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Implementing Applied Dynamics

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Abstract

The programs in mechanical engineering education programs and related fields do not provide adequate training to enable graduates to conduct analytical investigations of actual problems in dynamics. An analytical approach to the improvement and development of mechanical systems allows the purposeful control of the parameters and in the same time saves time and resources. The analytical investigation in applied dynamics comprises the following three steps:

1) Composing the differential equation of motion of the system and determining the initial conditions of motion;
2) Solving the differential equation of motion for the initial conditions of motion;
3) Analyzing the solution according to the goal of the investigation.

As it is well known, the differential equation of motion consists of loading factors: forces or moments. Active loading factors cause the motion, while resisting loading factors oppose the motion. The left side of the differential equation of motion consists of the sum of resisting loading factors including the force or moment of inertia, while the right side of the equation includes the sum of active loading factors (in the absence of active factors the right side of the equation equals zero). During the beginning of the analytical approach to solve a problem, the investigator, based on the information of the problem, should figure out the characteristics of the loading factors that are applied to the system. Obviously, insignificant loading factors could be ignored. It should be emphasized that the results of the investigation depend upon the measure of accuracy that the differential equation reflects the working process of the system.

The engineering programs do not offer a straightforward universal methodology of solving linear differential equations of motion. However, as it is known, the Laplace Transform allows solving any linear differential equation of motion or a system of two linear differential equations of motion for a two-degree-of-freedom system. The Laplace Transform methodology comprises the following three steps:

1) Converting the differential equation of motion into a corresponding algebraic equation of motion based on Laplace Transform Pairs that are presented in tables. There is no need to memorize the fundamentals of Laplace Transform in order to use the tables that are available in numerous publications;
2) Solving the obtained algebraic equation of motion for the displacement in the Laplace domain.
3) Inverting the obtained solution in the second step from the Laplace domain into the time domain. This inversion is also based on the Laplace Transform Pairs and actually represents the solution of the differential equation of motion.

The solution represents the law of motion of the system or its displacement as a function of time. The analysis of the solution includes taking the first and second derivatives of the displacement in order to determine the velocity and acceleration. By applying conventional mathematical

actions to the obtained parameters of motion, it is possible to determine the role of the parameters of the system in achieving the goal of the investigation.

Michael Spektor's book Solving Engineering Problems in Dynamics published by Industrial Press could be helpful in getting familiar with the considerations presented above. In addition, it should be mentioned that all these methodologies are fully applicable to electrical engineering and other fields.

Introduction

Modern technology is rapidly changing, requiring the application of the most effective methods of improvement and development of engineering systems. These methods include comprise the purposeful analytical investigations of mechanical and related systems in the area of applied dynamics. Computer programs play a very important role in performing stress analysis and solving other problems. The input data for stress analysis should include information regarding the characteristics of the loading factors that in general could be obtained from an appropriate analysis in applied dynamics. The main role of the analytical investigations in the area of dynamics related to engineering systems consists in providing parameters of the systems in order to obtain the required performance of the system during the working process.

Practicing engineers very seldom apply the analytical investigations in solving problems in dynamics. In most cases, the job descriptions for engineers do not emphasize the analytical approach as a required step in the improvement and development of an engineering system. The engineering education programs do not provide to the graduates with an adequate training in this regard. However, analytical investigations in dynamics very often could save a lot of time and eliminate significant expenses.

As an illustration of the importance of applying the analytical methods in engineering dynamics, it would be interesting to relate to the following real-life episode that happened in the mid-seventies in the former Soviet Union.

There was a severe need to improve the safety harnesses for the personnel working at high elevations in the construction and related industries. At those times in all countries the safety harnesses had a similar structure consisted of a belt made of technical fabric and a six-foot long steel rope attached to the belt and having on its end a hook for securing to an appropriate support. Fatal accidents occurred during the accidental fall of a person equipped with a safety harness. In many cases, the person slipped out from the belt. Examining the harness after the accidents, the experts did not find any damage to the belt or to other components of the harness. So it was assumed that the victim did not properly buckle the belt. The severity of the problem required a drastic improvement of the harness. An appropriate governmental agency hired one of the most reputable research and development intuitions in the country to solve the problem. Researchers with Ph.D. degrees, along with engineers, and technicians were involved in the project to development a new harness that would solve the problem. The project involved a few years of work and when completed was scheduled to be reported at a national conference related to industrial safety. Two days before the conference, the leaders of the research and development team arrived at the agency that facilitated the project, in order to present the results.

