AC 2007-405: INCORPORATING A RESEARCH PROBLEM IN A NUMERICAL METHODS COURSE FOR MECHANICAL ENGINEERS

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Abstract

This paper presents an example of incorporating a research problem in a course - Numerical Methods for Mechanical Engineers. In bascule bridges, the fulcrum is assembled by shrink-fitting a trunnion into a hub. In one case, the trunnion cooled in a dry-ice/alcohol mixture for shrink fitting got stuck halfway in the hub. Answering the question why the trunnion got stuck in the hub and finding a solution to the problem, involved numerical solution of mathematical procedures including nonlinear equations, simultaneous linear equations, interpolation, regression, integration, and ordinary differential equations. Students and faculty highly appreciate using this problem-centered approach to teaching the course.

Introduction

National scientific agencies such as the National Science Foundation[1] are continually encouraging integration of current research topics into undergraduate education. As our nation's engineers increasingly face global competition[2], bringing the state-of-art research into the classroom is becoming increasingly important. Education and research are of equal value and should be viewed as complementary parts of any STEM education system[3].

Incorporating a research problem into a graduate level course[4] presents challenges, and to incorporate a problem into an undergraduate level is even more challenging. These challenges include

1. the research problem may not address most of the topics of the course to justify the use of class time,
2. specific skills may be needed that are too time consuming to teach,
3. the problem may be out of scope for the education level of the students.

In the Numerical Methods course, we were either able to meet or not have to face these challenges. In the next sections, the description, implementation, and assessment of the problem are discussed.

Description of Problem

Amongst movable bridges, bascule bridges are the most popular type as they are simple and speedy to operate. The pivot assembly (called the trunnion-hub-girder (THG) assembly) of a bascule bridge consists of a trunnion shaft attached to the leaf (girder) via a hub, and supported on bearings to permit rotation of the leaf (Figure 1).

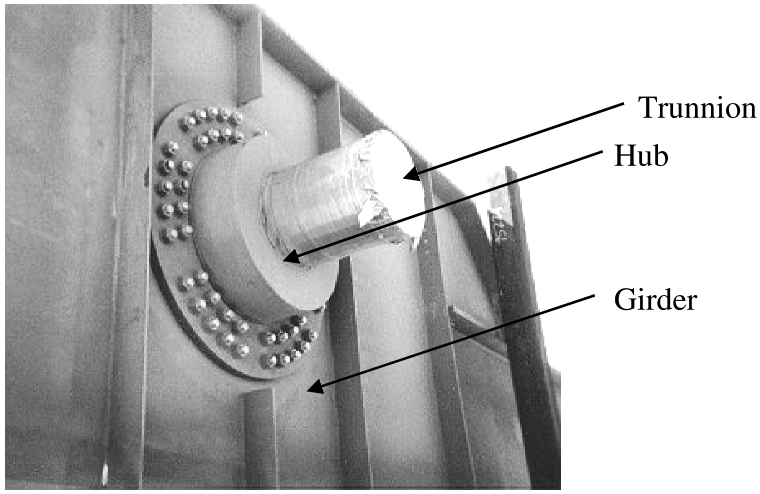
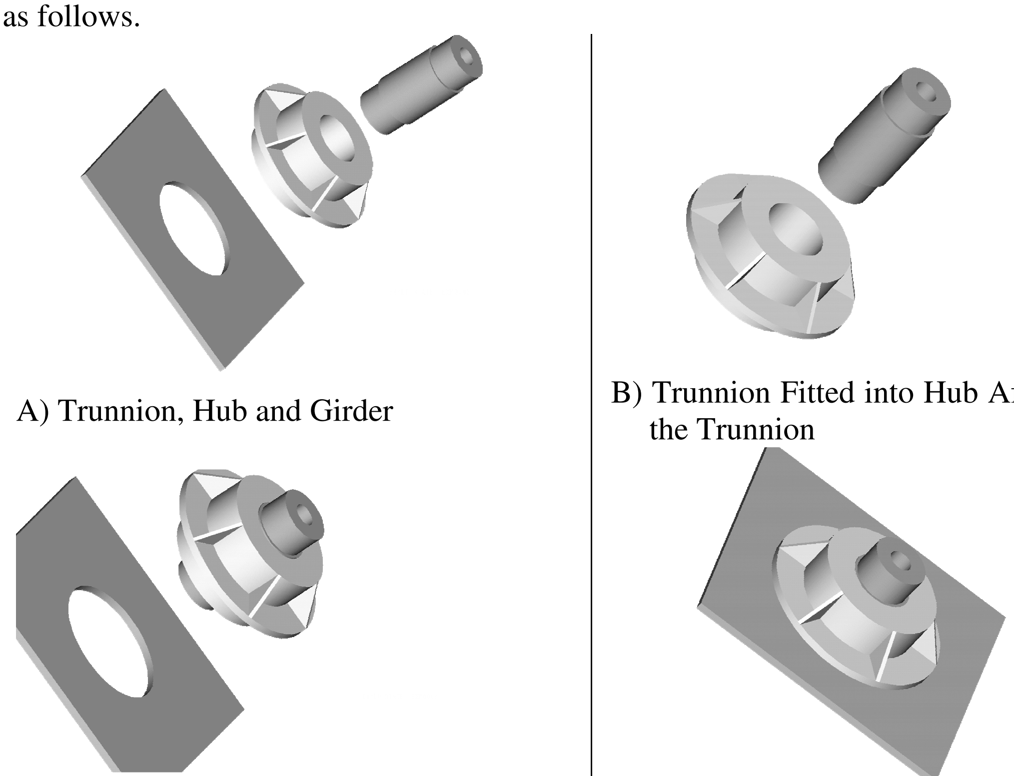


