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JUNE 22 - 26, 2020 #ASEEVC
Paper ID #32019

Improving STEM Education by Analyzing the Design of a Bottle

Alexander Henderson, San Jose State University

Alexander Henderson is an undergraduate student attending San Jose State University to attain a bachelor's degree in mechanical engineering along with a minor in aviation. Since starting his college years as a freshman in 2015, he has participated in a wide variety of engineering courses that have helped him achieve multiple accomplishments. Two of these accomplishments include being recognized as a dean scholar by San Jose State University and collaborating in the development of an aeroshell for Spartan Hyperloop. After participating in the CSU system for five years, he knows the strengths and weaknesses that are present in the American education system today.

Alexander Garcia, San Jose State University

Alexander Garcia is a student attending San Jose State University to attain a bachelor's degree in Mechanical Engineering. In his third semester he and his group presented the functionality of LEDs to early elementary school groups from another school. Before that, from 2008 to 2017, he attended community college, at first not knowing what his final goal was before deciding to pursue mechanical engineering and earning an associates degree in physics at Mission College.

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Abstract

STEM education for students is an essential aspect of today's education since it leads to greater development in the modern world and more technological achievements. Achieving this sort of education can be done by performing an engineering analysis to determine the volume content of a wine bottle. The project conducted here was to determine the volume of a wine bottle by measuring it at various points and using mathematics to perform this real-world analysis. Its objective is not only to reflect on what was done but to introduce it as a STEM-related project to encourage new growth within the STEM fields. There was also extended reasoning as to why STEM is important in today's world through small examples of what already exists. This is done as a step toward increasing the standing that the United States holds currently within the world's STEM community, which is low compared to other countries. This is partially due to a lack of interest that many American students have in STEM-related fields. To fix this, the STEM-related activities that students perform in schools today need to incorporate what they do in their everyday lives.

1. Introduction: Math in Engineering and STEM

The first question to ask here is, "What is STEM?" STEM is an acronym for "Science, Technology, Engineering, and Mathematics." It is an educational curriculum dedicated to the subjects stated in the name. The term was first coined by Judith Ramaley in 2001, where she was an Assistant Director of Education and Human Resources at the National Science Foundation [1]. It was initially called "SMET," but she and others agreed that "STEM" was better because it eliminated the implied level of importance of science and mathematics over-engineering and technology [2]. After this event, the focus on STEM started to extend out to the curriculum used in many other countries.

As more countries started to implement a STEM-based curriculum into their education system, the United States (U.S.) began to lag further behind. This can first be seen in a survey done in 2015 by the Programme of International Student Assessment, which ranked the U.S. 38th among 71 other countries surveyed in mathematics, and 24th among the same countries in science [3]. These low scores triggered a National Academies of Sciences report, which argued that the U.S. needed to strengthen K-12 education programs in areas relating to science and mathematics under the America Competes Act [3]. These results show that many American students that are within K-12 education programs are struggling with subjects that are STEM-related, whether they are a first grader who is learning how to add or a sophomore in high school who is taking algebra.

Unfortunately, this is not a new problem that the U.S. has faced. Even before the early 2000s, the U.S. was suffering from a lack of education in both mathematics and science. In 1965, the International Association for the Evaluation of Educational Achievement conducted a test on math students in 12 different countries to solve 70 problems, one which the U.S. ranked last [4]. In 1989, a test of similar purpose ranked the U.S. last and then second-to-last in 1990 for mathematics [4]. It was through the first implementations of STEM education in the U.S. that helped ordinary U.S. children and teenagers to learn more material relating to science, mathematics, technology, and engineering. It helped push more students to pursue a STEM career when they grew up. Despite these improvements, U.S. students are still behind the rest of the world in these areas.

