

## **Implementation and Evaluation of Visual Algorithm to Teach Benefit-to-Cost Ratio Analysis**

**Dr. Hector E. Medina, Liberty University**

Dr. Medina is an Assistant Professor of Mechanical Engineering at Liberty University (Lynchburg, Va.), where also he leads the development of the Mechanical Engineering Program. He obtained a B.Sc. in Engineering from the Colorado School of Mines, and both M.Sc. and Ph.D. in Mechanical and Nuclear Engineering from the Virginia Commonwealth University. Prior to graduate school, he worked in industry and as a high school teacher for several years in his native Venezuela and Aruba. Since 2012, he has published more than a dozen articles in peer-review journals and conference proceedings. Journals include Applied Mechanics Reviews, Polymer, and International Journal of Solids and Structures. He has also presented at both national and international podiums and won presentation awards at ICONE20 and ICONE21.

**Mr. Benjamin T. Scoville, Liberty University**

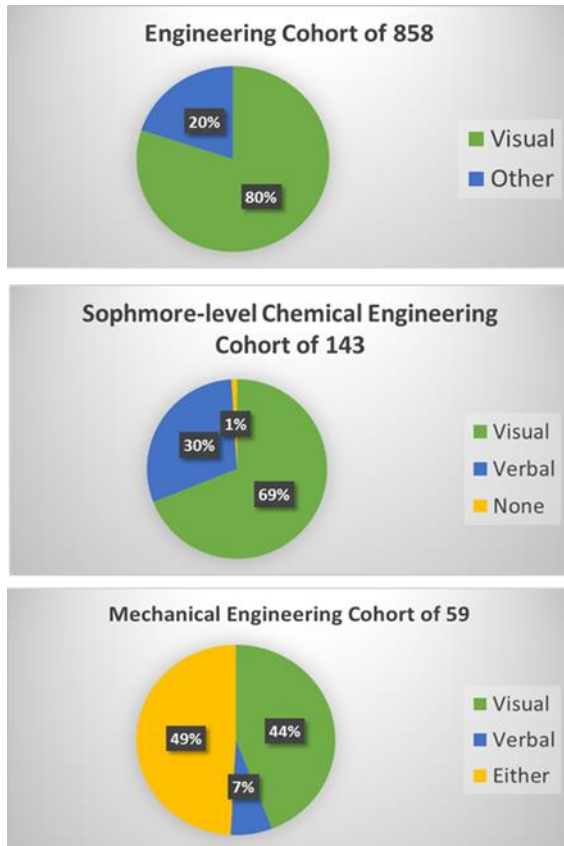
Benjamin Scoville is a third year student pursuing a degree in Electrical Engineering at Liberty University. His topics of intrigue are control systems, communication in automated systems, and cyber physical systems (CPS). Engineering education and CPS are his research interests. His other interests include piano, exploring the outdoors, baking, and marine aquaculture.

## **Implementation and Evaluation of Visual Algorithm to Teach Benefit-to-Cost Ratio Analysis**

*In the recent past, we developed a novel, visual, simple algorithm to teach incremental benefit-to-cost ratio (BCR) analysis to first- and second-year engineering students. The impetus behind that endeavor was twofold: (a) BCR analysis is the most used technique for economic analysis and decision making in the public sector, and (b) to accommodate to the visual learning style that dominates in the engineering student demographics. In the present follow-up work, we: (1) carried out statistical analysis to assess the reception and performance of students from two different semesters. Comparison is made versus the traditional incremental technique. Two null hypotheses were tested, Ho1: There is no difference in the true average levels of performance between the visual method and the traditional method; Ho2: There is no difference in the true average degree of acceptance between the two methods. (2) A simple mathematical proof is carried out to show the soundness of the method. The results corroborate, to a high confidence level, that students find the visual algorithm easier to use. Additionally, the data showed that there is no strong evidence to conclude that the performance is different, once a student has voluntarily selected a method.*

## Introduction

It has been established that a considerable majority of engineering students are either visual learners or exhibit no preference between verbal and visual styles. Figure 1 shows pie charts from three different studies carried out with 858 engineering students<sup>1</sup>, 143 sophomore-level chemical engineering students<sup>2</sup>, and 59 mechanical engineering students<sup>3</sup>, respectively. Due to large numbers of engineering students preferring visual methods for learning new information, unprecedented approaches to teaching concepts visually are needed in order for engineering



**Fig1.** Studies carried out, (*Top*): at the University of Ontario with 858 engineering students; (*Center*): in a sophomore-level chemical engineering class with 143 students; (*Bottom*): at two Midwestern universities with 59 mechanical engineering students.

students to receive the greatest benefit from the teaching-learning transactions<sup>4,5,6,7</sup>.