Figure 1 Trunnion-Hub-Girder (THG) Assembly

The THG assembly is made by using interference fits[5] between the trunnion and the hub, and the hub and the girder. The procedure[6,7] for assembling THG assemblies involves shrink-fitting the trunnion into the hub and then shrink-fitting the trunnion-hub onto the girder (Figure 2) as follows.



A) Trunnion, Hub and Girder

B) Trunnion Fitted into Hub After Cooling the Trunnion

C) Trunnion-Hub Fitted into Girder After Cooling the Trunnion-Hub Assembly

D) Completed THG assembly

Figure 2. Procedure for THG assembly.

Step 1: The trunnion is shrunk by immersing it in a cooled medium such as liquid nitrogen or dry-ice/alcohol mixture.

Step 2: The shrunk trunnion is then inserted into the hub, and allowed to warm up to ambient temperature to develop an interference fit on the trunnion-hub interface.
Step 3: The trunnion-hub assembly is shrunk by immersing it in a cooled medium.
Step 4: This shrunk trunnion-hub assembly is then inserted into the girder, and allowed to warm up to ambient temperature to develop an interference fit on the hub-girder interface.

On May 3, 1995, during the immersion of the trunnion-hub assembly in liquid nitrogen (Step 3 of the assembly procedure) for the Christa McAuliffe Bascule Bridge in Florida, a cracking sound was heard. On removing the trunnion-hub assembly out of liquid nitrogen, it was found that the hub had cracked near its inner radius. In a separate instance, during the fulcrum assembly of the Venetian Causeway Bascule Bridge in Florida, the trunnion was cooled in a medium of dry-ice/alcohol mixture and while inserting the trunnion into the hub (Step 2 of the assembly procedure), the trunnion got stuck halfway in the hub.

Each failure resulted in the loss of several hundred thousands dollars in material, labor, and construction delay. To prevent the recurrence of these assembly problems, Florida Department of Transportation (FDOT) decided to conduct an investigation[8] by providing a research grant to the Department of Mechanical Engineering at the University of South Florida. For three years, USF conducted a thermal stress analysis[6,7] of the fulcrum assembly procedure by using analytical[9], numerical[10], and experimental techniques[11].

Problem Centered Assignments

Since 2003, the THG assembly procedure problem has been introduced on the first day of class in the Numerical Methods course. Background information about bascule bridges and the fulcrum assembly is explained via an audiovisual presentation. The problem presented to the students is limited to analyzing the Venetian Causeway Bridge case where the trunnion got stuck in the hub.

The problem posed on the first day of class is as follows. A hollow trunnion of outside diameter 12.363" is to be fitted in a hub of inner diameter 12.358". The trunnion was immersed in a dry-ice/alcohol mixture at –108°F to contract the trunnion so that it can be slid through the hole of the hub. To slide the trunnion without sticking to the hub, a diametric clearance of at least 0.01" is specified between the trunnion outer diameter and the hub inner diameter. Now, solve the following problems.

