Incorporating Engineering Applications into Calculus Instruction[1]

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1. Introduction

The purpose of this paper is to present two multimedia modules in the areas of differential calculus, industrial engineering and industrial management as well as preliminary results on incorporating one of the modules in calculus instruction. These two modules are developed through a collaboration between the mathematics and industrial engineering departments at North Carolina A&T State University under the NSF funded project "Enhancing Mathematics Courses through Engineering Applications."

The first module addresses cost curves and optimization analysis related to the car replacement problem and inventory control problem, and is incorporated in Calculus I instruction. The car replacement problem discusses how long to retain a vehicle of your choice before trading in or selling it for a new one, while the inventory control problem is to make a decision on how a retailer determines economically the quantity of a product ordered from a distributor or manufacturer. Various cost components are introduced, and the total cost is then formed. The objective is to minimize the total cost. Both application problems can be solved by differential calculus. The second module studies the problem of location of facilities in a geographical area, and is to be incorporated in Calculus III instruction. The objective of the problem is to reduce the total transportation cost. When the distance measure used is Euclidean distance, finding a solution to the problem involves evaluating partial derivatives and solving a set of equations. We have recently solved a real problem in location for a company and have data to provide realism to the exercise.

Through these applications, students will have increased awareness of the importance of calculus they study and hence will have a stronger motivation to understand the materials. Faculty will also benefit in having easy access to up-to-date applications of the topics covered in calculus that are not provided in current textbooks.

[1] This work was supported in part by the National Science Foundation under the Grant DUE-9752266

The paper is organized as follows. Section 2 explains the need and motivation for incorporating engineering applications in calculus and other introductory mathematics courses. Section 3 presents the modules. Section 4 describes the teaching methodology used in incorporating the modules in calculus instruction. Section 5 describes the current status of the project. Section 6 summarizes the paper.

2. Motivation

The motivation for incorporating engineering applications in calculus and other basic mathematics courses is described in this section.

A question that often puzzles freshmen and sophomore college students concerns the applicability of the mathematics they are exposed to in standard introductory college mathematics courses. Many students regard these courses as a series of numbers and symbols that will never be used in their major field and in so-called "real-life" applications. Consequently, they find these courses dull, uninteresting, and often fail to see their importance. Hence, they lack the motivation necessary to obtain a thorough understanding of the material.

In addition, many mathematics faculty also lack knowledge of the applications of the introductory mathematics materials they teach. Many textbooks still do not provide a good source of up-to-date applications of the concepts that are covered. In addition, many mathematicians have had little exposure in their education to such application fields such as engineering and other sciences and thus have little knowledge to offer students in these areas.

In an attempt to alleviate this problem, a collaborative effort among the mathematics, electrical engineering, industrial engineering, and mechanical engineering departments at North Carolina A&T State University entitled, "Enhance mathematics courses through engineering applications," is currently being implemented with support from the National Science Foundation through a Course and Curriculum Development (CCD) grant. This project emphasizes the development of multimedia modules designed to demonstrate practical applications of topics covered in a typical engineering calculus and differential equations course sequence. A team of seven Co-PIs is divided into four groups. Each group consists of one mathematics faculty and one engineering faculty, assisted by a graduate student, and is charged to develop two multimedia modules and to design four computer laboratory exercises, two per module. The modules will be web-based and interactive, and will be created with the ToolBook II Assistant software. In a typical module, students are introduced to an interesting application topic through the use of text, audio and video in a multimedia software application. In addition, through two computer labs, students have the opportunity to explore and discover important aspects of the topic, and gain hands-on experience. As one of the four groups, this team of three authors is charged to develop one module for Calculus I, and one for Calculus III, and to design two computer labs for each module.

This project is a logical continuation of resources developed through recent pilot studies, such as "Early Design Experiences" [1], "Introducing Biology into Engineering

Curricula" [2], "IMPEC: An Integrated First-Year Engineering Curriculum" [3,4], "Mathwright Library" [5], and "Mathematics Across the Curriculum" [6], conducted at other institutions. These studies on integration of subject matter in engineering or science curricula can be classified as either vertical or horizontal integration. The "Early Design Experiences," "Introducing Biology into Engineering Curricula," and "Mathematics Across the Curriculum" are three examples of vertical integration. These efforts have attempted to provide linkages across the curriculum among courses placed in different years of a curriculum. For example, in the "Early Design Experiences" effort, freshmen are provided the opportunity of working with seniors in a capstone design project team, giving the freshmen a "glimpse" of a capstone project. Horizontal integration efforts involve courses that are placed in the same year of a curriculum. Most efforts here have involved courses in science and math at the freshman level. An example of horizontal integration is "IMPEC: An Integrated First-Year Engineering Curriculum" conducted at North Carolina State University. In that effort, linkages between mathematics, physics, chemistry and engineering have been conveyed to the student via courses that integrate the subject matter at the freshman level.