The primary motive behind this lack of effort among K-12 students today is mainly due to their lack of interest in STEM-related subjects. It must be remembered that the interest in STEM education and careers varies from personal reasons to conditions outside the control of potential and current students. The Pew Research Center did a survey in 2017 and found that 52% of the surveyed adults agree that the STEM-based curriculum for today's youth (around 21%-24%) was too hard for them [5]. These courses normally became hard between the ages of thirteen to seventeen [6]. Other reasons included that STEM education did not relate to their career goals or it did not catch their interest. Due to the indifference in these subjects, these students flocked to other subjects. In 2009, a report showed that two competing subjects against STEM were visual and performing arts indicating people were more interested in entertainment over mathematic, engineering, and other STEM-related topics [3]. This influx of people entering the entertainment industry has had extensive effects on STEM education. The lack of interest among students in STEM fields has caused the U.S. to currently rely on immigrants, who have STEM-qualifying degrees and training of variable levels from other countries, to help keep our technology and engineering industries at a standard level with the rest of the world [7]. It has also caused more students to move towards degrees in the liberal arts while pushing more students away from degrees and careers in STEM fields, which can have lasting effects in the U.S.

The curriculum categorized as STEM is essential to maintain the level of technology that the U.S. currently has and to make further advances for the benefit of people worldwide. This STEM education improvement is especially true since there has been an increase in STEM job demands between 2000 and 2010, along with the increased benefits that pursuing a STEM educational approach has on students [8]. Some of these benefits include a better understanding of the environment that people live in today, better preparation for a college education, greater creativity, and its vital importance for both STEM and non-STEM jobs alike [8]. Despite the greater demands and the benefits that STEM education has on students, many schools and students in the United States still lag in STEM fields.

So, what can be done to improve STEM education in the U.S.? One step that can be taken is to introduce projects that have a more practical implication in a student's everyday life. Mathematical courses alone are not elaborating on how relevant these subjects are to other future curriculums, courses, and applications. The same can be said of lower-level science courses. By introducing more group projects involving real-world problems to these students, the educational system may catch the interest of students in how they can help in STEM-related projects run by technological and engineering groups. This type of method can be seen when we took an engineering approach to perform a volumetric analysis of a wine bottle.

2. The Volume Calculation on the Bottle

Performing this volumetric analysis on a wine bottle has allowed us to determine if the mathematical functions derived from simple measurements done on the bottle were similar to the volumetric calculations that the manufacturer provided. When deriving the volume of this glass bottle, several assumptions had to be made since the process of glass blowing these bottles will create some slight variances in the final shape of each bottle. The first assumption comes from the wine bottle itself not being filled to maximum capacity. When wine bottles are distributed to consumers, they are usually partially filled to the 750 ml volume mark. In reality, these bottles can hold up to 800 ml of wine. The main reasons for this include obtaining a better wine and allowing space for the wine bottle cork to be placed [9]. The second assumption made for our computation is from the fact that the wine bottle has a punt on the bottom. It is placed here so that the wine bottle is more natural to hold, has a more prominent resistance to higher pressure, and to make the bottle larger than it appears [10]. By doing this, the likeliness of people buying these bottles goes up. The last assumptions to be made when calculating the volume includes the glass bottle having a constant thickness and a smooth surface all the way around. This assumption helps make determining the individual equations for the wine bottle a lot easier to derive with only a minimal amount of error.

Using a problem from Dr. Hsu's book "Applied Engineering Analysis" as a baseline [11], the project entailed the use of calculus to calculate the volume of the four outermost sections of the bottle and the punt. This included deriving the equations for two straight sections as well as the three curved sections. The straight sections were assumed to be simple cylinders, while the curved sections of the outermost section of the bottle and the punt were assumed to be parabolic equations to get the closest approximation for the volume. These curves can be seen in Figures 1 and 2.

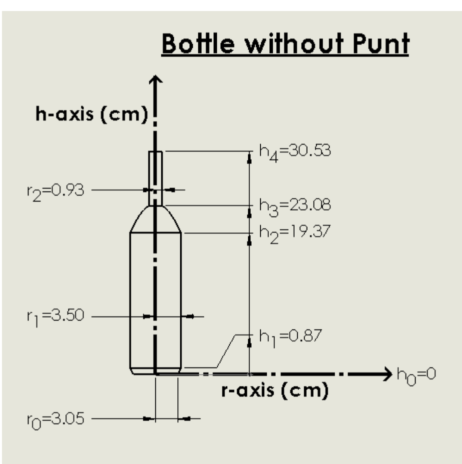

Figure 1. The four curves that were derived to determine the volume of the bottle without the punt.