It happened that a few hours before this presentation, which was to serve as a rehearsal for the national conference, a scientist, familiar with dynamics, met with the chief of the agency. This meeting was not related to the issues of the safety harness. But at the end of the meeting, the chief asked the scientist to attend the safety harness presentation.

The safety harness presentation lasted an hour while the investigators submitted a written report and demonstrated prototypes of new harnesses emphasizing very smart buckles and other related components. The report included a survey of all international standards related to safety harnesses and the analysis of their structures and components. It was emphasized that the British standard indicated that a free fall from the height of as little as six feet can be fatal even while the person remains in the belt. The investigators conducted numerous experimental tests of the harness and were satisfied with the results. The researchers also developed a new national standard for safety harnesses. The newly developed harness had the same structure as the existing one. During the discussion, the appointed reviewers approved the results of the research and development project and recommended to implement the new safety harness in the industry. After the presentation, the chief of the agency invited the scientist to his office and asked him about his opinion related to the outcomes of this project. The scientist emphasized that in his opinion the most important aspect that should determine the scope of such an investigation was missing, and therefore there was no justification for the recommendations proposed by the project. He explained that the missing part consists of the analytical investigation of the deceleration process of a human body in this kind of a safety harness. The scientist also mentioned that the investigators did not address the important statement of the British standard regarding the fatal outcome even while the body remained in the belt. The chief asked if the scientist could perform this analysis, and if yes, how much time this work would take. The answer was that the analysis would take only a couple of hours. The next day the scientist, having already completed the analysis, met again with the chief. The scientist presented the steps of the analytical investigation that included composing of the differential equation of motion of the human body equipped with a safety harness, solving of this equation, and analyzing the solution. The analysis had shown that during the deceleration of the human body in the harness, the fabric belt elongates to the extent that the body could slip out from the belt. In this case the elongation represents an elastic deformation of the belt and instantly disappears after the unloading process. This is why the experts did not find any damage to the belt. Thus, the analysis did not support the assumption that the person slipped out from the belt because that person did not buckle the belt properly. In addition, the analysis had shown that in case of a relatively stiff belt and a certain position of the body the overload on the body due to deceleration could be critically injure a person. Finally, the analysis had shown that the deceleration of the body depends on the ratio between the combined stiffness of the system body-harness and the mass of the body. Therefore, the stiffness of the harness must be properly adjusted to each person depending on his or her weight. In other words, the same harness should not be used for people having significantly different weights. Based on this analysis, the scientist recommended first to attach to the belt straps that go around the shoulders and legs in order to prevent the possibility of slipping out from the belt. The chief of the agency accepted the recommendation of the scientist. The presentation of the research and development team on the national conference was cancelled and the project was abandoned! The chief asked the scientist to present to this conference the results of his analytical approach and propose the recommendations regarding the straps. These recommendations were accepted. It is interesting to mention that the current safety harnesses in the United States now have these straps and attachments that adjust the stiffness of the harness to the depending on weight of the person. More details regarding the contemporary structure of the safety harness in USA can be obtained from the OSHA Documents [1]. This real-life story speaks for itself. What a waste of time and resources! Just a couple of hours of work based on the analytical approach versus a few years of work of a team of professionals diligently following methodologies that had a very limited analytical background changed everything.

Thus the scientist composed a differential equation of motion that reflects the deceleration process of the accidental fall of a person equipped with a safety harness. He solved the equation and analyzed the solution of the equation.

Actually, the methodology of the analytical investigation of engineering problems in dynamics comprises these three steps:

1) Composing the differential equation of motion and determining the initial conditions of motion;
2) Solving the differential equation of motion;
3) Analyzing the obtained solution.