This project addresses vertical integration among topics in mathematics and engineering courses at freshman and sophomore levels. Specifically, development of examples that address applications of engineering in a form and format that is suitable for integration into mathematics courses will be the focus of the project. Examples will be developed for eight topics for four courses: Calculus I, II, and III, and Introductory Differential Equations, two topics for each course. We desire to develop multimedia modules which follow the same version as the *Mathwright* project approach, that is, modules that are easy to implement for both students and faculty while being powerful enough to encourage student experimentation. Our primary goal of this project is to increase motivation, retention, and interest in mathematics among students at North Carolina A&T State University.

Why do we choose multimedia format in our development of application topics? The following comments may give some hints.

1. "Good educational software is *active*, not *passive*. Students ought to be doing something, not watching something." [7]
2. Key elements of an learning environment include [8]:
   (a) parameters of a database or simulation which can be altered,
   (b) system knowledge of the components in the environment (for example, a model of the mathematical formulas which describe the behavior of the application), and
   (c) system reasoning about user actions, and immediate response.
3. "Multimedia is less restrict than written text. Many people come to understand text better with border media support for its interpretation" [9].
4. An example of a multimedia tool that strongly encourages student exploration is the *Engineering Tutor,* developed by Corrado Poli at the University of Massachusetts [10]. This software teaches the concepts of design for manufacturing to first-year engineering undergraduate students.

The multimedia modules developed in this project will incorporate the philosophy represented by the above; not only will the student be introduced to an interesting application topic but also be required to explore and discover on their own, through nontrivial accomplishable tasks, about important aspects of the problem.

3. Two Application Modules

This section describes two multimedia modules of engineering applications of calculus, one is already developed, and the other is in progress.

The first module addresses cost and revenue curves, and optimization analysis related to the car replacement problem and inventory control problem, and is incorporated in Calculus I instruction. First let us give some background information about these two application problems, and mathematics content related to these two problems.

We all know that in both our personal life and in the work environment, we sometimes have to make economic decisions wisely. One such problem we encounter is in the area of capital expenditure (or expenses that can be characterized as being "large" and having to be made less often). The problem is of making a decision on how long to retain the "asset" purchased via the capital expenditure. An example of the capital expenditure problem in our personal life is the car replacement problem—how long to retain a vehicle of your choice before trading in or selling it for a new one. Another such problem is in the area of inventory control. The problem is to make a decision on how a retailer determines economically the quantity of a certain item ordered from a distributor or manufacturer. One typical example in the inventory control problem is about a car dealership—how many cars of a certain maker and model should be ordered from the manufacturer to maintain an adequate stock as well as the minimum maintenance and insurance cost.

The functional forms that different components of cost take on are linear, constant, and inversely proportional to a positive power of the decision variable of interest, namely, retention period of a vehicle for the car replacement problem, and order quantity for the inventory control of a retailer.

The total cost for the car replacement problem consists of components such as purchase price, maintenance cost, insurance, property taxes, and fuel consumption. It is a superposition of the following three costs: **initial cost (IC)**, **operating cost (OC)**, and **maintenance cost (MC)** (cf. Figure 1). The functional forms of the three cost components and the total cost are: $IC(t) = k$, where k is a constant, $OC(t) = a + bt$, where a and b are two constants, and $t > 0$ is the retention period, $MC(t) = c/t^{1/3}$, where c is a constant, and $TC(t) = IC(t) + OC(t) + MC(t) = k + a + bt + c/t^{1/3}$. The total cost for the inventory control of a retailer consists of components such as ordering related costs, maintenance cost, insurance, salesman's salary, and value depreciation. It is a superposition of the following three costs: **purchase cost (PC)**, **order cost (OC)**, and **holding cost (HC)** (cf. Figure 2). The functional forms of the three cost components and the total cost are: $PC(q) = p$, where p is a constant, $OC(q) = kr/q$, where k is per order cost, r is demand of units per year, both being constant, and q is the order quantity, $HC(q) = hq/2$, where h is the carrying cost and is a constant, and $TC(q) = PC(q) + OC(q) + HC(q) = p + kr/q + hq/2$.

