ASEE 2022 ANNUAL CONFERENCE
Excellence Through Diversity
MINNEAPOLIS, MINNESOTA, JUNE 26TH-29TH, 2022 ASEE

Paper ID #37810

Improving understanding of reaction forces in free body diagrams using a paired vector object in Prairie Learn

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Abstract

A foundational skill in mechanics is the ability to draw complete and correct free body diagrams (FBDs). Students benefit from extensive practice and feedback as they build their FBD skills, but in large enrollment courses the ability for course staff to give detailed individual feedback is limited. That mismatch between need for feedback and capacity to provide it drives adoption of teaching software with automated grading, such as PrairieLearn (PL). In the sophomore level Statics and Mechanics of Solids course at Cornell University we have recently introduced limited use of PL to give students additional practice with FBDs.

One particularly challenging application of FBDs for new learners in a statics course is for "frames" or "mechanisms." These consist of multiple members, both two-force and multiforce members, pinned together and loaded with forces and/or moments at any location. A major point of confusion for students in the course is how to treat reactions on members separated from each other at pins, including identifying two-force members and correctly showing equal and opposite reactions on the separated members. When we began using PL for our course, there was no mechanism in that platform to grade "exploded" FBDs for frames, which significantly limited the utility of the platform for our students.

We hypothesized that students would perform better when drawing FBDs by hand outside of the PL platform if they could practice placement of two vectors in PL as the equal and opposite reactions where members are separated from each other. To test this hypothesis, we devised and implemented a new object in PL, a "paired vector" object, that allows students to practice drawing exploded FBDs and simultaneously reinforces the concept of equal and opposite reactions. We then randomly divided the 240 person course into two groups. Each group had two PL assignments during the statics unit of the course. The first assignment was identical for all students, but for the second one, the intervention group had frames problems including the paired vector object and exploded FBDs, while the control group had modified versions of the same problems that did not involve drawing equal and opposite reactions. On one midterm exam and the final exam we included problems requiring exploded FBDs. We compare performance on drawing FBDs, including frequency of specific mistakes, between the intervention and control groups. Our results suggest that practicing with the paired vector object led to better outcomes on drawing exploded FBDs and decreased the incidence of incorrect reactions at joints where members are separated from each other.

Introduction

Drawing complete and correct free body diagrams (FBDs) is widely considered a foundational skill in solid mechanics and is typically one of the first topics taught in a statics course, after introducing forces and moments as vectors. For such a basic concept, there are many subtle aspects and decisions that go into drawing proper FBDs and it can be challenging for new learners to master this skill. Cornwell and Danesh-Yazdi[1] identified errors and lack of clarity in instruction on FBDs in physics and statics textbooks. Various instructional aids such as mnemonic devices[2], and supplementary animations[3,4] have been developed. Ultimately, the only way for students to master FBDs is by practice with expert feedback. The difficulty of scaling up individualized grading or feedback on FBDs to large enrollment courses has led to many efforts over the years to develop automated grading software. Some examples include: Newton's Pen[5,6], which uses a stylus and handwriting recognition to evaluate hand drawn FBDs and equilibrium equations; ARCHIMEDES[7], which allows students to add forces and moments to a given FBD, isolate individual members to reveal internal loads (or make "exploded" FBDs), and use MATLAB-like syntax to write equilibrium equations; Mechanix[8], which is a web interface allowing handwriting recognition of truss and rigid body FBDs; and PrairieLearn (PL)[9,10], which is an HTML-based platform that allows students to add predefined elements (e.g. force and moment vectors) to diagrams. Each of these has strengths and weaknesses, but all are trying to address the need for early feedback on this critical skill.

