



## **Improving Student Understanding and Efficiency through Technology Use in the Differential Equations Classroom**

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## Introduction

Ordinary Differential Equations (ODE) is often a difficult course for students, with solution processes that are often long and tedious, drawing heavily on material from calculus and linear algebra. Typically, the course has been taught in a classical manner at many institutions, with the professor developing a variety of algorithms to solve ODEs and working out examples by hand on the board. Solutions tend to be time-consuming, and so the instructor must often limit the examples to simpler problems. This method leaves also little time for classroom interactions with the student.

In recent years, there has been a movement towards inquiry-based and guided discovery instruction methods in ODEs<sup>1,2</sup>. While the research has illustrated that inquiry-based methods can often result in a deeper understanding of the concepts<sup>3</sup>, many instructors shy away from this instructional method. It is a difficult pedagogical transition for the instructor to make, and can result in fewer topics being covered. This can negatively impact subsequent courses, particularly in engineering, if students do not get to some of the more advanced topics in the course.

Unlike lower-level courses such as College Algebra and Calculus, the online course management systems, along with their accompanying video lectures, applets and other resources, are not readily available for Differential Equations textbooks. This means that, like most upper-division mathematics and engineering courses, students are heavily dependent on the instructor and class time to help them understand the material for the course. The more examples the instructor can work and the more problems students can work on their own in class, the more confident students become in their own understanding. Instructors may often skip the simpler steps (simplifying, integrating, etc.) that are required for the solution process in order to present a useful number of examples. In the author's experience, however, this leaves many students feeling lost, like they have missed some important step in the process.

Using Computer Algebra Systems (CAS) such as Maple to help students solve ODEs is one solution to this, and has also gained acceptance in the last decade<sup>4</sup>. There are many textbooks and supplements that include projects in MATLAB, Maple or Mathematica<sup>5</sup>. There is a fine line to walk, however, between having the CAS solve the ODE for the student, and the student using it as a tool to help them solve the exercise. The author will share how she has implemented Maple use into her classroom. The paper will also discuss the advantages and disadvantages of using

programs such as TechSmith's Camtasia to record lectures to post online and to answer student questions.

## Background

Clayton State University is a small liberal-arts institute that has a strong emphasis on technology use. Students are required to have access to a laptop computer that they can bring to class with them each day. Enrollment in the Ordinary Differential Equations class is comprised of mathematics majors and students in our dual degree and transfer programs for engineering. A significant portion of the students are non-traditional and/or have substantial work obligations that they need to balance with their coursework. Roughly 55% of our student body is enrolled full-time (Figure 1), and about 45% falls into the traditional college student age range (Figure 2).<sup>6</sup> Differential Equations is a 3 hour course with no recitations and no teaching assistants. While class size is thankfully small, many students have to work before and after class time, making it difficult for them to take advantage of the instructor's office hours.

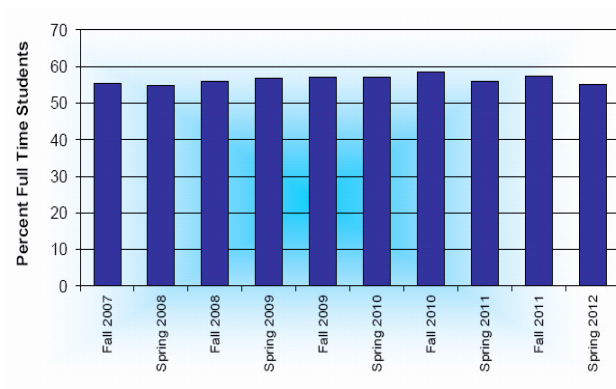


Figure 1: Percent full-time students, 2007-2012

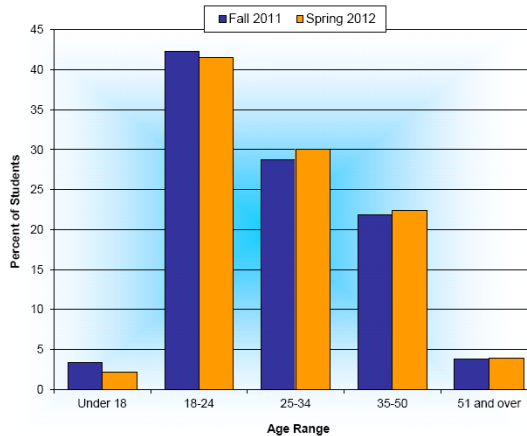


Figure 2: Breakdown of student population by age

Maple has been a tool available to our students for several years. Recently, we began requiring some basic use of Maple in our Calculus sequence, for things such as graphing, algebraic manipulations and function evaluations. This, in turn, has greatly facilitated more extensive use of the program in the Differential Equations course.

