AC 2008-2957: INCORPORATING EXPECTATION FAILURES IN AN UNDERGRADUATE FINITE ELEMENT COURSE

Vince Prantil, Milwaukee School of Engineering
Vince Prantil is an Associate Professor in Mechanical Engineering at the Milwaukee School of Engineering. Dr. Prantil received his BS, MS, and PhD in Mechanical Engineering from Cornell University. His research interests lie in micro-structural material modeling, finite element and numerical analysis. He was a senior staff member at Sandia National Laboratories California in the Applied Mechanics and Materials Modeling departments for eleven years. He joined the mechanical engineering faculty at MSOE in September 2000.

William Howard, East Carolina University
Ed Howard is an Assistant Professor in the College of Technology and Computer Science’s Department of Engineering at East Carolina University. He holds a B.S. in Civil Engineering and an M.S. in Engineering Mechanics from Virginia Tech, and a PhD in Mechanical Engineering from Marquette University. He has 14 years of industrial experience, mostly in the design and analysis of composite structures.
Incorporating Expectation Failures in an Undergraduate Finite Element Course

Vincent C. Prantil  
Milwaukee School of Engineering

William E. Howard  
East Carolina University

Abstract

In teaching an introduction to the finite element method at the undergraduate level, a prudent mix of theory and applications is often sought. At the Milwaukee School of Engineering (MSOE), the four year course of study culminates in the seniors addressing most aspects of the design process in the context of a year long capstone design project. In many cases, students use finite element analysis to perform parameter studies on potential designs to size parts, and weed out less desirable design scenarios. In this paper, we discuss common pitfalls encountered by many finite element analysts, in particular students encountering the method for the first time. We present two very simple problems in beam bending that distinguish the students’ knowledge of theoretical mechanics, the numerical method and approximations particular to the finite element method itself. We also present efforts to incorporate experimental laboratories in which analyses are coupled with the experiments to address how students’ interpretations of numerical results can be led astray and what can be done to allay such tendencies. Challenges in presenting the necessary mix of theory and applications in the context of a 10 week course are discussed. We also discuss a proposal for a follow-on course addressing such advanced topics as three-dimensional applications, transient and nonlinear analyses, and thermal analysis.

Introduction

In many undergraduate engineering curricula, a first course in finite element analysis is required [1]. The focus of such a class is often an overview of the procedural aspects of the method and development of the finite element theory for a variety of relatively simple one and two-dimensional element formulations. This is necessarily coupled with performing finite element analysis on relatively simple, linear, static boundary value problems. More and more often, these courses have exposed students to the use of commercial finite element software for solving these same boundary value problems. At the Milwaukee School of Engineering (MSOE), the undergraduate curriculum culminates in a senior-level capstone design experience wherein students integrate their accumulated learning with design intent foremost in mind. While all students have been exposed to the
commercial finite element software, as many as half of these students exercise it substantially in some element of their capstone design projects.

Recently, Chalice Engineering [2] compiled a subjective assessment of common mistakes in finite element analysis routinely performed in many industrial sectors. After 5 years of collecting anecdotal evidence in both teaching undergraduates and advising capstone design projects, we found this list to be nearly inclusive of the most common and more serious errors encountered by novice users of the finite element method. Here, we add several additional mistakes commonly observed in the classroom and in capstone design numerical analyses and present the augmented list in Table 1. While it may come as no surprise that novice users commit many, if not all, of these errors, they appear to routinely and repeatedly encounter a particular subset of them.

<table>
<thead>
<tr>
<th>TABLE 1. COMMON MISTAKES IN FINITE ELEMENT ANALYSIS</th>
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<tbody>
<tr>
<td><strong>Mistakes Most Often Made by First Time Undergraduate (Novice) Users</strong></td>
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<tr>
<td>(1) Lack of Verification: inadequate verification information to bridge the gap between a known benchmark solution and one’s own finite element strategy.</td>
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<td>(2) Ignoring geometry or boundary condition approximations: need to understand how inappropriate restraint conditions can affect results.</td>
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<td>(3) Not understanding the best dimensional space in which to perform an analysis: inadequate understanding of two-dimensional elasticity as approximations to three-dimensional analyses.</td>
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<td>(4) Ignoring errors associated with the mesh: sometimes these errors cancel out those associated with (2) which can confuse the user into thinking the model is more accurate than it is.</td>
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<tr>
<td>(5) Comparing to inadequate theory: novice analysts sometimes choose to verify finite element results with inadequate theory.</td>
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<td>(6) Wrong elements: use of inefficient element types or unreliable elements.</td>
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<tr>
<td>(7) Assuming conservatism: because one particular finite element analysis is known to be conservative, a different analysis of a similar structure under different conditions may not be so.</td>
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<td>(8) Poor post-processing: not post-processing results correctly or consistently.</td>
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**Additional Mistakes Consistently Made by Undergraduate Users in Capstone Design Analysis**