According to Haddad and Kalaani<sup>8</sup>, “visual learners learn best by engaging their visual senses as much as possible”. The use of visual aids can increase the learner’s remembrance of what was presented, while also providing an enhanced learning situation that can potentially increase understanding<sup>9</sup>. If the topic is not visually presented, but merely verbalized in class, there is a high probability that the visual learner would not retain it, but, even worse, could potentially be deterred from the concept<sup>10,11</sup>.

Jonassen and Grabowski<sup>12</sup> found that visual learners prefer graphs, diagrams, or pictures added to text-based material. In order for these students to excel to the fullest in their engineering courses, visual illustrations of relevance to course content should be utilized whenever possible<sup>13</sup>. Baukal and Ausburn<sup>4</sup> recommended that highly visual instructional content be incorporated when teaching students who identify as visual in the visual-verbal spectrum. In our particular context, it can be concluded that the integration of engineering economy concepts with visual teaching tools could be highly beneficial to optimizing retention and understanding.

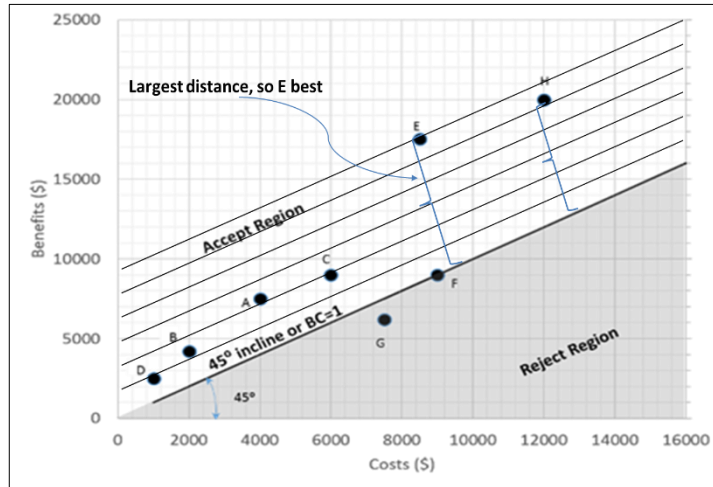
In response to the aforementioned need, an innovative, simple, visual algorithm for teaching benefit-to-cost ratio analysis to first-year engineering students has recently been developed and reported<sup>14</sup>. In the current follow-up work, results from the implementation of such technique are presented. In addition, a mathematical proof is presented to demonstrate that the traditional incremental BRC algorithm does indeed reduce to our visual technique, or vice versa. This paper is distributed as follows. First, a summary of the visual algorithm is briefly recapitulated—in order to make the current manuscript self-standing. Next, a mathematical

derivation is reported, which shows that the visual algorithm is mathematically identical to the incremental step-by-step traditional approach. Then, statistical analyses carried out with a cohort of students from two different semesters are reported. The paper is concluded with some recommendations.

### Visual Algorithm

For clarity in the sequel, following is a very brief recapitulation of the novel visual algorithm recently derived and reported—for details, see Medina and Ceffaratti<sup>14</sup>. In contrast to the traditional method<sup>15</sup>, the visual algorithm relies on visual aids throughout its development and teaching. Figure 2 shows a schematic of a typical complete solution using the novel visual algorithm (NVA) for a particular problem involving eight alternatives. First, all alternatives are plotted in a Cartesian

coordinate system with benefits (B) and costs (C) in the vertical and horizontal axes, respectively. An incline line of 45° from the C axis represents BCR = 1. The alternative whose distance from that incline is largest is the best alternative. In the following section, it is proven that this approach can be mathematically derived from the step-by-step method.



**Fig 2.** Typical visual aid to present the solution of a particular problem with eight alternatives. Note that the best alternative corresponds to the highest distance from the incline BCR = 1 line.

### Mathematical Derivation

Our visual approach claims that the alternative with the highest distance from the incline, BCR=1, is the best one. Obviously, this must be the case since incremental BCR analysis must lead to the same conclusion as the well-known PW analysis—and any other of the traditional economic analysis techniques—and it can be easily verified that the alternative heights corresponds to the their PW's. Furthermore, our visual algorithm should mathematically be derived from the traditional algorithm. In the sequel, this derivation is shown.