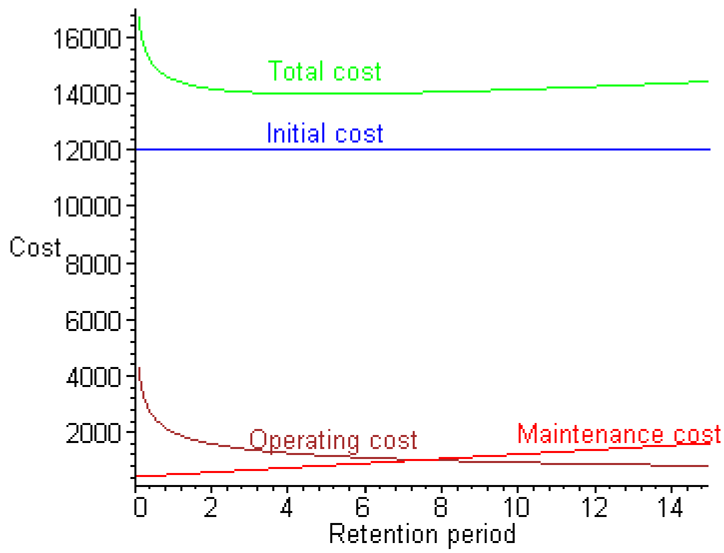


Figure 1: Cost Components of the Car Replacement Problem

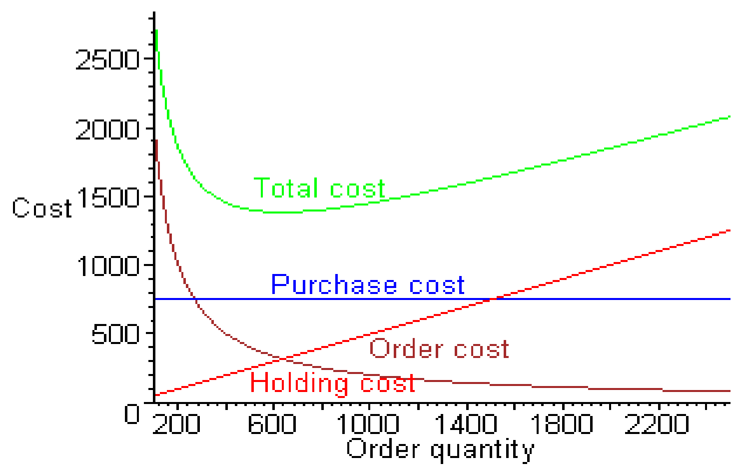


Figure 2: Cost Components of the Inventory Control Problem

The total cost may be annualized (expressed as an annual cost) and expressed as a function of either the retention period for the car replacement problem, or the order quantity for the inventory control of a retailer. This function is a superposition of three functions which are linear, constant, and inversely proportional, respectively, and is convex. The “economic” retention period for the vehicle or order quantity for the particular maker and model car can be obtained by application of differential calculus.

The module consists of four components: **Math Review**, **Background on Application Area**, **Car Replacement Problem**, and **Inventory Control Problem**.

In the Math Review part, we discuss/review several differential calculus topics related to the car replacement and inventory control problems:

- Functions and their graphs
- Superposition of several functions
- Curve fitting: least squares approach
- First and second derivatives
- Increasing and decreasing functions and the first derivative test
- Convexity and the second derivative test
- Optimization

Those calculus concepts are enhanced by examples, and graphical illustrations. This part prepares students for exploring the two application problems.

In the Background on Application Area part, we discuss the following topics:

- A widget factory
- Widget-making processes
- Costs involved in widget-making
- Different types of costs
- Revenue and profit
- How many units to make in order to maximize profit?

In this part, students are challenged to answer the questions: how to determine the fixed cost and variable cost; how to form the total cost; given the revenue function, how to form profit function and maximize it to reach the highest profit; what is the relationship between a local extremum and an absolute extremum? Under what conditions are two the same? The Background on Application Area part serves as a transition from the Math Review to two real application problems.

In the Car Replacement Problem part, we discuss the following topics:

- What is the problem?
- Costs involved
- Types of costs
- Deciding when to replace a car

In this part, an overview of the car replacement problem and various costs incurred in owning a car are discussed. Those costs are then classified into three cost components, and the total cost is then formed. The solution procedure is proposed to determine the best time to trade in or sell the car.