In this study we seek to leverage the PL platform to address one specific aspect of FBDs that our students have struggled with: correctly dealing with internal loads revealed by "exploding" a frame or mechanism by separating sub-parts at joints. Correctly dealing with reactions at joints or connections is an important higher-level use of FBDs[11]. Understanding how FBDs of the exploded system help identify efficient solution plans for equilibrium analysis can improve student success[12]. The aspects of exploded FBDs that we most sought to emphasize are: (i) showing equal and opposite reactions on the separated bodies, and (ii) distinguishing two force members (with a known line of action of the reaction force based solely on geometry) from multi-force members (where any reaction force acts in an arbitrary direction that must be determined through equilibrium analysis, based on the other applied forces). We restricted ourselves to 2D frames and mechanisms connected by pins (no reaction moments), but included frictionless rollers or collars as possible system components. Common mistakes we have observed related to these aspects include: showing forces acting in the same direction on both bodies separated at a joint; including reactions at internal locations on the overall FBD before separating members at pins; not noticing a two force member and treating the reactions at the pins on that member as acting in an arbitrary direction[*]; incorrectly treating reactions on a pin on a multi-force member as if they must align with the geometry of the member.

[*]Technically, this is not an error, since equilibrium analysis on the two force member would quickly lead to recognizing that the forces on the pins must be co-linear. However, recognizing two force members saves a significant amount of time and effort in equilibrium analysis and we therefore chose to require students to practice this skill.

Methodology

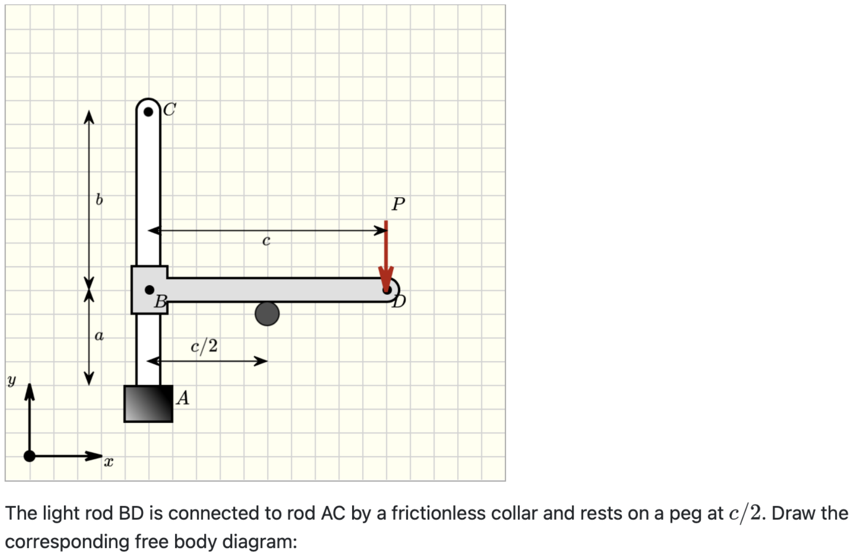
We recently began using PL on a limited basis in our Statics and Mechanics of Solids course, with a focus only on 2D FBDs and equilibrium analysis. The limitations of PL include only having simple drawing objects such as force and moment vectors, but no ability for students to draw the "bodies" of FBDs. But one important flexibility of the platform is that it recognizes an unknown force reaction as correct based on the line of action but regardless of whether it is drawn "pushing" or "pulling" on the body, and regardless of whether it points in the positive or negative direction. While we decided to require certain conventions (such as aligning reactions with two force members, and drawing arbitrary direction reactions as having one exactly horizontal and one exactly vertical component), we wanted to minimize the number of limitations we imposed so that we could emphasize to students the range of correct FBDs. While this directional agnosticism is a clear benefit for single-body FBDs it prevented grading exploded FBDs based on whether the equal and opposite reactions at a separated joint are indeed depicted as opposite. In the implementation of FBDs in PL when we began using it, it was capable of addressing our outcome (ii) related to two force vs. multi-force members, but not outcome (i) related to equal and opposite reactions at joints.

To address this deficiency, we developed a new PL object: the paired vector (PV) object. This object adds two force vectors to the diagram, each of which can be placed completely independently of each other, including location, angle, and direction of arrow head. Grading of the object depends on the vectors sharing an angle *and* having arrow heads in opposite directions. This object design is intended to emphasize that when you separate objects from each other you generate *two* of every reaction component: one on each body acting in equal and opposite directions. We hypothesized that having to literally place reactions in these equal-and-opposite pairs would reinforce that concept to students.