Let's start by stating the traditional incremental algorithm for any two alternatives *i* and *j* as:

$$\frac{B_i - B_j}{C_i - C_j} = \Delta BCR_{ij} \tag{Eqn. 1a}$$

$$\forall i, j: i, j = 1, 2, \dots, N, \text{ if } \begin{cases} \Delta BCR_{ij} \geq 1 \text{ select } i \text{ as the better alternative} \\ \text{else, select } j \text{ as the better alternative} \end{cases} \tag{Eqn. 1b}$$

Now let's consider the following pure rotation of the C-B coordinate system to a new C'-B' system, such that:

$$\begin{bmatrix} C' \\ B' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} C \\ B \end{bmatrix} \quad (\text{Eqn. 2a})$$

Notice that (2a) represents a pure rotation of  $\theta$  about the origin, such that if  $\theta$  is positive the rotation is CCW.

In polar coordinates, the incline BCR=1 corresponds to the line  $(r, \alpha) = (r, \pi/4)$ . Making  $\theta = \pi/4$  in (2) rotates the C-B axes  $45^\circ$  CCW, as follows:

$$\begin{bmatrix} C' \\ B' \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} C \\ B \end{bmatrix} \quad (\text{Eqn. 2b})$$

Which leads to expressions for C' and B' as:

$$\begin{aligned} C' &= \frac{\sqrt{2}}{2}(C + B) \\ B' &= \frac{\sqrt{2}}{2}(B - C) \end{aligned} \quad (\text{Eqn. 3})$$

Expressions (3) can be reversed to obtain expressions for C and B:

$$\begin{aligned} C &= \frac{\sqrt{2}}{2}(C' - B') \\ B &= \frac{\sqrt{2}}{2}(C' + B') \end{aligned} \quad (\text{Eqn. 4})$$

Next (4) can be plugged into the traditional algorithm (1), which leads to:

$$\frac{\frac{C'_{i+B'_i}}{\sqrt{2}} - \frac{C'_{j+B'_j}}{\sqrt{2}}}{\frac{C'_{i-B'_i}}{\sqrt{2}} - \frac{C'_{j-B'_j}}{\sqrt{2}}} \stackrel{? \rightarrow (i,j)}{\geq} 1 \quad (\text{Eqn. 5})$$

Where the symbols " $? \rightarrow (i, j)$ " denote a conditional such that if the inequality is true then select alternative "i", else select alternative "j".

One can easily verify that (5) can be expressed as:

$$\frac{(C'_i - C'_j) + (B'_i - B'_j)}{(C'_i - C'_j) + (B'_j - B'_i)} \stackrel{? \rightarrow (i,j)}{\geq} 1 \quad (\text{Eqn. 6})$$

And (6) can be further reduced to simply:

$$B'_i \stackrel{? \rightarrow (i,j)}{\geq} B'_j \quad (\text{Eqn. 7})$$

In a series of N alternatives, (7) clearly implies that the best alternative is the one with the highest B' value. This is graphically equivalent to selecting the highest height from the incline BCR =1 line.

The foregoing result, although simple, is remarkably important because it shows that the incremental analysis problem can be reduced to a single-step procedure.

Finally, we briefly show that (7) reduces to the present worth in the original C-B coordinate system.

Let k be the best alternative, that is, the one with the highest B' value. Also, let  $B'_k$  be the corresponding highest value. Substituting  $B'_k$  into eqn. (3b), we get:

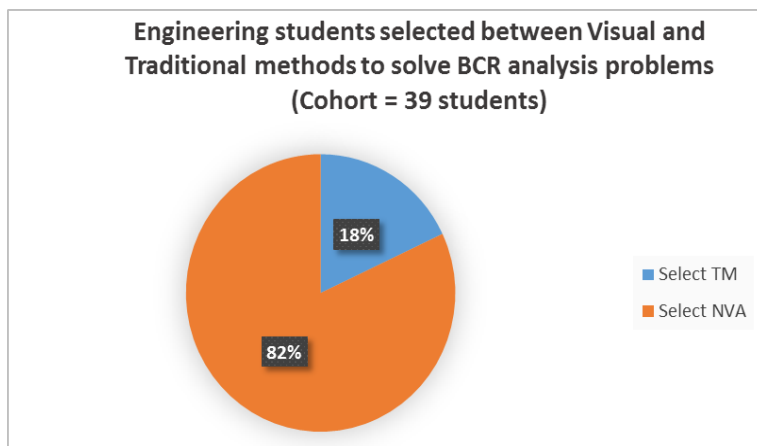
$$B'_k = \frac{\sqrt{2}}{2} (B_k - C_k) = \frac{\sqrt{2}}{2} PW_k \quad (\text{Eqn. 8})$$

Which simply implies that best alternative correspond the one with the highest present worth.