In the Inventory Control Problem part, we address the following topics:

- What is inventory control?
- Costs involved in inventory

- The inventory control problem

In this part, a number of industrial management concepts are discussed. Different types of inventory and the need to have inventory are presented. Various costs associated with inventory control are defined. Classification of these costs is stated. Finally, a mathematical solution to the problem is given.

Besides text presentation, we also include audio and video components in the module. For example, to illustrate a widget factory which produces one product made up of two different parts, we create an animation of a widget factory assembly line in which parts are produced by a set of three different processes A, B, and C. After each of these parts is produced, it is assembled in process D to make the product, which is then shipped out (cf. Figure 3).

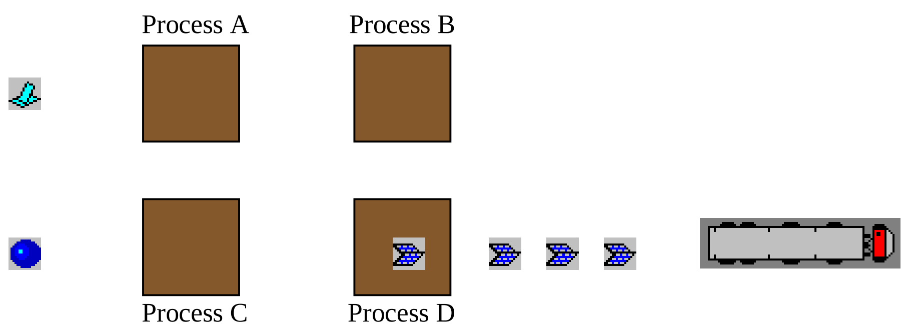
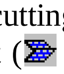


Figure 3: A Widget Factory

The four processes can be further specified: ***Power Press***, ***Cutting***, ***Lathe***, and ***Assembly***. Part A () is made as it goes through three processes in the following sequence: power press, cutting, and lathe. Part B () is made as it goes through three processes in the following sequence: power press, lathe, and cutting. Next, Parts A and B are assembled in Assembly Process to make the final product (). The completed products are shipped out of the factory. Sound effects of producing parts and assembling products are also illustrated through audio chips. Widget-making processes are illustrated in Figure 4.

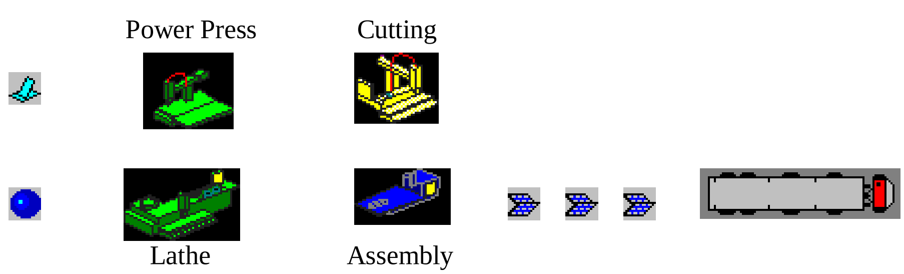


Figure 4: Widget-making Processes

For this module, we develop two computer labs. In the first lab, 20 application- related quiz problems and 20 math review quiz problems are given in Toolbook II Assistant format. The lab session is conducted by the instructor right after the presentation of the module. Students will be asked to answer these multiple-choice problems on-line. Scores will then be generated by the software, and individual report will then be mailed to the student and instructor. The second lab is designed in a project style. This lab design consists of three parts: introduction to basic Maple commands, a curve fitting technique: least squares fitting, and an assignment of completing one of the two Maple projects in car replacement and inventory control problems, respectively. This will be hands-on experience. Independent thinking and teamwork will be required. Maple source codes will be provided and can be downloaded from the project web server.

The second module is in planning and is to be developed in the Spring semester of 1999. It addresses the selection of a potential location of facilities in a geographical area. Such a problem is often referred to as the facility location problem. This class of problems can have several objectives but the most common one is to reduce the total transportation cost and the delivery time. When the distance measure used is Euclidean distance, finding an optimum solution to the problem involves finding partial derivatives and then solving a set of equations to locate stationary point(s). We have recently solved a real problem in location for a company and have data to provide realism to the exercise. This is an appropriate problem for a module on partial differentiation and optimization covered in Calculus III course. We plan to develop a module to address this learning objective in Calculus.

The two application modules will be available for evaluation and adoption by written request to one of the authors.