1. Assuming the room temperature is 80°F, is immersing the trunnion in dry-ice/alcohol mixture a correct decision?

To calculate the contraction in the diameter of the trunnion, consultants hired by FDOT used the thermal expansion coefficient at room temperature. In that case, the reduction, ΔD in the outer diameter of the trunnion is given by

$$\Delta D = D\alpha\Delta T \qquad (1)$$

where

D = outer diameter of the trunnion,

α = coefficient of thermal expansion coefficient at room temperature, and
ΔT = change in temperature,

Given

D = 12.363"

$\alpha = 6.817 \times 10^{-6}$ in/in/^{o}F at 80 ^{o}F

$$\Delta T = T_{fluid} - T_{room}$$
$$= -108-80$$
$$= -188^{o}F$$

where

T_{fluid} = temperature of dry-ice/alcohol mixture,

T_{room} = room temperature,

the change in the trunnion outer diameter is calculated as

$$\Delta D = (12.363)(6.47 \times 10^{-6})(-188)$$
$$= -0.01504"$$

But, is this enough reduction in diameter? As per specifications, we need the trunnion to contract by

= trunnion outside diameter - hub inner diameter + diametric clearance
= 12.363" – 12.358" + 0.01"
= 0.015"

So according to the consultant calculations, immersing the steel trunnion in dry-ice/alcohol mixture gives the desired contraction of at least 0.015" as the predicted contraction is 0.01504".

However, as shown in Table 1 and Figure 3, the coefficient of thermal expansion of steel decreases with temperature and is not constant over the range of temperature the trunnion is subjected to. Hence, Equation (1) overestimates the thermal contraction.

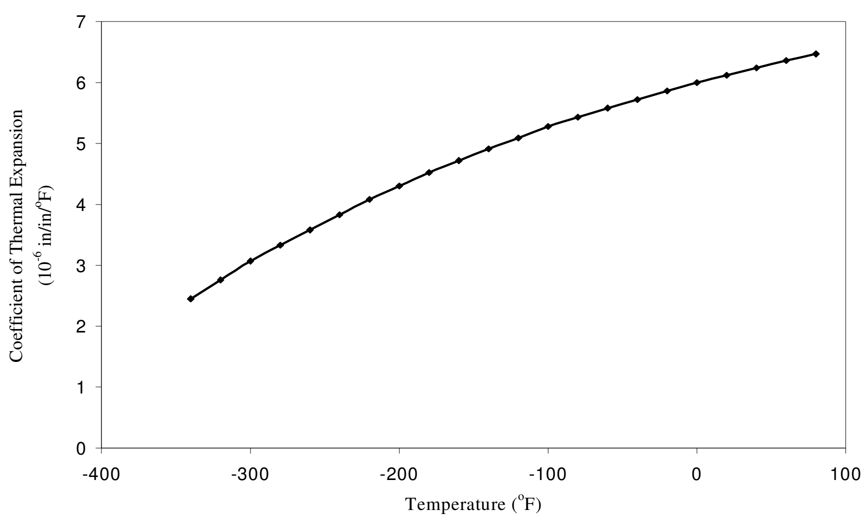


Figure 3. Coefficient of thermal expansion of steel as a function of temperature.

Table 1. Coefficient of thermal expansion of steel as a function of temperature

Temperature	Instantaneous Thermal Expansion
^{o}F	μin/in/^{o}F)
80	6.47
60	6.36
40	6.24
20	6.12
0	6.00
-20	5.86
-40	5.72
-60	5.58
-80	5.43
-100	5.28
-120	5.09
-140	4.91
-160	4.72
-180	4.52
-200	4.30
-220	4.08
-240	3.83
-260	3.58
-280	3.33
-300	3.07
-320	2.76
-340	2.45

To find correctly the amount of contraction, ΔD in the diameter, D of the trunnion, instead of Equation (1) we need to estimate the integral

$$\Delta D = D\int_{T_{room}}^{T_{fluid}} \alpha dT \qquad (2)$$

On the first day of class, the methods used by students to solve Equation (2) are limited to using average thermal expansion coefficient with Equation (1) or graphical methods based on Riemann's sum[12]. These become the foundation of why we need to develop scientifically more rigorous numerical methods.