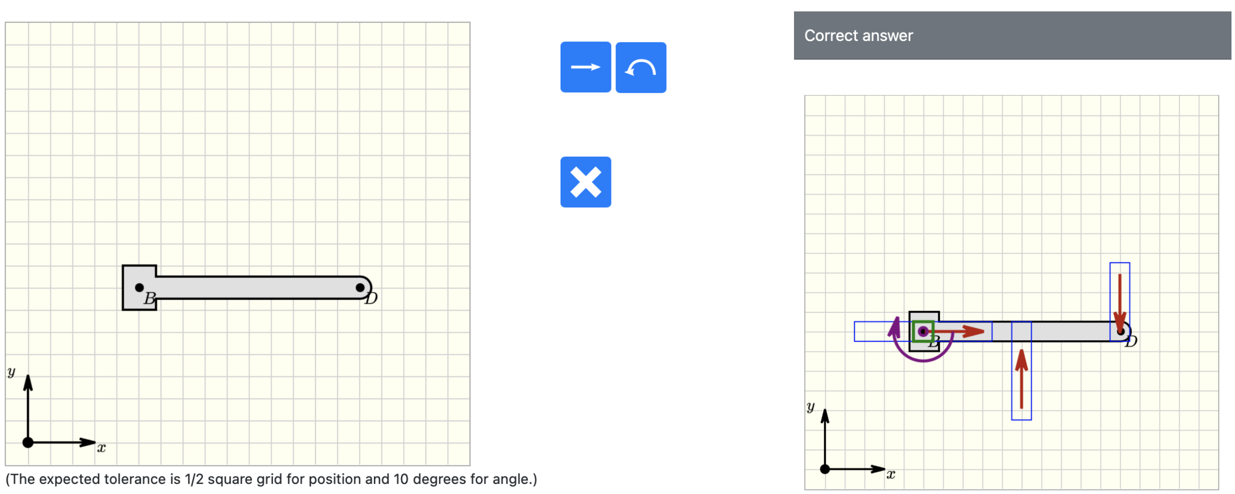
To study the effect of using this new PV object we randomly divided our class of 240 students into two groups.When we first introduced FBDs for 2D rigid bodies all students in both groups were given the same PL homework assignment which did not involve exploded FBDs. This ensured that all students had the same familiarity with the platform. When we moved on to introduce frames and mechanisms, the groups had different PL assignments. The intervention group had three problems using the PV object, while the control group had variations of two of the problems which did not involve the PV object. We then compared student performance on a related problem on the first midterm exam and the final exam. The PL assignments and exam problems are shown in the following sections.

PrairieLearn assignments

An example of a problem from the first homework is shown in Figure 1. The problem statement is in Figure 1(a), the canvas where students develop their solution is in Figure 1(b), and the correct solution is shown in Figure 1(c). Some things to note: the grid is given in both the problem statement and the canvas to help students with proper placement of reactions; the canvas has the body of interest provided; the blue icons to the right of the canvas show the types of objects that can be added to the canvas, in this case force or moment vectors; the blue outlines on the correct solution show the range of acceptable locations for the force arrows; even though a visual



(a) Problem statement



(b) Student canvas

(c) Correct solution

Figure 1: Example problem from the first PL assignment, which all students shared in common. This helped familiarize students with the PL interface.

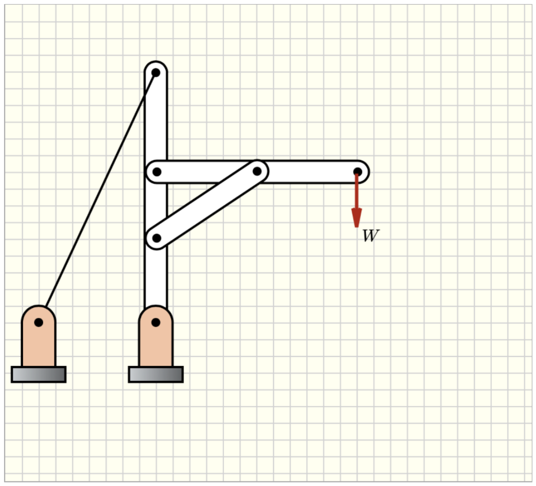
inspection shows that the reaction force at point B must be zero, the force must still be included on the diagram because an equilibrium analysis is required to discover the (zero) magnitude of the reaction force. Note that the applied load must be copied exactly to be counted correct, but the reaction force at B would be counted correct no matter whether it points right or left and no matter whether it's drawn to the right or left of point B, as indicated by the blue outline.