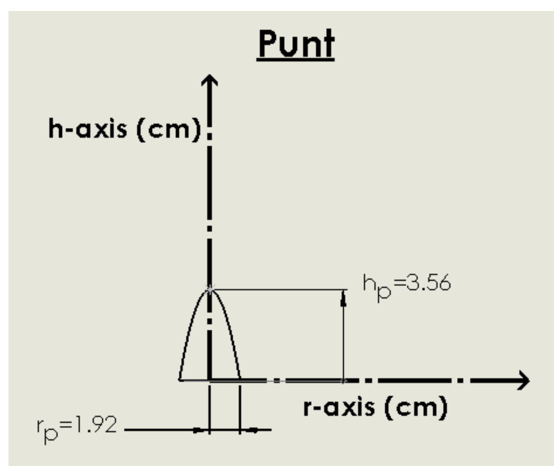

Figure 2. The shape of the punt used to determine the derivation of this curve.

To determine what the equations are for the three assumed parabolic curves and two assumed straight lines, the different heights and inner radii at different points on the bottle needed to be found since the measurements that were taken were based on the diameter of the outer shell of the bottle. Since the assumption that the bottle has a constant thickness is made, Equation 1 is used to determine the inner radius value. This equation states that

$$r_n = (d_n/2) - t \quad (1)$$

where r_n is the inner radius being measured (cm), d_n is an outer diameter that is being measured, and t is the thickness of the glass bottle (cm). Once these inner radii and heights at different sections of the bottle have been found, the equations for the different parts of the bottle can then be determined based on these values. The straight sections of the bottle can be assumed to follow the format of the equation $r=r_n$ where r is on the x-axis and r_n is the value on the x-axis itself.

The curved sections follow the format for a quadratic equation (which states $y=ax^2+bx+c$). The constants a, b, and c of this equation can be determined based on at least three random points off of each curve assuming that the y-axis goes straight through the middle of the bottle. The equations for each of these curves can then be used to determine the volume of each individual section by rotating these curves around an axis (in this case, around the h-axis). This can be done by using the integral seen in Equation 2, which finds the volume of an infinite number of cylinders between two points [11].

$$V=\int_a^b [\ \pi r^2] dh \quad (2)$$

Determining the different height and radius measurements as well as the different equations for all the different curved sections would be needed to determine the total volume of the wine bottle. This required the use of measuring tools such as rulers and measuring cups, and water and clay to fill the bottle and punt for volumetric measurements. The first step in determining the total volume was to determine what the volume of the wine bottle was without the punt. This was first done by using the calipers to measure the diameter at the lip of the neck and main body of the bottle. After using a ruler to measure the different heights of the bottle, the mathematical functions were derived which allowed for the total volume of the wine bottle to be calculated without the punt.

The second step in determining the total volume was to find the volume of the punt. This was accomplished using a clay mold where the clay was packed into the punt itself. Once the clay was removed from the punt, the height and the diameter at the lowest point of the mold were used to derive the necessary quadratic equation of this mold. This allowed for the volume of the punt to be calculated. Once this value was found, the total volume of the wine bottle could be determined by subtracting the volume of the punt itself from that of the wine bottle without the punt. This can be directly seen in Equation 3 which states

$$V_{Tot} = V_N - V_P \quad (3)$$

where V_{Tot} is the total volume (cm^3), V_N is the volume of the bottle not including the punt (cm^3), and V_P is the volume of the punt (cm^3). This procedure was used to determine the volume content when the wine bottle was filled to 800 ml and when it was filled to 750 ml. Using the curves seen in Figures 1 and 2, the equations for these curves, as well as the volume amounts that they represent, can be seen in Tables 1, 2, and 3 on the next page. The work to attain these values can be seen in the appendix. After comparing the calculated values to the measured values using wine bottles, water (with red dye for better visualization), and measuring cups, it was determined that the percentage error at 750 ml was 6.39% and 1.93% at 800 ml.