## Implementation

The author has adopted a mix of traditional ODE instruction, in-class group-work and guided discovery methods. The problem of the length of examples and necessary “simple” problems is addressed through careful use of a CAS, Maple in this specific case. Using technological tools (other options might be Mathematica or Wolfram|Alpha) allow the use of more complicated and

interesting examples while still keeping the problem accessible to the students and doable within a short time frame. When using Maple or other CASs in mathematics courses, the instructor must be careful to use the computer aid as a tool and not simply to get to the end result. Maple's *dsolve* command can solve most of the ODEs in the typical Differential Equations textbook. Using such 'high-level' commands, however, does the student no good, as it gives them no insight into how the problem is actually solved or why the ODE behaves as it does. The author uses a mixture of Maple and traditional board work to set up, graph, and solve traditional differential equations and application problems.

The solution is to stick to 'low-level' commands, using the CAS to perform the basic computations, such as algebraic manipulations, integrations, and determinants. These are skills that the student has demonstrated and mastered in pre-requisite courses. Instead of having them repeat these type calculations over and over, we allow the use of Maple to carry out these tasks. So the student is still completing the entire solution process, but using a tool, Maple, to carry out the time-consuming (yet "basic") calculations. The instructor uses these same low-level commands to carry out the solution of the examples in class, working both in Maple and on the board as needed. Using Maple in the lecture allows the instructor to quickly do required integrations, graphs, and algebraic manipulations without having to skip steps for the sake of brevity.

For example, consider using the eigenvalue method for solving homogeneous problems. A typical class lecture might begin with a simple  $2 \times 2$  example with distinct real eigenvalues, which can be quickly completed by hand. This demonstrates the solution process and provides a quick reminder for processes, like finding a determinant, that were learned in prior courses. A second reinforcement, using a  $3 \times 3$  would be done both with Maple and by hand, to illustrate that the steps are equivalent. Once these are done, the instructor can move on to more involved problems, such as larger systems and/or ones with "messy" coefficients, which are rarely done as class examples. At this point the problem would be worked almost completely with Maple. While Maple does have a higher-level built-in command for finding eigenvalues and eigenvectors, this skill is not always covered sufficiently in our linear algebra course, so it is important that the students practice going through each step of the process themselves, and that it be covered in the class examples each time. The instructor would define the coefficient matrix in Maple, then use it to calculate the characteristic equation of an eigenvalue problem. The determinant is then readily taken using Maple (something that could be quite time-consuming and prone to algebraic errors by hand for larger systems), and the eigenvalues found. The next step, of course, is to find the eigenvectors. This part of the process, while fairly basic linear algebra, can be quite time-consuming when done by hand. The author uses the *ReducedRowEchelonForm* command to do this step and then has the students solve the resulting equations to find each eigenvector. Once those are found, the general solution is formed and can be easily graphed using the program.

Using Maple as a tool like this also allows for a nice transition into the next topic, systems with complex eigenvalues, as the process stays the same for much of the problem. This is not to say that the author does not use *dsolve* or other similar high-level commands at all, though. Students are encouraged to use those to check their work, but also reminded that Maple may not always give the answer in the same form. Some class time is also taken to use tools such as *DEplot* and *dsolve* to investigate how small changes in a model or equation can change the end result.

This approach to the more traditional instructional method allows ample time to work several examples in each class, both the typical “nice” problems and larger (or “messier”) problems that would normally be saved for homework. Students are expected to input the Maple commands in each example on their own laptops alongside the instructor, increasing engagement and creating them a copy of the example. New Maple commands can be introduced “just in time” and reinforced through their use in multiple examples. Because the in-class examples take less time, the instructor is able to allot significant class time for students to work in groups on solving and presenting additional exercises. While a few students do initially resist using Maple to help them solve ODEs, preferring to solve equations entirely by hand, most of them do come to realize that it is a very useful tool.