### Implementation and evaluation

As previously mentioned, data regarding student preference and performance has been collected over two semesters (a third semester cohort is currently under study). A total of 39 engineering students enrolled in a 200-level engineering economic analysis course were considered for this study. The information was gathered as follows:

- During a 50-minute class session the traditional step-by-step method was introduced during the first 30-35 minutes. The remainder of the time was dedicated to introducing the NVA method. An examination was applied to the students during the following class session. In general, the students were given the option to select the method to solve the problem. However, in one question they were



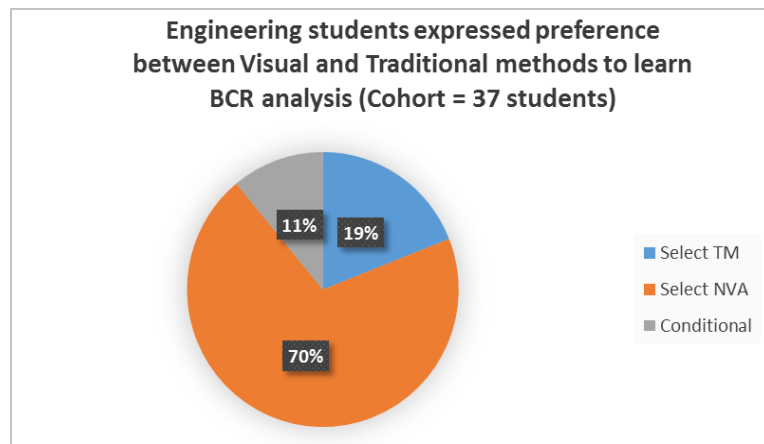
**Fig 3.** Pie chart showing the distribution of selected method between Visual (NVA) and Traditional (TM) methods. Students were given the option to select the method to solve the problems. A total of 39 students were tested.

explicitly asked to use the TM and in another the NVA. Graphing isometric paper was provided to the students.

- A survey was carried out after the students were tested. The survey contained only one question: “In class, we learned two methods to carry out BCR incremental analysis: (a) The traditional step-by-step method, and (b) the graphical method. State which method you prefer and explain why. (Since this assignment has only one question, make sure you write a comprehensive answer.)”

Figure 3 presents a pie chart showing the selected method the students use to solve the problems when the method was optional. Note that as much as 82% of the students (32/37) arbitrarily selected the NVA method to solve the given problems. This is in complete agreement with the studies mentioned previously.

Figure 4 shows a pie chart that summarizes the results from the survey. First, note that 2 out of 39 students did not answered the survey at all. From the survey, it is observed that 70% of students do explicitly express preference towards NVA while almost 20% are inclined towards the TM. Furthermore, about a tenth of the surveyed cohort expressed a conditional preference. This answer, although not explicitly asked in the survey, meant that students would select the visual method if the problem involve a large number of alternatives. However, for shorter problems, students expressed that the step-by-step method was more convenient to use.



**Fig 4.** Pie chart showing the expressed preference between visual and traditional methods. This information was gathered via a single-question survey. The conditional answer meant that students preferred the visual method for large-number-of-alternative problems; otherwise, they would chose the other method.

Statistical analysis was also carried out to complete this study. Two null hypothesis were tested:

(a) Null hypothesis  $H_{01}$ : *There is no difference in the true average levels of performance between the visual method and the traditional method.* With a high confidence level (>95%), no evidence was found to reject null hypothesis  $H_{01}$ . This implies that once a student decides a particular method, the level of performance (i.e., grade obtained) is independent of the method.

(b) Null hypothesis  $H_{02}$ : *There is no difference in the true average degree of acceptance between the two methods.* With a high confidence level (>95%), this null hypothesis was rejected. This implies that, in fact, there is a significant difference between the preferences of the two methods. (This result was expected based on the previous analyses.)

Finally, a representative collection of comments from the surveys were consolidated and shown in table 1. Note that both positive and negatives comments were included.

Typical comments about NVA	Typical comments on favor of traditional method
(+)“Easier to visualize and get answer from graph”	(+)“More effective to keep track and verify answers”
(+) “I am visual learner, so graphical method is better for me”	(-) “It is long and tedious”
(+) “It saves time	
(+) “ Visual is very helpful”	
(+) “Less room for errors”	
(-) “Some uncertainty if numbers are too close to one another”	
(-) “it requires graphics skills”	

**Table 1.** Typical comments about both visual and traditional method consolidated from the survey. Included are both positive (+) and negative (-) comments.