Table 1. The equations used to derive the volume values at 750 ml and 800 ml for each individual section

Bottle without Punt			
	Equation	**Volume at 750 ml**	**Volume at 800 ml**
Top	$r = 0.925$	2.55 ml	20.03 ml
Curve Dome	$h = -0.327r^2+23.364$	76.28 ml	76.28 ml
Bottom	$r = 3.495$	709.93 ml	709.93 ml
Curve Foot	$h = 0.299r^2-2.776$	29.54 ml	29.54 ml
Total	x	818.30 ml	835.78 ml

Table 2. The equation for the punt (height as a function of radius) and the calculated volume

The Punt	
Equation	**Volume**
$h = 35.6 - 0.09627r^2$	20.38 ml

Table 3. The total calculated volume at 750 ml and 800 ml

Overall Volume		
Equation	**Volume at 750 ml**	**Volume at 800 ml**
$V_{Tot} = V_H - V_P$	797.80 ml.	815.40 ml

3. Why perform volume measurements on a wine bottle?

The purely mathematical volume was verified in two ways. It was first verified using a virtual model created using SolidWorks. After creating the exact curves in SolidWorks (using the equation spline tool), and rotating these curves around a central axis, a virtual replica of the volume was created while taking the thickness into account. A volumetric analysis of that model confirmed that the final equations and the calculus-based math performed on these curves were correct.

The values were then verified by filling up the bottle with red dyed water to the calculated values and seeing how they compare to the actual values. These methods helped determine the error percentage between our findings and the measurement from the real bottle was minuscule. Several factors could have caused the primary sources of this error. These include (but are not limited to) inconsistent thickness, risen graphics, and the grilled ring at the bottom of the bottle. The factors mentioned here are due to the fact that glass wine bottles are blown from hot glass material into a mold, which causes each bottle to be adherently different from one another in some way. Therefore, the volume that each bottle will have will be slightly different from one another. Some other minor factors for error include the presence of surface tension and air bubbles in the water, both of which might help increase the apparent volume size of the bottle by a small amount.

One way to improve these results includes adding more points to solve for various kinds of curves, including elliptical curves, trigonometric curves using functions like sine and cosine, and multi-level polynomials. A third-level or even a fifth-level polynomial, for example, would work better to calculate the transitive volume going from the main body to the neck because of its shape with the "corners" between these two sections not being ninety-degree angles on a real bottle. By using a higher-level polynomial, the correlation between the mathematical model and the actual shape of the bottle would also be more precise since a polynomial function is working with more degree values. This higher correlation value is exactly what is desired when determining what should and shouldn't be used in a model. Another way to improve the results is to choose another type of container to fill with water for volumetric analysis. By choosing multiple models of container, we can challenge ourselves with a variety of methods to find their volume. This is important because engineers don't take shortcuts and assume that something is going to be what they precisely calculated using mathematics. However, this isn't always the case. For this reason, engineers must analyze everything mathematically and physically to make sure their models work.

After performing this mathematical analysis, there will still be many U.S. students who will be wondering as to why these volume measurements needed to be taken for a wine bottle. Again, this is to clarify to all people that the mathematics that they use in their everyday lives will be helpful to them. Right now, many U.S. students are struggling in mathematics. Many students learn how to perform these mathematical skills from a professor whose background is only in mathematics [12], which results in students learning how to perform basic mathematical skills with excellence. However, this limited amount of knowledge creates a problem where students have a harder time applying their mathematics in real life. In other words, students do not have the practical knowledge required to be able to understand, enjoy, or apply what they've learned into STEM fields; therefore, they shun the field completely out of lack of interest.

All of this results in a substantial amount of discouragement for students to move towards a career in engineering or any other STEM-related job opportunities. This lack of confidence can be supported by the 50% of US students (who try to earn a degree in a STEM field) that end up choosing a non-STEM career even though STEM careers earn 25% more [13]. Again, students might be learning the material well, but there is a struggle when they try to apply it in a real-world situation. They lack the practical knowledge to be prepared to take on a real-world situation, and instead wonder how learning mathematics will help them in real life.