Later during the course, the students are asked to find this contraction by several methods such as

1) Trapezoidal rule for discrete and unequally spaced data,
2) Regress the coefficient of thermal expansions vs. temperature data in Table 1 to a polynomial model and then use Equation (2),

3) Find polynomial splines to relate the coefficient of thermal expansions vs. temperature data in Table 1 and then use Equation (2).

These methods show that the contraction ($\Delta D \approx -0.0137"$) is well below the required amount of 0.015". This leads to the next problem.

2. From the above, the student can now determine correctly if the contraction is enough to slide through the hub with the specified clearance. Since that is not the case, one needs to find what should be the temperature of a cooling medium should be so that the trunnion that is originally at a room temperature of 80°F contracts enough so as not to get stuck in the hub. In Equation (2), since we know the desired contraction ($|\Delta D| \leq 0.015"$), and using a second order polynomial regression curve ($\alpha = -1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.0150\times10^{-6}$) as an approximation for the coefficient of thermal expansion, the resulting equation is given by

$$-0.015 = 12.363\int_{80}^{T_{fluid}}\left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$

to result in a cubic equation

$$f\left(T_{fluid}\right) = -0.50598\times10^{-11}T_{fluid}^3 + 0.38291\times10^{-7}T_{fluid}^2 + 0.74363\times10^{-4}T_{fluid} - 0.021168 = 0 \qquad (3)$$

Equation (3) is solved numerically. Also, since Equation (3) is cubic, it has three roots, and the physics of the problem need to be discussed to find the acceptable root.

3. The data given in Table 1 needs to be regressed to develop a relationship between the coefficient of thermal expansion and temperature. Questions include choosing the optimum degree of polynomial for the regression model by plotting $S_r/(n-[m+1])$ vs. m, where

S_r = the sum of the square of the residuals,
n = the number of data points, and
m = order of the polynomial.

The order of polynomial for which $S_r/(n-[m+1])$ is minimum or does not change appreciably is the optimum order of the polynomial.

4. Regression models obtained using default Excel® format do not properly account for significant digits, and hence create artificial large differences between the observed and the predicted values. For example, if Excel® is used to regress the coefficient of thermal expansion vs. temperature data (Table 1) to a second order polynomial, the result is

$$\alpha = -1\times10^{-5}T^2 + 0.0062T + 6.015, \qquad (4)$$

where

α = coefficient of thermal expansion coefficient at room temperature (μin/in/°F), and
T = temperature (°F).

At T=-300 °F, the value of the thermal expansion coefficient calculated from Equation (4) is 3.26 μin/in/°F, while the actual value of the data is 3.07 μin/in/°F (Table 1), which is a 6% difference. This is not evident, as the regression model visually agrees very well with the data (Figure 3).

This discrepancy is quite unsettling to the student, and when the format of the regressed line is changed from the *default* to *scientific*, the resulting curve is shown differently as

$$\alpha = -1.2278\times10^{-5}T^2 + 6.1946\times10^{-3}T + 6.015\,, \tag{5}$$

At the same value of T=-300 °F, the value of the thermal expansion coefficient is calculated as 3.05 μin/in/°F, while the actual value of the data is 3.07 μin/in/°F, which is now only a 0.6% difference as opposed to 6% when Equation (4) is used. This clearly shows the effect of significant digits on accuracy in a real-life problem. Now, one can change the number of significant digits in Excel scientific format and quantitatively study the effect of number of significant digits on the difference between predicted and observed values.

5. When one is solving to find the coefficients of the polynomial regression curve for the coefficient of thermal expansion vs. temperature data $(T_1,\alpha_1),(T_2,\alpha_2),...,(T_n,\alpha_n)$, it requires one to solve simultaneous linear equations. For example, for a second order polynomial regression curve $\alpha = a_0 + a_1T + a_2T^2$, you need to solve three simultaneous linear equations.

$$\begin{aligned}
&na_0 + a_1\sum_{i=1}^{n}T_i + a_2\sum_{i=1}^{n}T_i^2 = \sum_{i=1}^{n}\alpha_i \\
&a_0\sum_{i=1}^{n}T_i + a_1\sum_{i=1}^{n}T_i^2 + a_2\sum_{i=1}^{n}T_i^3 = \sum_{i=1}^{n}\alpha_iT_i \\
&a_0\sum_{i=1}^{n}T_i^2 + a_1\sum_{i=1}^{n}T_i^3 + a_2\sum_{i=1}^{n}T_i^4 = \sum_{i=1}^{n}\alpha_iT_i^2
\end{aligned} \tag{6}$$

Equation (6) is solved by numerical methods such as Gaussian elimination. In addition, solutions of equations for higher-order polynomial regression are used to illustrate the effect of relative large differences in the order of coefficient matrix elements on the accuracy of the solutions.