### Conclusions and Recommendations

- Having previously developed a novel algorithm for performing BCR analysis, in this follow-up work, its implementation was carried out and its reception was evaluated. Specifically, performance and preference of the visual algorithm by engineering students from two different semesters were studied. It was found that a large majority of the engineering students selected the visual method when given the option. It might be worth mentioning that, when surveyed, about a mere ten percent of the students revealed that part of the inclination towards the visual method was due to the fact that problems given in the test had a large number of alternatives. More importantly, over half of this cohort—without being asked—identified themselves as visual learners, while also noting the visual algorithm to be an easier and quicker process. However, it was found that once a student selected a method, the expected level of performance was comparable for either method.
- Undoubtedly, it is clear that engineering students prefer visual learning methods. This fact should pose a strong motivation to instructors to develop new visual teaching aids and to incorporate existing ones into class activities.
- The mathematical proof presented in this document, although simple, could have significant consequences. Since the long step-by-step method is the most used technique for economic analysis and decision making in the public sector, one wonders if the extra effort invested has been unnecessary. The fact that the long incremental algorithm reduces mathematically to a simple maximization problem should lead to the conclusion that the extra calculations are redundant.
- Additional investigation is recommended to see if reducing the number of alternatives affects significantly the results. Further studies are currently being carried out.



## References

- <sup>1</sup> P. Rosita, Specific Differences and Similarities in the Learning Preferences of Engineering Students, *29th ASEE/IEEE Frontiers in Education Conference*, San Juan, Puerto Rico, pp. 12c1.17-12c1.22, 1999.
- <sup>2</sup> S. Montgomery, Addressing Diverse Learning Styles Through the Use of Multimedia, *Frontiers in Education Conference, 1995 Proceedings*, Atlanta, GA, pp 3a2.13-3a2.21, 1995.
- <sup>3</sup> R. Mayer and L. Massa, Three Facets of Visual and Verbal Learners: Cognitive Ability, Cognitive Style, and Learning Preference, *Journal of Educational Psychology*, Vol. 95, No. 4, pp. 833-846, 2003.
- <sup>4</sup> C. Baukal & L. Ausburn, Learning Strategy and Verbal-Visual Preferences for Mechanical Engineering Students, *121<sup>st</sup> ASEE Annual Conference and Exposition*, Indianapolis, Indiana, June 15-18, 2014, Paper ID #8682.
- <sup>5</sup> T. Nguyen & I. Khoo, Learning and Teaching Engineering Courses with Visualizations, *World Congress on Engineering and Computer Science*, San Francisco, California, October 20-22, 2009, ISBN:978-988-17012-6-84.
- <sup>6</sup> M. McGrath and J. Brown, Visual Learning for Science and Engineering, *Computer Graphics and Applications*, *IEEE*, Vol. 25, No. 5, pp. 56-63, 2005.
- <sup>7</sup> Y. Haik, and K. Moustafa, "Thinking and Learning Preferences for a Sample of Engineering Students at the United Arab Emirates University", *Emirates Journal for Engineering Research*. Vol. 25, No. 1, pp. 65-71, 2007.
- <sup>8</sup> R. Haddad and Y. Kalaani, Adaptive Teaching: An Effective Approach for Learning-Centered Classrooms, *ASEE International Forum*, Indianapolis, Indiana, June 14, 2014, Paper ID #11049.
- <sup>9</sup> R. Felder and L. Silverman, Learning and Teaching Styles in Engineering Education, *Engineering Education*, Vol. 78, No. 7, pp. 674-681, 1988.
- <sup>10</sup> R. Felder, Reaching the Second Tier: Learning and Teaching Styles in College Science Education, *J. College Science Teaching*, Vol. 23, No. 5, pp. 286-290, 1993.
- <sup>11</sup> D. Raviv, An Intuitive Approach to Teaching Key Concepts in Control Systems, *121<sup>st</sup> ASEE Annual Conference and Exposition*, Indianapolis, Indiana, June 15-18, 2014, Paper ID #9179.
- <sup>12</sup> D. Jonassen and B. Grabowski, *Handbook of Individual Differences, Learning and Instruction*, Lawrence Erlbaum, Hillsdale, NJ, 1993.
- <sup>13</sup> R. Felder, D. Woods, J. Stice and A. Rugarcia, The Future of Engineering Education II. Teaching Methods That Work, *Chem. Engr. Education*, Vol. 34, No. 1, pp. 26-39, 2000.
- <sup>14</sup> H. Medina, and K. Ceffaratti, "Novel visual algorithm to teach benefit-to-cost ratio analysis", *Proceedings of the 122<sup>nd</sup> ASEE Annual Conference and Exposition*, Seattle, WA, June 14-17, 2015, Paper # 14237.
- <sup>15</sup> D. G. Newnan, T. Eschenbach, and J. P. Lavelle. *Engineering Economic Analysis*. Vol. 2. Oxford University Press, 2004.