Session 3365

How Students React to Formulations of the Straight Line Used in Engineering Courses

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Abstract

Straight lines are used to model and illustrate a variety of concepts in engineering courses. The formulations that are used depend upon the model or the concept that is being presented. However, some formulations are more difficult for students to recognize, to understand, and to use than others. The purpose of this paper is to illustrate four different formulations of the straight line in a variety of engineering courses and show the difficulties that student typically encounter in using them.

I. Introduction

When one teaches the same course over a period of time, one notices that different classes of students experience the same type of difficulties with certain sections or concepts of the material that forms the content of the course. A habit of noting what these sections and the related concepts are can be useful when the instructor wishes to revise the way and the order in which the subjects are presented.

Straight lines are used in a variety of purposes in engineering courses. These include the formulation and illustration of natural and empirical laws, curve fitting using linear regression, linear interpolation of data, and the modeling of a variety of behaviors over short intervals. Accordingly, different features are emphasized for different purposes.

Four features of the straight line that are commonly encountered are the following:
1)A line that is uniquely defined by two distinct points through which it passes.
2)A line that is uniquely defined by specifying one point through which it passes and a direction.
3)A line that has zero curvature everywhere along its length.

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4)The addition of two straight lines gives a third straight line.

In this paper, we illustrate four different formulations of the straight line and use examples to show the kinds of difficulties students typically encounter in using them.

II. Four Formulations

The four formulations discussed in this paper are discussed below.

Formulation #1: Slope-and -intercept formulation
Perhaps the most familiar formulation of the straight line is one that uses a slope and an intercept. This is typically used to help visualize the linear behavior of a system. Examples are Hooke's law, Ohm's law, Newton's law of viscosity and Newton's law of cooling.

Difficulties
There are three principal, but subtle, difficulties that students have with this basic formulation. All three seem to arise from biases students have developed from using this formulation only in one context.

a) The first-quadrant bias
In our experience, students appear to have a lot of experience working with lines and segments thereof that have positive slopes and are located in the first quadrant. However, they have limited experience with other quadrants. Exercises involving lines in the other quadrants are perceived as " more difficult". This bias may be due to two factors. First, faculty and authors of textbooks appear to favor the use the first quadrant for illustrations and derivations. Secondly, the convenience of working with positive quantities favors the use of the first quadrant as well. Thus, more work is done with this quadrant than with others in math and physics textbooks as well as in the engineering classroom. Indeed, many natural and empirical laws governing natural systems are typically written to make them easy to be represented graphically in the first quadrant. The student is expected to carry what was learned in the first quadrant over to the others.

b*) The positive- angles bias*
In mathematics textbooks, angles are typically zero on the positive x-axis and defined as positive in the counterclockwise direction. Many students have learned their circular functions well using this convention. They associate the cosine and sine of an angle with the horizontal and vertical sides of a right triangle, irrespective of which angle they are considering. Difficulties arise then, when they are using angles that are measured, say from the y-axis as shown in Figure 3, or from inclined planes, Figure 1.

c) The bias of the orientation of x-y coordinates wherein the x-axis is horizontal and the y-axis is vertical.
For many students, this is the only orientation of cartesian coordinates that they have ever used-- Perhaps with the exception of one exercise in Physics where a mass was sliding down an inclined plane--. Thus, when other coordinate axes are chosen so as to be inclined to these familiar ones,

difficulties arise with the evaluation of circular functions. For example, this occurs in doing stress transformations for rotated coordinate systems. Here, one needs to find the coordinates of a point relative to a coordinate system that has been rotated through some arbitrary angle.

Formulation #2: A line defined by two distinct points or by a given point and a specified direction.

This formulation is typically used in vector analysis wherein vector quantities are viewed as directed segments of straight lines. This is common in modeling forces and moments in mechanics and electrostatics. Examples include internal forces on two-force members of pinned trusses, flexible cables and guy wires.

a) *Position vectors:* A vector, **r**, that is fixed in space and used to locate a point, A, relative to another point, B. In cartesian coordinates, for example, one has:

$$\vec{r} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k} \quad \text{.....(1)}$$

Where i, j, and k are unit vectors along x, y, and z, respectively.

Difficulty: Typical errors here arise from matching this formulation to the correct direction of the resulting vector, when the name of the vector has been changed. Usually, that means knowing which coordinate to subtract from which. Although the student knows that

$$(x_B - x_A) \neq (x_A - x_B)$$

there is ambiguity in the students' mind about which difference one should use to get, say, the vector **AB** as opposed to the vector **BA**.

b) *Force vectors directed along a line.*

In problems involving three-dimensional vectors, the direction of a force is often specified by two points, say A and B, through which the line of action of the force passes. It is common to use the fact that the force sought has the same direction as one of the two possible position vectors[1],

$$vector = \overline{A}\vec{B}; or, vector = \overline{B}\vec{A}?.$$

Thus, the force vector can be formulated, using the unit vector, as the product of its unknown magnitude and the unit vector that can be obtained from the position vector of one the points relative to the other. Thus,

$$\vec{F} = F\vec{u} = F(\frac{\vec{r}}{r}) \quad \text{.....(2)}$$

$$\vec{u} = \frac{\vec{r}}{r} = unit.vector$$

Difficulty:

The process of dividing a vector by its own length appears to be very confusing to some students and a sterile operation to others who either fail to grasp it, or, perhaps, who steadfastly refuse to learn it. By contrast, although it is equivalent to finding a unit vector, the use of direction cosines seems less problematic to these same students. Discussion with students has revealed two important things to us. The first is that they view the computation of direction cosines as a useful idea because it is an extension into three dimensions of how they use trigonometry to compute components of a vector in the plane. The second one is that students view direction cosines as "formulas" that make sense because they can visualize what they are doing physically. That is, they can see that they are finding a vector one component at a time. By contrast, the computation of the whole unit vector in one step appears very abstract for them and does not relate easily to what they already know.

$$\vec{F} = F\vec{u} = F\cos\alpha\vec{i} + F\cos\beta\vec{j} + F\cos\gamma\vec{k} \qquad (3)$$

Where α, β, γ are coordinate direction angles.

$$\cos\alpha = \frac{F_x}{F} = \frac{r_x}{r}$$
$$\cos\beta = \frac{F_y}{F} = \frac{r_y}{r} \qquad (4)$$
$$\cos\gamma = \frac{F_z}{F} = \frac{r_z}{r}$$

Formulation #3: A line whose curvature is zero everywhere along its length.
This formulation is used in the study of deformable bodies and the stresses and strains to which they are subjected under loads of various kinds. Examples are deflections due to bending. The longitudinal strains, ε, are directly proportional to the curvature of the beam, κ, and the distance, y, from the neutral surface. Thus,

$$\varepsilon_x = \frac{y}{\rho} = \kappa y \qquad (5)$$

In this formula, the undeformed shape of the neutral surface is a plane. This makes the reference state for the deformation of the beam's neutral axis a straight line. Thus, when κ=0, the beam is straight and the longitudinal strain is zero for all positions y. Typically, the difficulty that students experience here is in visualizing both the strains, ε, and the curvature, κ, on the physical beam itself.

Formulation #4: The straight line as a first order Lagrange's interpolation polynomial.
In this formulation, the straight line is written as the sum of two constituent straight lines. It is commonly used in the following: a) the study of phase diagrams in thermodynamics; b) Mechanics to teach the principle of superposition; c) Numerical methods to do linear interpolation and extrapolation; and d) the study of the method of finite elements.

This first order Lagrange polynomial allows one to write a straight line between two points (x_1, f_1) and (x_2, f_2) of known coordinates as the algebraic sum of two constituent straight lines. The equation of the resultant line can then be written using the coordinates (x,y) of some point of that line as follows:

$$y(x) = \frac{x - x_2}{x_1 - x_2} f(x_1) + \frac{x - x_1}{x_2 - x_1} f(x_2) \quad \text{...(6)}$$

This formulation is commonly used in

a) *The study of binary phase diagrams* in thermodynamics; consider the simple equilibrium phase diagram of the copper-nickel system that demonstrates complete solubility. All compositions are in the liquid state for all temperatures that are above the liquidus line; similarly, all compositions are in the solid state for all temperatures below the solidus line. However, in the area that lies between these two lines, the liquid and solid phases coexist. A horizontal line drawn in this area to connect the liquidus and solidus lines is sometimes called a tie line. The amount of each phase that is present at any point along the tie line can be computed using the inverse lever rule. It says that the fraction of the total amount of substance that consists of each phase present at a point in a two-phase region is the ratio of the distance between that point and the end of the tie line opposite the phase to the distance between the two ends of the tie line[2]. If y (x) represents the total amount of substance at point x, and f(x1) and f(x2) the amount of substance at the liquidus and solidus lines, respectively, then, Lagrange's first polynomial that was written above states that y(x) equals the sum of the amount of liquid and the amount solid phases present at point x, where each constituent amount was computed using the inverse lever rule.

b) *Mechanics* to teach the principle of superposition; consider a horizontal beam of length, L, that is simply supported. Let the left end of the beam be called A (X_A, 0) and the right end of the beam be called B (X_B, 0). If the beam is carrying a concentrated downward load, W, locate at a point C (X_C, 0), and the reactions at the support are called R_A and R_B, respectively, then, from equilibrium, the magnitude of the load carried by each support is

given by the inverse lever rule, Figure 3. Thus,

$$W = R_A + R_B = \frac{(x_C - x_B)}{x_A - x_B} W + \frac{(x_C - x_A)}{x_B - x_A} W \quad \text{..(7)}$$

Difficulty:

The first quantity on the right-hand side of equation (7) is the reaction at support A; and the second one is the reaction at support B, Figure 2. The fact that the numerator of the ratio that is used to compute each support reaction does not contain the coordinate of its point of application is a common difficulty that students have with the inverse lever rule despite the fact that it is simply an application of Lagrangian interpolation.