Consideration of Some Details of These Steps

The differential equation of motion should reflect the actual working process of the mechanical system with a justifiable degree of accuracy. Composing the differential equation of motion is based on determining the characteristics of the loading factors playing the important roles in this working process. Current engineering education programs do not provide adequate training in composing differential equation of motion for real-life problems. It should be emphasized that in the majority of engineering education problems associated with dynamics, the differential equation of motion is the most important element of the analytical investigation. The differential equation of motion in this case is a second order differential equation, the structure and the components of which are based on principles of mathematics. This equation is not specifically targeted to dynamics and may have numerous applications in human activities. The parameters of motion and the factors that cause and oppose the motion become the components of the second order differential equation. It is obvious that in mathematics the components of the equation are dimensionless. As we know in dynamics the parameters related to motion such as time, displacement, velocity, and others, have units that differ from each other, and, consequently, cannot be put in one equation. However, multiplying these parameters by certain coefficients allows bringing them to the same units. This allows us to use the modified parameters, having the same units, as components of a second order differential equation of motion. The components of the differential equation of motion represent the loading factors (forces or moments.) Newton's second law stating that the force equals to the product of multiplying the mass by the acceleration has become the basis of structuring the differential equation of motion. In rectilinear motion the acceleration (the second derivative) is a parameter of motion, while the mass is the coefficient that makes their product has the unit of a force. Multiplying the velocity (a parameter of motion representing the first derivative) by the damping coefficient results in a product that is also force. And similarly other parameters of motion being multiplied by corresponding coefficients result in products representing forces. Similar procedures are applied in rotational motion, in which the angular acceleration is multiplied by the moment of inertia and their product represents a moment, etc. Therefore, in order to compose a differential equation of motion the investigator should determine the characteristics of the loading factors that support and oppose the motion of the mechanical system. The factors opposing the motion, including the force or moment of inertia, are usually placed in the left side of the differential equation of motion, while the factors that cause the motion are included in the right side of the equation. In the case when the system moves by inertia, the right side of the equation equals zero. The investigator, based on justifiable reasons, decides which factors should be included in the differential equation of motion and which factors could be ignored. The investigator should be familiar with all existing loading factors and their characteristics that are applicable to the differential equations of motion. However, it is difficult to find in published sources a definitive statement that makes it clear how many and what kind of loading factors could be related to applied dynamics. This may explain why the investigators sometimes are not sure that they addressed all relevant loading factors while composing the differential equation of motion. Without a complete understanding of the characteristics of all existing loading factors that could be included in a differential equation of motion, it is impossible to justify that a particular differential equation of motion composed by an investigator for a particular problem is actually correct. An attempt to describe and classify all existing loading factors that could be included in differential equations of motion is presented in the recently published book of Michael Spektor [2]. This book addresses many other related issues. The loading factors could be linear or non-linear causing the linearity or non-linearity of the differential equations of motion. Currently there are no methodologies for solving non-linear differential equations. The existing catalogs that offer solutions for some non-linear differential equations have a very limited applicability to engineering problems. The vast majority of common mechanical engineering problems in dynamics could be solved with an acceptable level of accuracy described by linear differential equations of motion. Obviously, an approximate solution is better than no solution. The investigator makes appropriate decisions regarding the characteristics of these factors and their role in the problem under consideration.

Sometimes the investigators are not sure that the particular differential equation of motion fits to or reflects the given engineering problem. The equation must be composed based on the characteristics of the particular problem and these characteristics should represent loading factors relevant in describing motion in dynamics. The differential equation of motion has just one argument, and that is the running time. Such important factors as the cost, the efficiency, the safety, the temperature and other things that may influence the problem cannot be directly included into the equation. Including such factors into an equation would make it as an equation with multiple arguments. And there does not exist a differential equation of motion with multiple arguments. It should be clear that for the required accuracy of the analytical investigation of a problem, it must be composed just one correct differential equation of motion.

The next step of the analytical investigation is the actual solving of the differential equation of motion for the determined initial conditions of motion. The engineering education programs offer some ways of solving differential equations of motion. However, these ways do not have a theoretical background (except when integrating very simple differential equations). Usually the solutions begin with some strategic assumptions that are based on the analysis of the components and the structure of the differential equation. To make an appropriate assumption one should have a certain experience in solving differential equations. It is interesting to mention that the differential equation describing vibrations could not be solved using these strategic assumptions until the famous scientist L. Euler, based on his intuition (not on any science), inserted the complex number into the solution. In the 19th century, Pierre-Simon Laplace developed a scientifically based straightforward universal methodology that allows the solving of any linear differential equation of motion [3]. The Laplace Transform methodology is briefly discussed in many engineering programs. It should be emphasized that there is no need to memorize the Laplace Transform fundamentals in order to use this methodology for solving the differential equation of motion. This methodology comprises the following three steps:

1) Converting the differential equation of motion from the time domain into a corresponding algebraic equation of motion in the Laplace domain.
2) Solving the obtained algebraic equation for the displacement in the Laplace domain.
3) Inverting the equation of the displacement from Laplace domain into the time domain. The inversion represents the solution of the differential equation of motion with the given initial conditions of motion.

The conversion and inversion are based on using tables of Laplace Transform Pairs. Numerous related publications contain these tables [4].