For the second PL homework, the intervention group (which we label PV for paired vector) was assigned a frame problem which was a direct copy from a solved example in the textbook. The purpose of this problem was to teach them how to use the new PV object on a problem where they knew the right answer. Figure 2 shows the problem introducing the PV object. Notice in Figure 2(b) that instead of a moment vector, the students now have access to the paired vector (PV) object, denoted by the two arrows icon. In Figure 2(c) you can see that the two vectors from each PV object share the same color. In particular, students had to learn that, for example, on the two force member the PV object should *not* be put at either end of the two force member, but rather each end gets its own PV object, the other half of which appears on the member from which the two force member was separated. You can also see that as long as the location and angle of each force is chosen correctly, the direction of the arrow head does not affect the grading, as desired.

The rest of the assignment had two problems for each group: one linkage with a sliding collar and one hand tool/loppers. Figure 3 shows the problem statement which both groups shared and the unique canvases available to the PV and control groups for the linkage problem. Note that both groups had to include the overall FBD and the exploded FBD for the top link, but only the PV group also had to include the FBD for the lower link. Both groups got to practice drawing the reaction perpendicular to the correct link based on the frictionless collar, but only the PV group also got to practice drawing equal and opposite reactions on the lower link. It's possible that control group students could have thought the reaction should be perpendicular to *each* link on its own FBD, and this problem would not have disabused them of that notion. The remaining problem based on a hand tool had similar execution: both groups had to draw the overall FBD, the control group had to draw just one member in the exploded FBD, and the PV group had to draw all members in the exploded FBD.

Exam problems

On the first midterm exam and on the final exam we included problems that specifically required drawing exploded FBDs. Those problems are shown in Figure 4. For the midterm problem, only part a is considered for this study. We use Gradescope for exam grading which allows us to keep track of which rubric items were applied to each student's exam.

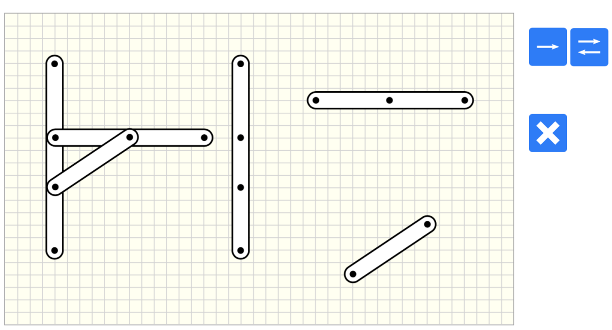


The frame in the figure is shown in section 6.3 of the textbook. Draw (copy) the corresponding free body diagram.

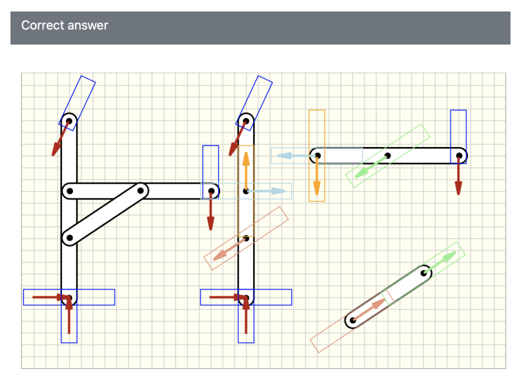
Use individual arrows to show external forces on the entire structure and the "exploded" components; use paired arrows of the same color to show forces internal to the structure in equal and opposite pairs. When using paired vectors, try to draw two-force members in tension.

Grading info: paired vectors are graded in the pairs that they were formed. All combinations are checked, but the pair is graded all-or-nothing. The colors cycle, but that's just a visual aid (delete any vector to also remove its paired vector, and get a fresh color).

