, the linear discretized equations can be found (equations 16 a, b and c).

$$\alpha = \frac{-U\ A_{ex}\ V_{ex}}{F_c V_w \rho_w Cp_w} \qquad [16a]$$

$$\beta = \frac{U \cdot A_{ex}}{n\ F_c \rho_c Cp_c} \qquad [16b]$$

$$\omega = 1 \qquad [16c]$$

The matrix $A$ remains unchanged, the vector of initial temperature values all $T = 1$, and because of our choice of reference temperature the vector $B = 0$. This formulation is easily adapted for use in any numerical ODE solver package.

We find a more significant pedagogical exercise is to recognize that in either the dimensional of non-dimensional forms the time constants for the cooling water are much faster that the time constant for the wort. This observation suggests that with respect to any one element of cooling water, the wort temperature is at steady-state, and the cooling loop behavior is essentially steady-state. The governing equations for cooling water (equations 7 and 15) are changed to equations 17 and 18.

$$\frac{dT_c}{dz} = -\frac{2\,U \cdot \pi \cdot r_{ex}}{F_{ex} \rho_c Cp_c} \left(T_w - T_c\right) \qquad [17]$$

$$\frac{dT_{NDc}}{dz_{ND}} = -\frac{2\,U \cdot \pi \cdot r_{ex} \cdot L}{F_{ex} \rho_c Cp_c} \left(T_{NDw} - T_{NDc}\right) \qquad [18]$$

Students are asked to recognize this as a linear first order differential equation with the well-known exponential solution. For this system with a 25 foot coil length, the characteristic dimension (inverse of the eigenvalue) is 30 to 40 feet so it is reasonable to assume the temperature profile is linear. We show that the much simplier system of non-dimensional equations defined by equation 19a, 19b, 20a and 20b.

$$\alpha = \frac{-U\ A_{ex}\ V_{ex}}{2F_c V_w \rho_w Cp_w} \qquad [19a]$$

$$\beta = \frac{U \cdot A_{ex}}{2F_c \rho_c Cp_c} \qquad [19b]$$

$$A = \begin{bmatrix} 2\alpha & -\alpha \\ 2\beta & -1-\beta \end{bmatrix} \qquad [20a]$$

$$TT_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad [20b]$$

The numerical solution to this problem given by equation 21.

$$TT_j = TT_{j-1} + \left(A \cdot TT_{j-1}\right) \cdot \Delta t \qquad [21]$$

Equation 21 is essentially equivalent to the more complex discretized version above (equation 12). The key learning we wish to make clear is that we could have foreseen this by looking at the characteristic dimensions of the system first. The numerical results for both the complex discretization and simplified cases can be seen in Figure 3.

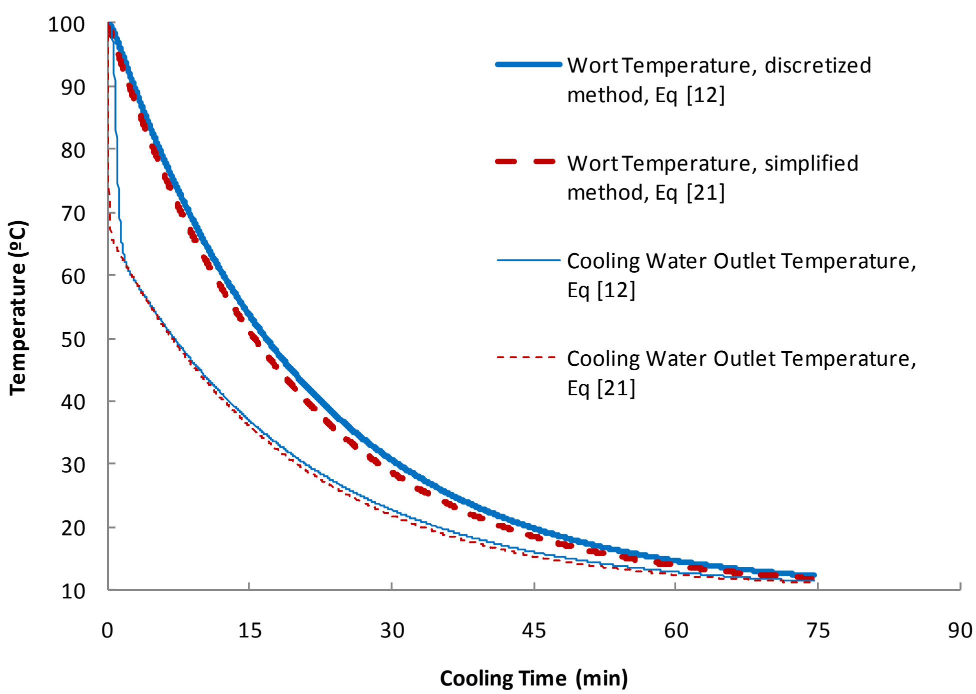


**Figure 3.** The numerical results for solving the complex discretized and simplified examples of the wort chiller problem.

This simpler system is also more amenable to introducing variation in the overall heat transfer coefficient *U,* which varies significantly due to changes in the dominant natural convection external film resistance. The resulting system would be non-linear, and it would then be appropriate to reintroduce the $\Delta T_{LM}$ as the appropriate driving force in the model equations, closing the loop the concepts introduced in Heat Transfer.

Again relating this to the "scale" of the design, here we observe that since volume scales by some characteristic length cubed and area by length squared, a simple doubling of tank diameter results in an 8-fold increase in volume, but only a 4-fold increase in area, thus the resulting area is proportionally only half of the original. The effect is considerably more with scale up from a homebrewer's 5-6 gallon batch to even a modest microbrewers 7 or 9 barrel brew house (one US barrel of beer is 31 gallons).

### *Conclusion*

In this paper the authors have shown how a simple home brewer's immersion-coil wort chiller provides rich and fertile material for investigating the nature of convective heat transfer in a Junior year heat transfer course, as well as a offers a system to demonstrate some important process analysis and modeling concepts one year later. The same basic problem is picked up in at least two different classes one year apart but with different focus. The authors believe students will grasp the essential difference in the two approaches to this demonstration and will gain a fuller understanding of both heat transfer and process modeling. The authors believe this demonstration can be extended for use in many different chemical engineering departments, and recognize further assessment of the student's improved knowledge of heat transfer and modeling should be implemented in future use of this demonstration.

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