These two examples are the most basic; and they illustrate the form of the formulation and the difficulty that it presents to some students. Similar relations are used in numerical methods for interpolating polynomials, in general, and in finite-element methods, in particular. Indeed, trial functions, sometimes called chapeaux functions, that are used in finite elements for interpolation between adjacent nodes are Lagrange's interpolations.

III. Conclusions

In this paper, we chose four basic formulations of the straight line. We stated each formulation and explained some of the difficulties that students encounter with each one. In our experience, these difficulties are primarily due to students' lack of familiarity with the physical meaning of the different formulations.

Although the concepts discussed in this paper are rather elementary, the issues that they raise impact what faculty do with their time and how students react to the way faculty teach.

The engineering curriculum attracts a variety of students. They may vary in the depth, the strength and the level of preparation they have had in the basic prerequisite courses as well as in their personal life's experiences. Naturally, their perception of what happens in the classroom affects how they react. Many persist and succeed; some get discouraged, or disenchanted, and change majors; others stop out; and still others drop out altogether.

I remember getting an unusually long letter from a student I "taught" many years ago, as part of a course evaluation of a Statics course. Excerpts of this letter are shown below.

" Dear Professor,
You asked for our honest opinion of your course. This is mine… I know that I failed your course. … I do not blame you for it. … I suspect that you wished I had been a better student in your class. Frankly, so do I!

However, in all fairness, I do not believe that it was all my fault. … I know that I am not as stupid as your grade will suggest. … I wish that I had had a better teacher, too- you know. Nothing against you personally. You are probably a nice guy and all that!

I feel that **IF** the university admits us, it should help us succeed. … I am not asking for handouts or freebies. …

I own my own small business; and I learned years ago that selling is great; but you have to service after you sell. I am not sure that this university does both as well as it could.

I am not unhappy; I am just disappointed. Thank you. …"

IV. Figures

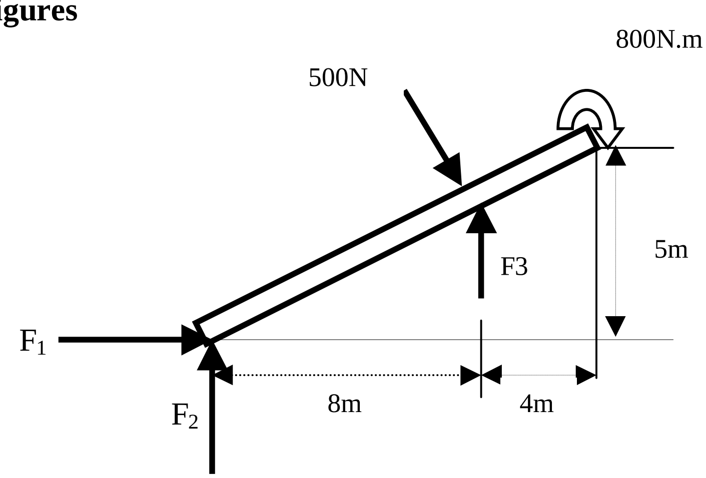

Figure 1. Neglecting the weight of the beam, find forces F_1, F_2, and F_3.

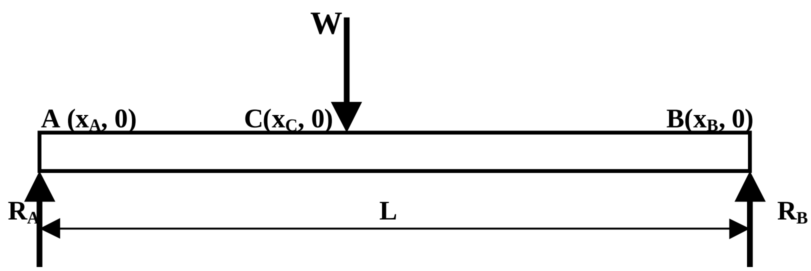

Figure 2. Reactions at the supports of a simply supported beam.

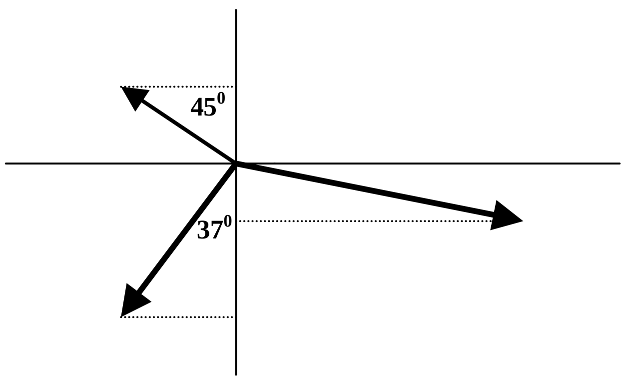

Figure 3. Examples of vectors located relative to the vertical axis.

References.

1. R.C. Hibbler, *Engineering Mechanics: Statics*. Eigth Edition,Prentice-Hall, Upper Saddle River, New Jersey , 1998, chapters 2-5.
2. Joseph Datsko, *Materials for Design and Manufacturing: Theory and practice*, Marcel Dekker, Inc., New York, 1977, pages 76-80.

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