| (9) Doing numerical analysis for the sake of it: not being aware of the end requirements of a finite element analysis |
| (10) Thinking a ten week course qualifies one to perform more general finite element analysis: the so-called expert phenomenon |

**Additional Common Mistakes Listed by Chalice Engineering**

| (11) Attempting to predict contact stresses without modeling contact |
| (12) Not standardizing finite element procedures: all analyses should follow a documented standard modeling procedure; not doing so is often a cause of repeated or lost work. |
| (13) Inadequate archiving: all analyses should follow a master model of detailed instructions about what and how to archive results; not doing so is a common source of lost work. |

Table 1. Common Mistakes Made in Finite Element Analysis compiled from [1] and anecdotal classroom observations.

1 Items are listed in order of frequency of occurrence with the earlier items occurring more often.
Because the three most commonly committed errors are observed repeatedly and the two most common are continually observed in industry, it would be fruitful to design early modeling experiences that challenge students to focus on the circumstances that lead them to make these errors. Observations made in the classroom indicate at least two contributing factors. First, circumventing these obstacles predicates the need for student awareness. If circumstances can be provided within which students can convince themselves their methods are in error, they will be more likely to reconstruct their modeling methods. Second, research indicates that when students have an incorrect model of how the world works, they hold onto it with great fervor and are reluctant to surrender it. But constructing a more correct model of how something works is expressly dependent on students first deconstructing their previously incorrect model. A good instructor can help students in doing this by creating for them “an expectation failure”[3]. It has been shown that when students are faced with a specific problem that does not conform with their model and they are forced to work it out with judicious questioning and investigation, their learning is deeper, and their recall and critical thinking skills are enhanced. With this in mind, two relatively simple examples in beam bending are offered which present students with such an expectation failure. The first is a failure of their interpretation of the dimensional space needed to model the bending behavior, while the second boundary value problem is a classic example of why a student’s typical underestimation of the importance of boundary conditions leads them to an incorrect prescription of the beam fixity conditions.

**Simply-Supported Point Loaded Beam**

In the first problem, a simply-supported beam of rectangular cross-section is point loaded at some arbitrary point along its length as shown in Figure 1.

![Figure 1: Simply supported beam.](image)

$P = 10,000 \text{ lb}, \quad L = 100 \text{ in}, \quad a = 75 \text{ in}, \quad b = 3 \text{ in}, \quad \text{and} \quad h = 8 \text{ in}$

While, in general, a finite element analysis will more accurately predict deflections than, say, internal stresses, this problem presents a potential case study for students to investigate a situation in which even the deflections can be poorly modeled. Before being assigned this problem, students have been introduced to one-dimensional beam elements, two-dimensional analysis of plane strain and plane stress problems using continuum elements and three-dimensional analysis using solid elements. They are advised to...
examine the possibilities of analyzing the problem with one, two, and three-dimensional element formulations. Students are further told they must choose and defend their method of analysis.

In the absence of knowing the correct answer, students often assume Euler Bernoulli beam theory applies, presumably because it is the theory with which they are most familiar. While this assumption by students is not particularly surprising, it leads them to make further errors in finite element judgment. For instance, in this case the maximum transverse deflection that results under plane stress conditions underestimates the predictions of Euler Bernoulli beam theory while those using an assumption of plain strain overestimate simple beam theory. Here, the two-dimensional assumption of plane stress is the more appropriate two-dimensional approximation as verified by a full three-dimensional analysis. However, the maximum deflection predicted using a relatively coarse mesh and assuming plane strain conditions is more in agreement with the result from Euler Bernoulli beam theory. Here, in addition to inappropriately applying simple beam theory, students often do not perform sufficient mesh refinement studies. Invariably they will most often accept a solution assuming plane strain conditions that overestimates the deflections, use too coarse a mesh, often with stiffer elements such as the triangular continuum element (rather than the quadrilateral) minimizing the over-estimation and bringing the predicted deflection into reasonable agreement with simple beam theory. While simple beam theory is not sufficiently accurate this geometry with pinned boundaries at the bottom surface of the beam cross section, students accept it as verification for an incorrect numerical simulation of the boundary value problem.