Let us consider an example.
It is required to solve the following differential equation of motion that describes free vibrations in the horizontal direction:

$$m\frac{d^2x}{dt^2} + Kx = 0 \tag{1}$$

where m is the mass of the system, K is the stiffness coefficient of the elastic link, x is the displacement of the system as a function of running time t.

The initial conditions of motion are:
for $t = 0 \quad x = 0; \quad \frac{dx}{dt} = v_0$ (2)

Applying the conventional actions to the equation (1), we may write:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{3}$$

where
$$\omega^2 = \frac{K}{m} \tag{4}$$

Now we perform the first step of the methodology consisting of converting the differential equation (1) with its initial condition (2) from the time domain into the Laplace domain. Here is an example of a table with Laplace Transform Pairs; in this table, the letter l represents the complex argument in the Laplace domain.

Table 1

Time domain functions	Laplace domain functions
1. Constant	1. Constant
2. x or $x(t)$	2. $x(l)$
3. $\frac{d^2x}{dt^2}$	3. $l^2x(l) - lv_0 - l^2s_0$
4. $\sin \omega t$	4. $\frac{\omega l}{l^2+\omega^2}$

Using the pair 3 from the table, we convert the first term (the second derivative) of the equation (3) with the initial conditions (2) from the time domain into the Laplace domain. Based on the pairs 1 and 2, we convert the second term of the equation (1) into the Laplace domain (zero in both domains is the same). As a result of the conversion we obtain the corresponding algebraic equation of motion in the Laplace domain:

$$l^2x(l) - lv_0 + \omega^2x(l) = 0 \qquad (5)$$

The initial displacement s_0 in our case equals zero). The second step consists of solving the equation (5) for the displacement in Laplace domain $l(x)$. Using conventional algebraic actions, we have:

$$x(l)(l^2 + \omega^2) - lv_0 = 0 \qquad (6)$$

Finally we obtain:

$$x(l) = \frac{lv_0}{l^2+\omega^2} \qquad (7)$$

During the third step, using the pairs 1, 2, and 4, we invert equation (7) from the Laplace domain into the time domain and obtain the solution of the differential equation of motion (1) with the initial conditions of motion (2):

$$x = \frac{v_0}{\omega}\sin \omega t \qquad (8)$$

This concludes the consideration of the example related to the use of the Laplace Transform methodology for solving differential equation of motion. It should be mentioned that most of the published tables of Laplace Transform Pairs are not targeted toward solving engineering problems in dynamics, and often it is challenging to find the needed Transform Pairs.

The above mentioned Michael Spektor's book addresses the aspects of composing, solving, and analyzing linear and non-linear differential equations of motion including one and two degrees of freedom mechanical systems. This book contains a table of Laplace Transform Pairs that are appropriate for solving mechanical engineering problems. It should be mentioned that the Laplace Transform methodology is applicable for solving differential equations describing electrical and electronics circuits and for other engineering fields.

The last step of the analytical investigation consists of the analysis of the solution of the differential equation of motion that represents an equation of the displacement as a function of time (the equation (8) in the case considered above). This equation is often called as the law of motion of the system. Taking the first and second derivatives of the displacement, we determine the velocity and acceleration of the system respectively. The displacement, the velocity, and the acceleration are the three basic parameters of motion that should undergo purposeful investigations in order to determine the role of the parameters of the system. Actually, the result of this analysis represents a set of analytical expressions that allow to control the parameters of the system in order to achieve the goal, or in other words - to solve the problem.
The engineering programs adequately provide all the needed knowledge that is required to perform the purposeful analysis of the solution of the differential equation of motion.

Conclusion

It should be emphasized that in order to implement applied dynamics in the engineering education programs there is no need to reconsider the existing courses. The courses related to differential equations and dynamics in most programs are offered during the sophomore year when the students did not yet acquire sufficient knowledge in engineering. Applied dynamics is a combination of several sciences. Therefore, it can be recommended to offer an appropriate new course in applied dynamics for senior and graduate students. The Michael Spektor's book mentioned above could be helpful in implementing this course.

References:

[1] *Fall Protection in Construction*. OSHA 3146-05R 2015
[2] Spektor, M., *Solving Engineering Problems in Dynamics,* Industrial Press,
[3] Grossman, S. and Derrick, W., *Advanced Engineering Mathematics,* Harper & Row Publishers
[4] Churchill, R., *Operational Mathematics,* McGraw-Hill Book Company
[5] Bracewell, R.M., *The Fourier Transform and Its Applications*, Chapter 11, The Laplace Transform, *pp 219 to 240, McGraw*-Hill Book Company