The author does all of the traditional board work on a TabletPC projected onto the board. This allows the entire class lecture, both board work and Maple commands to easily be recorded using Camtasia and distributed to the students on the Internet. There are tools that allow this to be accomplished using iPads or other tablets, as well. This enables the student to access the course lecture at any time, which has proven useful both for review and students who miss class. This approach is useful for asynchronous instruction as well. The author has recorded several targeted tutorials for the course to help students get extra instruction and practice on some of the tougher concepts of the course. Camtasia records both audio as well as video of what’s going on on the computer screen, so explanations can be more comprehensive than what is typically viable in type-written notes. The author has also used recordings to address student questions for several students who were not able to come to regular office hours because of their work obligations.

## **Survey Results**

Students in the Fall 2012 semester class were surveyed to gauge their use of Camtasia recordings of class and class notes that were posted to the Internet. They were also surveyed as to their perception of the helpfulness of Maple use in class. Responses were anonymous, with all students who completed the course responding.

Class sizes are small, so there is a large amount of variability in student performance between semesters. Student performance shows a significant increase ( $z = 1.898$ ,  $P = 0.029$ ) in the

proportion of students passing the class (grade of A, B or C) from Fall 2010 to Fall 2012. In both courses, Maple was used in class, while Camtasia recordings and posted class notes were added for the 2012 class. Use of Maple was more extensive in the 2012 class. The P value gives the probability of obtaining a difference in sample proportions that is at least as large as what was actually obtained, if there is actually no difference in the population proportions. P-values below 0.05 are generally regarded as strong evidence of a difference in population proportions. The z-score value gives the number of standard deviations away from 0 (no difference) that the sample difference lies.<sup>7</sup>

Semester	Resources Used	%A	%B	%C	%D	%F	%W	n
Fall 2012	Camtasia+Maple+Notes	10	35	30	0	10	15	20
Spring 2011	Maple + Notes	33	11	44	0	11	0	9
Fall 2010	Maple	14.3	21.4	7.1	21.4	14.3	21.4	14

Table 1: Grade distributions, in percent. n gives the sample size for each semester.

Overall, student use of Camtasia recordings of class was good, with 64% of the respondents making at least some use of the recordings. Students mainly used class recordings for review purposes and to help them understand more difficult concepts, as shown in Figure 3 below. The mean rating of the helpfulness of the videos was 4.44 on a scale of 1 to 5, with 5 being the most helpful. Two-thirds of those who used the video recordings reported using them at least once a week. In general, nearly all students made use of at least some of the electronic resources provided to them, as shown in Figure 4.