(a) Problem statement

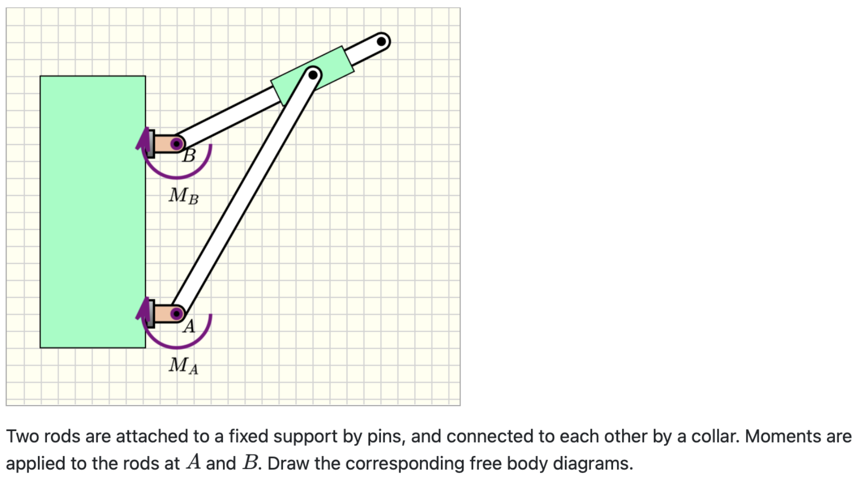


(b) Student canvas

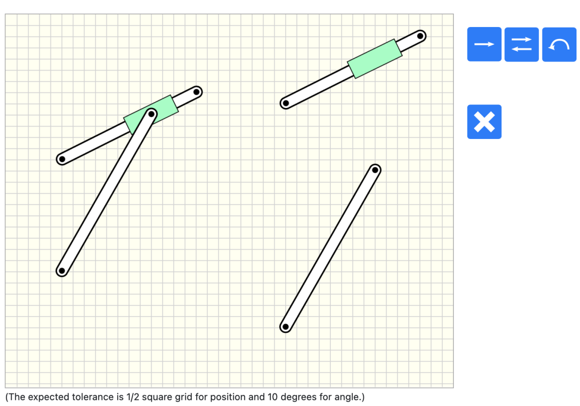


(c) Correct solution

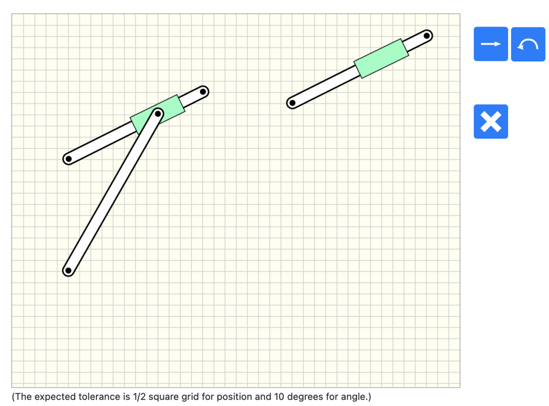
Figure 2: Training problem from the intervention group's second PL assignment. This is a frame problem that they already had the full solution for, and was included to give them practice with the PV object, which appears as the double arrow icon in the canvas and as two vectors of the same color in the solution.