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4. Teaching Methodology

The first module of cost and revenue curves is incorporated into Calculus I in the following manner. Following the completion of the materials of *Chapter 5: Analysis of Functions and Their Graphs* in the current Calculus textbook (Anton [11]), we use three classes to present the module. This is done in cooperation with the faculty from engineering. In the first class, the Math Review part of the module is presented by the math faculty to prepare students for the next two class presentations. It basically reviews what was covered in Chapter 5 and previews part of the materials of *Chapter 6: Applications of the Derivatives*. In the second class, the Background on Application Area part is presented. Finally, in the third class visitation, the car replacement and inventory control problems and their solutions are presented. Both the Background on Application Area and two application problems are presented by the engineering faculty. Students are then asked to give their perception and suggestion for improvement of the presentations of the module and to indicate whether the module has served to increase their overall interest in mathematics through a survey conducted right after the three classes.

One of the primary benefits of using multimedia as a teaching tool is the time saved for the instructor in teaching application topics as opposed to trying to teach these topics within a standard lecture format. This time savings alleviates the instructor's worry in covering the basic concepts within the time frame of a standard syllabus and allows more time for face-to-face interaction with students, what-if discussions, and the consideration of the "real-life" applications at hand. For example, it normally would require 5-6 class meetings to cover most of topics of Chapter 6. However, we here covered most of topics in 3 class presentations with the module. The saved time can be used to conduct two computer lab sessions.

The first computer lab is conducted one week after the last class, and serves as an exam to test students' knowledge and mastery of the application problems and their underlying mathematics concepts. Students are asked to visit the project web site and view the module during this period of time to prepare for the first lab session. Students are expected to answer a total of 40 multiple choice questions, 20 of which are math-oriented, and the rest are applications-oriented. This lab session could be conducted by either the instructor or a graduate assistant. A score will be generated after an answer to each of all 40 questions is checked, and is e-mailed to both the student and instructor. We would like to make a remark there that this computer lab session can also be executed as an assignment by students in an out-of-class setting on their own spare time if more time-saving is desired.

The second computer lab is designed in two parts. The first part will be presented by the instructor. It will cover the basic Maple commands and a curve fitting technique: least squares approach in Maple. The second part is a project to be completed by students. Two project topics will be provided, one of which is related to the car replacement problem, and the other is related to the inventory control problem. Students have an option to choose to work on one of them individually or in a team. Data for various costs for each of the two application problems will be provided. Students will be asked to approximate the cost functions, sketch the cost curves, and determine the optimum time to trade in or sell a car, or to compute the optimum quantity of goods of an order. A project report is required from each individual or team, and will be graded by the instructor.

This approach of combining in-class module presentations with computer lab sessions will not only increase students' awareness of the importance of calculus and motivate them to understand the material better, but also provide students with hands-on computer experience, cooperative learning opportunities, and reinforcement of lecture concepts.

The implementation of the second module in Calculus III instruction will follow a similar approach.

5. Current Status

The module on cost and revenue curves has already been developed and tested in the classroom. A run-time version of the module has been generated and installed in several computers in a PC lab in Marteena Hall of Physics and Mathematical Sciences for students to view. The module was presented in one Calculus I class in the week of November 16-19, and the first computer lab was conducted in the week of November 30-December 4. A survey of students was conducted simultaneously. Out of 27 participating students, 15 students answered "yes" to the question: *did the application examples help you learn calculus better*? 7 answered "no" to the question, 5 answered "don't know"; 25 students answered "yes" to the question: *would you like to see more applications in* *calculus classes*? 2 answered "no" to the question; 26 students answered "yes" to the question: *do you think multimedia format is useful*? Only one student answered "no" to this question. Suggestions for improving the presentations of the module in the four categories of content, examples, video and audio were also provided by the students for each of the four parts of the module.

At present, we are working on revision and web delivery of the module on cost and revenue curves. The design of the second computer lab for the module is also in progress, and is expected to be completed by the beginning the Spring semester of 1999. We are also in the process of planning and developing the second module on the facilities location problem to serve as an application of partial differentiation and optimization in Calculus III. We expect to complete the module by the end of the Spring semester of 1999 and incorporate it into Calculus III instruction in the Summer of 1999.

6. Summary

This paper describes a collaborative effort of developing two multimedia modules of engineering applications of calculus, and provides preliminary results of incorporating one of the two modules into calculus instruction. Four accompanying computer labs are being designed. Co-teaching by math and engineering faculty approach is implemented. Feedback on both the presentations of the module and its impact on student's learning is sought, and will be used for improvement of the presentations of the modules. Work currently in progress is also briefly mentioned.

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