![Figure 2: Predicted maximum deflection assuming 2D plane strain, 2D plane stress along with a full 3D analysis.](image-url)
More precisely, elementary beam theory would be reasonable if one were to take into account the nature of the support boundary conditions. Some students notice that the plane stress solutions converge to a maximum deflection nearly half that obtained by simple beam theory. In this case, they may investigate the possibility of pinning the end supports of the two-dimensional mesh at the mid-plane location of the neutral axis. This, of course, lowers the area moment of inertia by close to a factor of two bringing the theory and two-dimensional analysis is very good agreement. Alternatively, they can apply beam theory or use offset, one-dimensional beam elements with a moment of inertia about some point well below the neutral axis as will be the case for pin supports on the bottom edge of the beam. This exercise illustrates the rather strong dependence of the solution of the boundary value problem to the precise prescription of the boundary conditions, as well as the bounding nature of two-dimensional continuum approximations for truly three-dimensional problems.

While they were willing to accept that the three-dimensional analysis is most accurate in this case, students remained generally frustrated that a two-dimensional analysis was always an approximation the accuracy of which they had to be prepared to verify. While a three-dimensional analysis may be accurate for this particular problem, it is substantially more computationally costly than the corresponding two-dimensional plane stress approximation. Further, it can lead a large percentage of students to conclude the rather strong and necessarily incorrect conclusion that three-dimensional analysis is always better than two-dimensional analysis which is always better than one-dimensional analysis. This conclusion is then tested by a second benchmark problem.

**Variation on the Simply-Supported Point Loaded Beam**

As part of a previous course in strength of materials, students have already tested a T-beam section under a pair of symmetric transverse point loads. The beam section and loading are illustrated in Figure 3. The load was applied with a hydraulic cylinder apparatus. Strain gages mounted at several locations between the loading points (where the moment was constant and the transverse shear force was zero) were monitored during the test. Results were compared to simple beam theory, and finite element analysis [4].

![Figure 3: T-beam cross-section and loading diagrams.](image-url)
Although solid modeling is not a prerequisite for this class, many of the students were already comfortable using commercially available solid modeling packages. The geometry of the beam was simple enough so that new users could analyze the beam using solid, hexahedral elements after importing the geometry from a solid model developed in class. The distribution of axial stress at the mid-span of the beam is shown in Figure 4\(^2\). The linear distribution of stress through the depth of the section was, of course, the expected result.

![Figure 4: Axial stress distribution in the T-beam.](image)

The beam was also analyzed using simple, one-dimensional beam elements. Results of these analyses and the test data from the lab allowed for some interesting comparisons, as shown in Table 2.

<table>
<thead>
<tr>
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<th>Axial Stress, Bottom of Beam(^3)</th>
<th>Axial Stress, Top of Beam</th>
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<tbody>
<tr>
<td>From measured strains in mechanics lab</td>
<td>0.1108</td>
<td>-0.2464</td>
</tr>
<tr>
<td>From simple beam theory</td>
<td>0.1134</td>
<td>-0.2724</td>
</tr>
<tr>
<td>From finite element analysis with 1D beam elements</td>
<td>0.1134(^4)</td>
<td>-0.2724</td>
</tr>
<tr>
<td>From finite element analysis with 3D solid elements</td>
<td>0.0536</td>
<td>-0.2184</td>
</tr>
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</table>

\(^2\) Most of the elements have been hidden for clarity.

\(^3\) Dimensionless maximum stresses are reported here, normalized by \(\hat{\sigma} = \frac{PLh}{2I}\).

\(^4\) Linearly extrapolated from stress at bottom of beam.
Predictions using simple beam theory agreed exactly with the one-dimensional beam element results. This was expected, since the beam element is based on the same assumptions as simple beam theory. These results also agreed fairly well with the experimental results (about 10% error at the bottom of the beam). The results from the solid-element analysis were far off from the other results. Students were asked to consider what the differences might be. Some possible reasons discussed included:

- The simple beam calculations were made with a cross-section that neglected the fillets between the web and flange. The solid-element model includes the fillets, resulting in a stiffer structure. The effect of the fillets on the moment of inertia is to increase it by about ½ of 1%. Therefore, the error introduced by neglecting the fillets was insignificant.