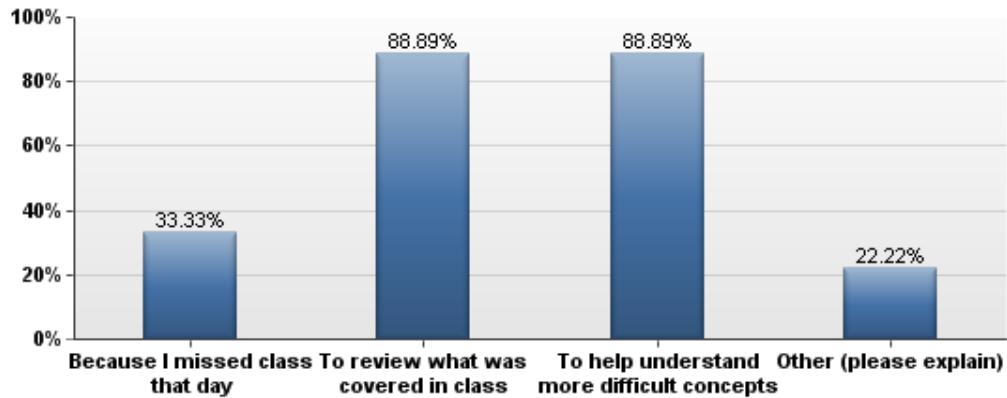


Figure 3: Reasons for student use of video recordings

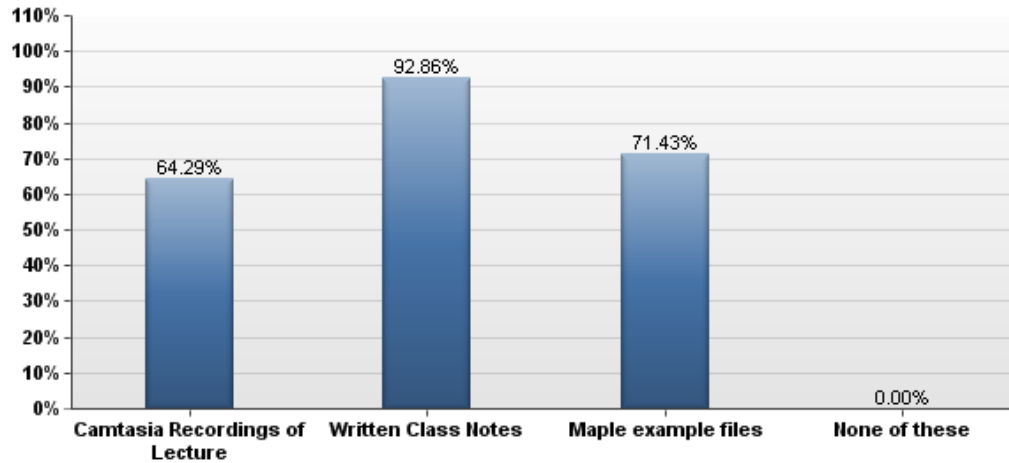


Figure 4: Electronic resources used by students

All of the survey respondents were at least somewhat familiar with Maple prior to starting the Differential Equations class, with a self-rated intermediate level of knowledge (mean of 2.93 on a 1-5 scale). Upon completing the course, students rated their knowledge higher, with a mean of 3.43. All respondents felt that the instructor’s use of Maple in class enabled the coverage of more material and examples, and nearly all (93%) felt that it had a positive impact on their understanding of the course material. Figure 4 shows student ratings of the helpfulness of the instructor’s use of Maple in class, which yielded a mean rating of 4.21 out of 5.

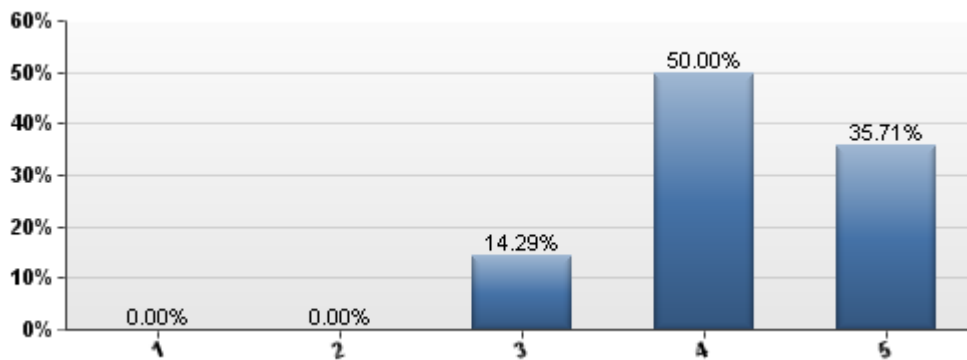


Figure 5: Student ratings of the helpfulness of Maple use in class

## Conclusions

Careful use of Maple in ODEs can allow the instructor to increase the number, variety, and difficulty level of examples worked in class. By using ‘low-level’ Maple commands, the instructor can effectively “fast-forward” over repetitive and tedious operations such as integrations and matrix operations without sacrificing the students’ understanding of the solution process. It can also help to increase the amount of time available for in-class group work, or for

guided inquiry problems. Students are receptive to the instructor's use of Maple. Students were also seen to make frequent use of audio/video recordings of class to help them review and master more difficult topics.

Video recordings of class are easily done in any size classroom, as is instructor use of Maple for examples. Student use of Maple in class group activities may require the use of teaching assistants to effectively check student progress in larger classes.

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