6. To cool the trunnion for shrink fitting into the hub, we need to estimate the time required for the trunnion to reach steady state temperature when it is immersed in a cooling medium. The trunnion is approximated as a lumped system and the cooling is modeled as a first order ordinary differential equation of temperature, θ as a function of time, t as

$$mC\frac{d\theta}{dt} = -hA(\theta - \theta_a), \tag{7}$$

where

h = convective cooling coefficient,
A = surface area,
θ_a = ambient temperature of the cooling medium,
m = mass of the trunnion,
C = specific heat of the trunnion.

Equation (7) has an exact solution. However, it is shown to the student that if the temperature change is large, the convective cooling coefficient, h and the specific heat, C are not constant but vary substantially with temperature. In such cases, the ordinary differential equation needs to be solved numerically by methods such as Euler's and Runge-Kutta methods. The results obtained using constant and varying convective cooling coefficient and specific heat are compared.

From the above posed problems, all major mathematical procedures, namely, nonlinear equations, simultaneous linear equations, interpolation, regression, integration, and ordinary differential equations that are taught in a typical numerical methods course are covered.

Assessment

Using a single real-life physical problem to illustrate every mathematical procedure covered in the course is highly appreciated by students and faculty alike. It was not possible to have formal assessment data that isolated the impact of the problem-centered approach because other synergistic improvements were being made simultaneously in the course.

In addition to problem-centered learning, these improvements[13,14] included 1) user controlled simulations in multiple computational systems such as Mathcad[15], Maple[16], Matlab[17], and Mathematica[18], 2) textbook notes and eBooks, 3) problem assignments and multiple choice tests based on Bloom's taxonomy, and 4) real world problems from multiple engineering majors. The summary assessment data for the overall improvement in student examination performance and student satisfaction in the course is given below to illustrate the synergetic effect of all improvements.

Final Examination Performance: The final examination consists of 32 multiple-choice questions based on Bloom's taxonomy[19]. Four questions are asked on each of the eight numerical methods topics. Two questions are asked from the lower three levels of Bloom's taxonomy and two from the higher three levels of Bloom's taxonomy. The percentage of correctly answered questions is given in Table 2. Also given are statistics based on t-test of two samples (before full implementation and after full implementation of problem-centered approach) assuming unequal variances with a level of confidence corresponding to α=0.05.

Table 2. Overall results (maximum of 100) for final examination results

	Before Full Implementation Summer 2004 (Mean $\pm$ SD)	**After Full Implementation Summer 2006 (Mean $\pm$ SD)**	**Difference**	**t-Stat**	**p-value**
Number	49	56			
Average	57 ± 11	63 ± 14	6	t(101)=-2.22	<0.03

Student Satisfaction Survey: Student surveys, which consist of three distinct sections (reading assignments, class presentation, and problem sets) with eight questions per section, are asked. Students answer questions on a Likert[20] scale of 1 (truly inadequate) to 7 (truly outstanding). The overall results are given in Table 3. Also given are statistics based on the t-test of two samples (before full implementation and after full implementation of problem-centered approach) assuming unequal variances with a level of confidence corresponding to α=0.05.

Table 3. Overall results (1- truly inadequate to 7- truly outstanding) for student satisfaction (Number of students is in parenthesis)

	Before Full Implementation Summer 2004 (Mean ± SD)	**After Full Implementation Summer 2006 (Mean ± SD)**	**Difference**	**t-stat**	**p-value**
Number	45	49			
Average	4.3 ± 1.5	4.9 ± 1.2	0.6	t(86)=-2.12	<0.04

Conclusions

The integration of a single research problem has been successfully implemented in a course in Numerical Methods. The breadth of topics covered using the research problem includes all the mathematical procedures taught in a typical Numerical Methods course. Coupled with other improvements, the effect of the problem-centered approach improved student satisfaction and student examination performance in the course.

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