To solve this problem, schools across the U.S. need to start implementing STEM programs, however, one of the difficulties in implementing STEM programs is making the material learned in the classroom relatable to real-life situations. This can change if the classroom environment is improved to implement the engineering and mathematical approach in STEM to help solve global issues. Some of these issues include climate change, energy efficiency, and hazard mitigation [14]. By taking this approach, it will allow more students to be more aware of what's happening around them and to push themselves to fix these problems. It will encourage students to learn how to create different machines and mechanisms so that they can apply what they learned in school. However, the reality is that schools have a hard time doing this and stick with a more mathematical approach without applying it in the real world. This approach psychologically discourages even more students to be pushed away from STEM-related fields, especially when more are needed in modern society. It creates a lack of purpose and resolve that many people today cannot afford to have.

This lack of purpose that students face is the reason why teachers need assistance in creating a curriculum that both inspires STEM-related activities and the implementation of these activities. Teachers need to show students that math can be used in a variety of disciplines and in multiple ways to benefit society. This can be accomplished by assigning students to perform projects using engineering and mathematical approaches. The projects do not have to be complicated; it just has to be relatable. It can be something as simple as finding the volume of a wine bottle or determining how long it will take to fill a bucket with water. If there is no relation between the student and the project that the student completes, the students will not be motivated to move toward a STEM-based career.

The project that a student chooses is also highly dependent on a student's grade level. There are already small lessons in Kindergarten classes that involve showing students the most basic perspectives on science such as how gases are created by chemical reactions, which can be demonstrated by a balloon inflating due to the expansion of said gases. What if that example could be expanded to include mechanical applications such as how expanding gases move simple machines like pistons on flywheels? For students in elementary school, they could do a poster presentation that incorporates the basic mathematical concepts that they are learning and how they are used. This might require a quick jump into algebra merely for the sake of timing how fast something goes when pulled by gravity. Doing so would make the concepts easier to understand as they advance in school, especially when they hit the middle school level. Here they can create a situational analysis project that applies to both the basic mathematical concepts that they learned in elementary school as well as the algebraic concepts that they continue to learn. When they hit high school, they can start to perform more advanced analysis projects (and maybe even some engineering-based projects) that incorporates what they learned in their multiple mathematics courses. Some of these courses include geometry, statistics, and even calculus. Examples of such projects might include a structural analysis problem in a geometry class, weather analysis in statistics, and analysis of force in calculus. This will prepare more students for what they will face both in college and in the real world. By taking a more project-based approach towards math courses early on, students will know that the mathematics they learn can and will be useful in a variety of STEM-related fields.

4. Other Applications for Mathematics in Engineering and Science

The information learned in a STEM education curriculum, such as determining the volume of a curve, can be used in a variety of applications. Some examples of STEM applications include the volume determination of a flask or for a medical IV bottle used in a hospital. However, just like when the volume of a wine bottle was determined, specific assumptions for each case will have to be made. Each case will have different assumptions that will need to be made depending on the external conditions. Also, making more assumptions about a problem will most likely cause the error value for the result to be higher than attended. Insufficient data will also do this. Whenever using mathematics to solve a problem such as the volume determination of a wine bottle, be sure that there are a reasonable number of assumptions, as well as enough data, to make the solution accurate and easy to derive.

There are plenty of other applications that mathematics can be used for aside from finding the volume of an object. One place they can be used for is aircraft manufacturing. Here, statistics can be used to determine if a specific part is held up to standards using control charts. Calculus can also be used to determine the magnitude and direction of velocity or acceleration that a jet might face under certain conditions or to determine the amount of power that a jet might need to produce. It can also be used to determine the natural frequencies in which a plane's wing will vibrate. These types of examples should be explained to students so that they understand how engineers use the mathematics they learned from going to school and participating in STEM-related activities.