- Experimental errors, including reading of the applied pressure, locations of the supports and load application points, inaccurate modulus of elasticity, and strain gage errors, caused the measured strains to be inaccurate. If only the solid-element model were being compared to the experimental results, this might have been the students’ conclusion. However, the agreement of the simple beam calculations and beam element model results to the experimental results may cast doubt on the accuracy some particular aspect of the solid-element model.

- There are not enough elements through the thickness in the solid-element model to allow for the bending stresses to be accurately calculated. While this is a possibility, closer examination of the maximum and minimum stresses predicted by the solid-element model shows that the neutral axis location (assuming a linear distribution of stress) is more than ½ inch away from the centroid of the cross-section. This result suggests that some other type of loading is being introduced into the beam. With this in mind, the boundary conditions are, again, suspect.

The model was analyzed with boundary conditions that allow only rotation about the y-axis permitted on the edge of the supported end, as shown in Figure 5. This boundary condition seems to be a good representation of the physical constraint, as the beam rests on a support that extends across the width of the beam. Note that the portion of the beam that extends beyond the support is not included in the finite element model.

Figure 5: Boundary conditions used in finite element model and end supports of the beam.
However, the boundary conditions restrict motions that are possible with the three-dimensional nature of the actual physical constraint. In particular, the flange of the beam does not remain perfectly flat. Since the axial strain varies with distance away from the neutral axis, the transverse strain due to the Poisson’s ratio also varies. This variation of transverse strain, not accounted for in one-dimensional analyses, results in curvature of the flange. Students can easily visualize this effect by bending a rubber eraser between thumb and forefinger and noticing the curvature transverse to the applied bending. To allow the model to curve in the transverse direction, boundary conditions were applied to the two corner nodes, as shown in Figure 6 along with the deflected shape of a slice of the beam section with these new boundary conditions applied. Although the deflections are greatly exaggerated, the tendency of the beam flange to curve rather than sit flat on the support is clearly evident.

![Figure 6: Modified boundary conditions applied to finite element model and predicted deflected shape of beam at these supports.](image)

As reported in Table 3, the new results are much closer to the experimental results than those of the previous analysis with solid elements.

An important lesson for the students to take away from this exercise was that solid elements are not always the best choice for an analysis when this choice is made irrespective of the boundary conditions. Often, realistic deformations result that are outside of the realm of their limited experience. Many students think that because they have a part or assembly modeled with a 3D solid modeling program, it is logical to analyze the structure with solid elements. In this example problem, an analysis with over 14,000 three-dimensional, solid elements produced no better results than an analysis with four simple one-dimensional beam elements.
Table 3  Results of Beam Analyses

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<tr>
<td>From simple beam theory and 1D beam elements</td>
<td>0.1134</td>
<td>-0.2724</td>
</tr>
<tr>
<td>From finite element analysis with 3D solid elements and loosely pinned supports</td>
<td>0.0946</td>
<td>-0.2230</td>
</tr>
<tr>
<td>From finite element analysis with 3D solid elements and fully pinned supports</td>
<td>0.0536</td>
<td>-0.2184</td>
</tr>
</tbody>
</table>

In addition, very often the part or assembly modeled with a 3D solid modeling program has been created without previous knowledge of where and how loads and boundary conditions will need to be applied in a subsequent finite element analysis. Often students will struggle with wanting to import these solid model part or assembly files, nonetheless. This often forces them to place less than optimal loadings and boundary conditions where they otherwise might not.

A frame made up of thin tubing sections, such as is typical for Mini-Baja and Formula cars and Human Powered Vehicles, all routinely analyzed and built for American Society of Mechanical Engineers (ASME) and Society of Automotive Engineers (SAE) student competitions, can require hundreds of thousands of elements to model with solid elements, when 100 to 200 beam elements will suffice. Typical example frames are shown in Figure 7, along with respective finite element models in Figure 8.
A Proposed Dual Finite Element Course Sequence

After having compiled the list of most commonly committed finite element crimes, several anecdotal observations repeatedly appear:

- Students, while initially frustrated by the problems with built in “expectation failures”, admit in course evaluations that struggling with the apparent lack of a single numerical solution, the exercise in prudent verification served them well.
- Students report a newly developed appreciation for the importance of boundary conditions and analyzing a problem with more than one element type or mesh.
- Students report a new found skepticism for the results of a single, preliminary finite element analysis.