(a) Problem statement



(b) PV group canvas

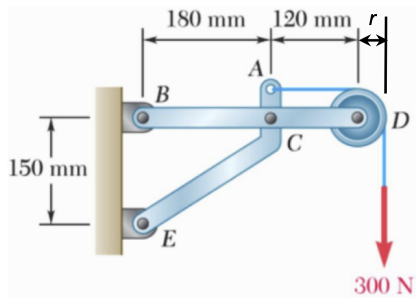


(c) Control group canvas

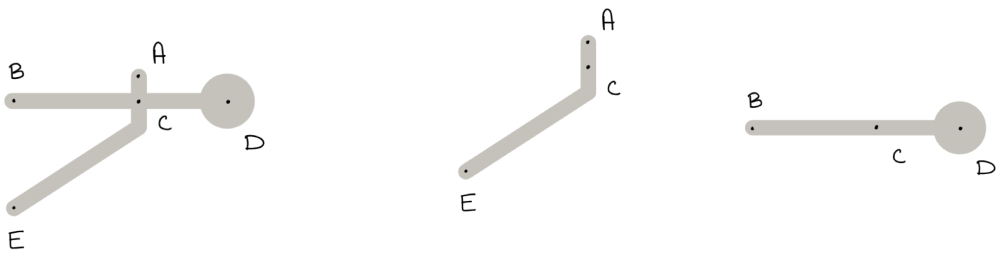
Figure 3: Problem from the second PL homework showing the versions that each group was assigned. PV group includes the PV object and the control group does not.

Problem 5 (25 pts). The frame shown here has a pulley with $r = 50$ mm and a cable supporting a 300 N load.

(a) Draw the free body diagrams of the entire assembly and members ACE and BCD on the provided bodies below.
(b) Determine the components of the reactions at B and E.



(a)

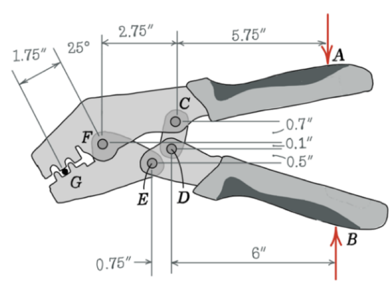


(a) Midterm exam problem

Problem 2 (10 pts). The tool shown here generates a crimping force at *G* due to the equal and opposite applied loads at *A* and *B*, which share a line of action. *C, D, E,* and *F* are all pins. Make an "exploded" free body diagram for this tool, including separate drawings of each of the four parts.

On your FBD, if you know the line of action of a force by inspection (e.g. a two force member) you must draw it with the known line of action. If you do not know the line of action without doing calculations, draw independent horizontal and vertical components. In the interest of time, *for this problem* you do not need to write dimensions on your exploded FBD.

Your drawings don't have to exactly match the geometry perfectly, but only need to be clear enough to capture the relative position of every load.



(b) Final exam problem

Figure 4: Problems from the exams to test exploded FBDs for frames and mechanisms, used to evaluate efficacy of the PV object for student learning.

Table 1: Median exam problem scores and p-values for Wilcoxon rank sum tests. While the PV group performed better on both exam problems, only the final exam problem showed statistical significance.

Exam problem	**PV group [%]**	**Control group [%]**	p-value
Midterm exam #5a	87	80	0.1255
Final exam #2	100	60	0.0012

Results and Discussion

Our analysis focused both on numerical scores on the two exam problems noted above, and on the incidence of certain rubric items related to our hypothesis. Before analyzing outcomes related to our hypothesis, we first looked at student performance on two concept inventories related to course content: the Statics Concept Inventory[13] and a Mechanics of Materials Concept Inventory currently under development by P. Steif and B. Aktas, each of which was administered as both a pre-test before the topics were covered in class, as well as a post-test after the concepts were covered. Because the students were randomly assigned to either the PV or control group, we used a Wilcoxon rank sum test on both pre-post gains on each concept inventory, and just post-test score. The results showed no statistically significant difference between the two groups. This is not conclusive, but is reassuring evidence that students in the two groups were relatively equivalent. With that reassurance in hand, we used a Wilcoxon rank sum test on exam problem scores, and Table 1 shows the results. While the PV group performed better on both exam problems, only the final exam problem score showed statistical significance with $p < 0.0012$.

In addition to scores on the exam problems noted above, here we compare the incidence of certain rubric items related to our hypothesis. The first author of this paper graded all of the exam problems under consideration in this study. While grading, we intentionally used descriptive rubric items to identify specific errors or approaches. After the end of the semester, we looked at the list of rubric items used and identified those that we expected to be most likely to be affected by the PV intervention. That list of rubric items we chose to focus on is given in Table 2. Note that because we chose the rubric items to focus on before looking at any student scores related to the study, it turned out that the last rubric item for the midterm exam problem in Table 2 was not assigned to any student in the study and is therefore omitted from further analysis. After identifying the rubric items of interest, we then analyzed the outcomes on those rubric items for each group using crosstabs. Those results are shown in Table 3.