First-order differential equations can be used in a variety of ways when performing heat exchange and fluid analysis. When observing these types of analyses, the three fundamental laws of physics need to be taken into consideration [11]. These include the laws of conservation of mass, energy, and momentum [11]. Using these laws, along with the principles of first-order differential calculus, allows for the first law of thermodynamics as well as many other relationships to be derived. These relationships include Fourier's Law of Heat Conduction, Newton's Law of Cooling, and Bernoulli's Law. Though other factors play a role in the development of these laws, all the laws used in heat exchange and fluid flow analysis are derived from basic mathematical principles that can be learned in high school.

While weather conditions are observed through computer systems, people had to develop the mathematical formulas that are used to show the weather and its everlasting effects. The use of computers for weather forecasting did not start until the 1980s, and it was not until after the end of WWII that numerical weather prediction was developed [15]. Today, meteorologists continue to contribute to the analysis by using functions that calculate and include wind pressure, wind velocity, temperature, and humidity [16].

Earthquakes can be either minor or major, and their effects can be destructive to our homes. This is why analysts are needed to help predict their effects. In a given location, it would be essential to find out where the epicenter of an earthquake is so that people could use it in a function of intensity where it is most active [17]. This location would help to determine what kind of damages could occur in areas where people live, assuming they live close enough to feel the effects.

Two other implications that the usage of mathematics pertains to includes the development of swimming pools and houses. Swimming pools and houses need advanced mathematics skills to be designed and constructed. Without them, each would not be able to meet safety standards and codes required by organizations like FEMA and NFIP for coastal areas. For pools, several factors are taken into account, depending on the kind of pool being constructed. These include the load which the water exerts on the structure of the pool, the mass of the soil in which the pool is being built, and the bending moments at the base of the pool [18]. For educational problems involving swimming pools, these factors can be added to expand the problem into a more involved concept. For houses, geometry is used to design the structures before the final construction. However, over the years, the way houses are constructed changed to conform to environmental conditions, including earthquakes, heavy snow, and hurricanes. Engineered designs use math formulas to determine the proper amount of materials and the construction of houses which have the potential to experience these kinds of conditions [19].

Besides the consideration of the volume of different vessels and different weather and earth conditions, safety factors must also be considered. Mathematically, a safety factor is the ratio between an ultimate condition and a maximum condition. While there are no rules set for determining the values of safety factors in engineering analysis, there are reasons in which they would be set. Such conditions include how credible and sophisticated the mathematical analyses are, the reliability of material properties used in the mathematical analyses, public safety, and environmental conditions [11]. There are other factors such as variations that may occur in a load-bearing member of a structure, types of failures that may occur in those members, deterioration due to poor maintenance, and the number of loadings expected during the structure's lifetime [20].

Known safety factors differ depending on the application. One example of a low safety factor is that used in aircraft production pertaining to the weight of the materials used to build the aircraft because airline companies want their planes to carry as many people as allowed on the planes to get the maximum amount of revenue per flight. Higher safety factors are used in the construction of wire ropes and lifting equipment among other applications. For wire rope, higher values would be crucial to account for weather conditions, dynamic forces, and the diameter of the pulley wheels that the wire ropes may be turned on [21].

5. Conclusion

The number of students interested in pursuing a career in any STEM field is decreasing. This is the result of the material being learned in classrooms today not being relatable to the student's life or the environment that U.S. students are living in today. To reverse these effects within the U.S. education system, the STEM material that is being learned needs to become relatable to the students. This can be done by performing real-life activities that require a mathematical approach.

One such example of this would be determining the volume content of a wine bottle for which the authors of this paper performed. Although many people have completed this mathematical analysis, we couldn't find anyone who has publicly documented it to this extent. We performed two basic steps in this analysis. These steps include deriving the quadratic and linear equations, and then using these equations to determine the volume of the wine bottle with calculus. This alone has a variety of applications in the field. Some of these activities include the volume determination of a flask or of an IV bottle used in a hospital. Other such activities where mathematics can be used in the real world include aircraft manufacturing, heat exchange, and fluid flow (where safety factor needs to be considered) as well as weather analysis, earthquake analysis, and home construction.