While this makes an admittedly modest dent in the list of finite element mistakes most commonly made, we feel it is a good place to start. The philosophy that we should address the problems that exist in preliminary instruction, is, we feel, well-founded. To this end, in a new curriculum revision to be implemented at the Milwaukee School of Engineering in the fall quarter of 2008, there are plans to introduce a required second course in finite element analysis. With an eye toward preparing students for the types of analysis they might most be interested in addressing in their capstone design projects, the follow-on course will be offered in multiple sections, each with a different focus area. It will be the intent of this course to further address commonly committed errors in the context of teaching further finite element theory in areas pertinent to their individual capstone design experiences.
We are tentatively considering the following list of topically-focused course offerings:

- Transient dynamics and vibration analysis
- Rudimentary nonlinear analysis\(^5\)
- Three-dimensional structural analysis of assemblies\(^6\)
- Heat transfer and thermal analysis, both steady state and transient
- Computational fluid dynamics and finite difference methods

**An Apprenticeship Model**

MSOE currently has an exchange program with the Fach Hochschule Lubeck in Lubeck, Germany. As part of the degree requirements for the Lubeck diploma, students are responsible for completing a thesis project in conjunction with their capstone design project. Students spend their junior year at Lubeck and their senior year in Milwaukee where they all take the finite element course together. It has been our experience that a fair amount of learning of the finite element method can be attained by a journeyman application of the method under the watchful tutelage of a more experienced user. The so-called apprenticeship model, while perhaps impractical for large class sizes, has distinct advantages. In the context of the Lubeck exchange program, it was deemed an ideal scenario for examining these advantages. In the spring of 2007, we offered an unofficial, one-on-one tutelage to students desiring to bring their capstone design analyses to us for advice in modeling these systems. Two projects were undertaken: dynamic response of the rear suspension of a Mini-Baja vehicle and equivalent static loading for a toothed gear in a transmission assembly, illustrated in Figures 9 and 10 respectively.

![Figure 8: Modeling tubing and plate assembly for the rear suspension of the SAE Mini-Baja vehicle competition [6].](image-url)

\(^5\) Focused on material nonlinearity as the large deformation aspects of geometric nonlinearity are beyond an undergraduate scope.

\(^6\) More complex structures with emphasis on importing solid models, mesh construction, refinement, boundary condition prescription taking into account the principle of statically equivalent loads and St. Venant’s principle.
The apprenticeship model offered students a variety of the advantages of apprentice-journeyman interaction, including:

- Discussion with someone more experienced in the details of more realistic finite element numerical analysis
- Discussion of simplified vs. more complex models
- Discussion of assumptions relevant to the problem at hand
- Detailed discussion of ramifications of their numerical model, i.e.
  - Choice of element type and mesh
  - Prudent boundary condition prescription
  - Use of statically equivalent loadings to reduce model size
  - Exploiting problem symmetry
  - Use of mixed element types in static analysis

While the number of students was admittedly small, the response was generally positive. We hope to offer this apprenticeship in modeling as part of the diploma thesis course in the Lubeck program track in Spring of 2008.

Conclusions

We have introduced a sequence of problems exposing students to “expectation failures” into an introductory course in finite element analysis at the Milwaukee School of Engineering. We have done this to emphasize common pitfalls encountered by many finite element analysts, in particular students encountering the method for the first time. We have presented very simple beam bending benchmark problems that distinguish their
knowledge of theoretical mechanics and numerical approximations particular to the finite element method itself. We have also presented efforts to incorporate experimental laboratories incorporated to provide students with data with which to verify their numerical simulations. Based on an anecdotal list of commonly made mistakes in applying finite element analysis, these problems have been introduced to get students to focus on incorrect choices they routinely make. It is our hope that exposing students to their common mistakes early on will reduce a number of such mistakes from being repeatedly made. In a new curriculum development, the mechanical engineering faculty is proposing a requirement for a follow-on course addressing such advanced topics as three-dimensional applications, transient and nonlinear analyses, and thermal analysis, wherein the commonly made mistakes will, again, be addressed, but in context of the students’ capstone design projects. Finally, a description of a potential apprenticeship model is offered for exchange students to experience one-on-one tutelage in applying the finite element method.

References