On the midterm exam problem, the two groups had near-identical performance on the overall FBD, but the PV group consistently outperformed the control group on the exploded FBD. While a minority of both groups (42% of PV and 30% of control) did the exploded FBD entirely correctly, the PV group had much better accuracy dealing with point C where the two multiforce members connected with each other, with statistical significance of $p < 0.005$. This is one of the concepts most directly explored by the paired vector object in PL, so the results on this rubric item are particularly relevant to determining the value of the PV object.

On the final exam problem the PV group outperformed the control group by more than 10

Table 2: Relevant rubric items from the exam problems. For some items we expect the PV group to have a higher incidence (e.g. overall correctness) but for others we expect the PV group to have lower incidence (e.g. Cx and Cy don't match). For that latter category, we have reversed the coding, or counted the number of students NOT earning that rubric item.

Rubric item	**Reversed coding?**
Midterm exam	
Overall FBD: All correct overall FBD	-
Overall FBD: Internal loads shown at C and/or D	reverse
Exploded FBDs: All correct exploded	-
Exploded FBDs: C correct	-
Exploded FBDs: Cx, Cy don't match	reverse
Exploded FBDs: CE or BCD treated as 2FM or other very wrong reactions at C	reverse
Exploded FBDs: Dx, Dy, or Md	reverse
Final exam	
Treating E correctly	-
Treating F correctly	-
Treating G correctly	-
2FM at CD correct	-

Table 3: Percentage of group members who earned each rubric item along with the p-value from crosstab analysis, all coded so that a higher percentage indicates a better outcome. The asterisk * indicates a reverse-coded rubric item whose wording has been changed to match the data in this table. The dagger † indicates rubric items that are only applied in the case of some prior mistake. Therefore, for those items the percentages shown include students who either had a positive result on that rubric item or solved the entire problem correctly. The double dagger ‡ indicates statistical significance with $p < 0.01$.

Rubric item	**PV [%]**	**Control [%]**	**p-value**
Midterm exam			
Overall FBD: All correct overall FBD	63.8	64.9	0.91
Overall FBD: Internal loads not* shown at C and/or D	96.5	97.3	0.80
Exploded FBDs: All correct exploded	41.8	29.7	0.18
† Exploded FBDs: C correct	82.3	59.5	0.003‡
† Exploded FBDs: Cx, Cy match*	90.8	73.0	0.004‡
† Exploded FBDs: CE or BCD not* treated as 2FM and no other very wrong reactions at C	92.2	86.5	0.28
Final exam			
Treating E correctly	81.6	70.3	0.13
Treating F correctly	80.6	67.6	0.08
Treating G correctly	78.7	67.6	0.15
2FM at CD correct	84.4	64.9	0.008‡

percentage points on each of the rubric items analyzed, but only correctly handling the two force member rose to the level of statistical significance. It's also worth highlighting that the median score on this problem for the PV group was full credit.

Conclusion

Platforms to provide automated feedback to students on drawing complete and correct FBDs are a useful tool to scale up detailed feedback to large enrollment courses early in the semester. While many platforms have been developed or are in development, they all have some drawbacks and room for improvement. In this study we added a capability to the PrairieLearn platform that allows students to practice exploded FBDs of frames or mechanisms. The paired vector object trains students that when two bodies are separated, any reactions revealed must happen equally and in opposite directions on the two bodies. This reinforces that critical aspect of FBDs, especially with multiforce members. Reinforcement through a PL assignment with the PV object in week 4 led to improved outcomes drawing exploded FBDs by hand on exams in weeks 6 and 16. Inclusion of this object in the PrairieLearn platform is encouraged as it broadens the types of problems that instructors can successfully implement in the platform.

Acknowledgments

This work was completed under Cornell University IRB protocol #2108010487. We thank Dr. Mariana Silva and Nicolas Nytko for their helpful conversations about how best to implement exploded FBD support in PL and technical assistance incorporating the new object into the platform. We thank also Dr. Erika Mudrak from Cornell Statistical Consulting Unit for invaluable assistance with the statistical analysis.

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