If the U.S. is to maintain or improve its standing in STEM education and careers, it needs to increase the general interest in its subjects. Students of all levels need to be introduced to the many opportunities that STEM careers provide and how they benefit the world. This can only happen if we can make the materials they learn in their mathematics and science classes more relatable to their everyday lives. By doing so, the U.S. can greatly improve the level of participation in STEM fields.

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Appendix

The following work that is seen below is based from the measurements seen in figures one and two.

Volume of the Top (V_1)

The following measurements were taken from the numbers seen in figure 1:

- r_2 =0.93 cm
- h_3 =23.08 cm
- h_4 =30.53 cm (for 800 ml)
- h_4 =24.03 cm (for 750 ml)

Remember that $V = \int_a^b \pi r^2 dh$ where

$$r = 0.93,$$
$$a = h_3, and\ b = h_4$$

After deriving the equation for this curve, the volume was determined to be the following if the bottle was completely filled to 800 ml:

$$V_1 = \int_{23.08}^{30.53} \pi\, r^2 dh = \int_{23.08}^{30.53} \pi\, (0.93)^2 dh \rightarrow$$
$$V_1 = \pi[0.865h]_{23.08}^{30.53} = 9.403\pi = 0.865\pi(30.53 - 23.08)$$
$$V_1 = 20.03\ cm^3 = 20.03\ ml$$

The volume was determined to be the following if the bottle was filled to the 750 ml:

$$V_1 = \int_{23.08}^{24.03} \pi\, r^2 dh = \int_{23.08}^{24.03} \pi\, (0.93)^2 dh \rightarrow$$
$$V_1 = \pi[0.865h]_{23.08}^{24.03} = 9.403\pi = 0.865\pi(24.03 - 23.08)$$
$$V_1 = 2.55\ cm^3 = 2.55\ ml$$

Volume of the Curve Dome (V_2)

The following measurements were once again taken from the numbers seen in figure 1:

- r_1 =3.50 cm
- r_2 =0.93 cm
- h_2 =19.37 cm
- h_3 =23.08 cm

Remember that $V = \int_a^b \pi\, r^2 dh$ where

$$h = -0.327r^2 + 23.364 \rightarrow r^2 = 71.45 - 3.058h,$$
$$a = h_2, and\ b = h_3$$

This equation was derived using three points, which included (-3.495, 19.37), (3.495, 19.37), and (-0.925, 23.084), and the following mathematical operations based off of the basic relationship for a quadratic equation ($y = ax^2 + bx + c$):

$$19.37 = 12.215a - 3.495b + c \qquad (4)$$
$$19.37 = 12.215a + 3.048b + c \qquad (5)$$
$$23.084 = 0.856a + 0.925b + c \qquad (6)$$

Subtracting equation 3 from equation 1, we get

$$-11.36a = 3.71 \ \rightarrow\ a = -0.327$$

Subtracting equation 2 from equation 1, we get

$$6.996b = 0 \ \rightarrow\ b = 0$$

Knowing what the values for a and b are allows for the calculation of c to take place by plugging a=0.299 and b=0 in either equation 1, equation 2, or equation 3. By plugging these values into equation 3, c turns out to be the following:

$$23.084 = 0.856a + 0.925b + c \rightarrow 23.084 = 0.856(0.299) + 0.925(0) + c \rightarrow$$
$$c = 23.364$$

After deriving the equation for this curve, the volume was determined to be the following:

$$V_2 = \int_{19.37}^{23.08} \pi r^2 dh = \int_{19.37}^{23.08} \pi (71.45 - 3.058h) dh \rightarrow$$
$$V_2 = \pi[71.45h - 1.529h^2]_{19.37}^{23.08} = 24.28\pi \rightarrow$$
$$V_2 = 76.28\ cm^3 = 76.28\ ml$$

Volume of the Bottom (V_3)

Again, the following measurements were taken from the numbers seen in figure 1:

- r_1 =3.50 cm
- h_1 =0.874 cm
- h_2 =19.37 cm

Remember that $V = \int_a^b \pi r^2 dh$ where

$$r = 3.50,$$
$$a = h_1, and b = h_2$$

After deriving the equation for this curve, the volume was determined to be the following:

$$V_3 = \int_{0.87}^{19.37} \pi r^2 dh = \int_{0.87}^{19.37} \pi (3.50)^2 dh \rightarrow$$
$$V_3 = \pi[12.22h]_{0.87}^{19.37} = 9.403\pi = 12.22\pi(19.37 - 0.87)$$
$$V_3 = 709.49\ cm^3 = 709.49\ ml$$

Volume of the Curve Foot (V_4)

The following measurements were once again taken from the numbers seen in figure 1:

- r_0 =3.05 cm
- r_1 =3.50 cm
- h_0 =0 cm
- h_1 =0.87 cm

Remember that $V = \int_a^b \pi r^2 dh$ where

$$h = 0.299r^2 - 2.766 \rightarrow r^2 = 3.34h + 9.30,$$
$$a = h_0, and\ b = h_1$$

This equation was derived using three points, which included (3.048, 0), (-3.048,0), and (3.495, 0.874), and the following mathematical operations based off of the basic relationship for a quadratic equation ($y = ax^2 + bx + c$):

$$0 = 9.29a + 3.048b + c \tag{7}$$
$$0 = 9.29a - 3.048b + c \tag{8}$$
$$0.874 = 12.215a + 3.495b + c \tag{9}$$

Subtracting equation 3 from equation 1, we get

$$2.925a = 0.874 \;\rightarrow\; a = 0.299$$

Subtracting equation 2 from equation 1, we get

$$6.096b = 0 \;\rightarrow\; b = 0$$

Knowing what the values for a and b are allows for the calculation of c to take place by plugging a=0.299 and b=0 in either equation 1, equation 2, or equation 3. By plugging these values into equation 3, c turns out to be the following:

$$0.874 = 12.215a + 3.495b + c \;\rightarrow\; 0.874 = 12.215(0.299) + 3.495(0) + c \rightarrow$$
$$c = -2.776$$

After deriving the equation for this curve, the volume was determined to be the following:

$$V_4 = \int_0^{0.87} \pi r^2 \, dh = \int_0^{0.87} \pi \,(3.34h + 9.30) dh \rightarrow$$
$$V_4 = \pi[1.67h^2 + 9.30h]_0^{0.87} = 9.403\pi \rightarrow$$
$$V_4 = 29.54 \; cm^3 = 29.54 \; ml$$

Volume of the Punt (V_P)

Basing the numbers seen in figure 2, the following measurements were taken:

- $h_p = 3.56 \; cm \; = \; 35.6 \; mm$
- $r_p = 1.92 \; cm \; = \; 19.23 \; mm$

Remember that $V = \int_a^b \pi \, r^2 dh$ where

$$h = 35.6 - .09627r^2 \rightarrow r^2 = \frac{35.6 - h}{.09627},$$

a=0, and b=h_p.

This equation was derived from three points to get

$$0 = 35.6mm - Ar^2 \rightarrow 35.6mm = Ar^2 \rightarrow \; A = \frac{35.6mm}{(19.23mm)^2} = .09627/mm$$

Once the equation was derived for the curve, the volume was determined to be

$$V_P = \pi \int_0^{35.6} \frac{35.6 - h}{.09627} dh = 20678.95mm^3 = 20.67895cm^3$$

$$V_P = 20.68\ cm^3 = 20.68\ ml$$

Total Volume

This was the main equation that was used to determine the total volume of the wine bottle including the punt:

$$V_{Tot} = (V_1 + V_2 + V_3 + V_4) - V_P$$

When the wine bottle was filled completely at 800 ml, the volume came out to be

$$V_{Tot} = (20.03\ ml + 76.28\ ml + 709.49\ ml + 29.54\ ml) - 20.68\ ml$$

$$V_T = 815.40 \text{ ml}$$

When the wine bottle was filled completely at 800 ml, the volume came out to be

$$V_{Tot} = (2.55\ ml + 76.28\ ml + 709.49\ ml + 29.54\ ml) - 20.68\ ml$$

$$V_T = 797.80 \text{ ml}$$