Incorporating Math and Design in High School Physics

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Introduction

Global competition is intense not only in business, but also in education. Although the most recent report (2003) from the Trends in International Mathematics and Science Study (TIMSS)\(^1\) reported progress for U.S. 8th-graders in their math and science performance, the newest report (2003) from the Program for International Student Assessment (PISA)\(^2\) showed that 15-year-old U.S. students rank near the bottom of industrialized countries in math skills. To promote inquiry-based learning of STEM (Science, Technology, Engineering, and Math) skills in K-12 education and cope with the rapid advancement in science and technology, the National Science Foundation has funded a number of research projects at dozens of universities nationwide. One of these projects is the Science and Technology Enhancement Program (STEP) currently being conducted at the University of Cincinnati. Graduate and undergraduate fellows of Project STEP are placed in different secondary schools to work with math and science teachers. The main responsibility of a fellow is to develop and implement hands-on activities that are creative, engineering-focused, and technology-driven. Activities are incorporated into lessons, demonstrations, laboratory exercises, and field experiences. By doing these activities, students will experience authentic scientific and engineering research and design practices that require higher-order thinking skills and problem-solving skills. This will enable students to develop a better understanding of science and engineering and hopefully foster a desire to advance their education in a related field.

The author, as a participant of project STEP, developed several modules to teach seniors at Hughes High School in Cincinnati, Ohio. One of these modules was designed to strengthen and test students’ knowledge about moment (torque) through studies of (mechanical) cranes. The overall objective of this module was to increase student interest in physics and engage them in the learning process. To this end, technology related to cranes was introduced at the beginning. The module included two sessions. Each session lasted 1.5 hours and contained a hands-on activity. These activities aligned with the science and math standards of Ohio and were designed to be attractive and challenging. All lectures for this module were delivered with PowerPoint slides. Most slides contain one or two graphics to illustrate the content being covered. Students were assessed by grading the worksheets they completed. They were also given an opportunity to evaluate the module, the implementation, and the performance of the instructor.

Students were given a model crane for the first activity. They were required to find the minimum tension force in the cable that held the jib in position. They were required to solve this problem experimentally first, then mathematically by drawing a free body diagram of the jib,
establishing the moment equilibrium equation, and using their differential calculus knowledge. Their second requirement was to improve the given model so that the tension force in the cable was kept to a minimum. Finally, they were given an improved model to compare with their own design and test whether the improved model was better. In the second activity, students were given a model tower crane. They were required to determine the weight and location of the counterweight for the crane. Again, they were required to solve this problem experimentally first. This provided them a good opportunity to get a taste of what are critical states and small disturbances according to Ohio science standards. Then, they were required to solve the same problem mathematically by a symbolic approach. Details of this module and some related thoughts will be presented in the following paragraphs.

Description

As mentioned above, activities in the “crane” module are both creative and challenging. The lesson plan for this module is presented below.

Mechanical Cranes

Objectives:

1. Students will experience authentic engineering research/design practice that requires higher-order thinking skills.
2. Students will make a scientific inquiry and construct a logical verification.
3. Students will be able to experimentally determine the minimum force in the string which holds the jib of a model crane in place. They will also be able to solve the same problem mathematically.
4. Students will learn how to improve the design of a given model crane.
5. Students will be able to solve a practical problem --- determine the weight and location of counterweight of a model tower crane. They will also be able to mathematically solve the same problem.
6. Students will understand the concept of critical state and small disturbance.
7. Students will know the scientific way to evaluate their findings.
8. Students will understand how the technology of designing, constructing, and erecting huge cranes is driven and developed by human needs.

Materials:

Each group of students needs:
Two model cranes as shown in the following figure, a spring scale, some weights, a protractor, a ruler, pieces of string, and several nails. Note: the bar with a pulley at one end can be taken away from the jib and will not be given to students until Step 7 in Activity One.
**Procedures:**

**Activity One:**
1. Show students pictures of different types of cranes, especially gigantic ones. Provide them with some data to show the incredible capacity a gigantic crane has. Talk about how modern technology, such as automatic control and high-strength materials, makes it possible to manufacture a crane with maximum lifting capacity as high as 1,600 tons at a radius of 22 meters.

2. Ask students to hold the jib of the given crane in a position such that the angle $\theta$ between the jib and the pedestal is $60^\circ$. The string attached to one end of the jib is the control to change this angle. Ask students to put a load of 100 grams at the free end of the jib.

3. Ask students to attach the other end of the string to points at different positions either on the pedestal or the wall (such as those indicated in the figure). The purpose of this step is to find the minimum force $F$ in the string.

4. Ask students to mathematically determine the minimum value of $F$. Ask them to compare the mathematical and experimental results, and calculate the relative error.

5. Ask students to repeat Step 2 but vary angle $\theta$ to a different value such as $30^\circ$, $45^\circ$, or $75^\circ$. Ask them how the variation of $\theta$ affects $F$.

6. Suppose angle $\theta$ is $60^\circ$. Ask students to figure out a way to improve the design of the given crane without using the wall (there is not a wall in a real crane) so that the force in the string will still be the minimum. Ask students to draw their improved design, if any, on the worksheet.

7. Give students the bar with a pulley on one end. Tell them they may use it to reduce the force in the string (the bar can be inserted on the jib).

8. With their improved crane, ask students to repeat Step 2. Ask them to compare the magnitude of $F$ with what they got in Step 2. They should get better results (smaller $F$). This step serves as a confirmation that the performance of their cranes is indeed improved.
Activity Two:
1. Show students pictures of tower cranes. Emphasize that preventing a crane from toppling is an important issue to be considered when designing and erecting it.
2. Guide students to experimentally determine the weight of the counterweight of the model tower crane and the exact location where the counterweight should be hung on the jib of the crane. Suppose the maximum lifting capacity of the model crane is 500 grams. Ask students to hang 500 grams of weight at the end of the longer arm of the jib and balance the crane with minimum weight on the shorter arm of the jib.
3. Tell students the crane is in critical equilibrium (state). If additional non-trivial weight is added on the longer arm, the crane will topple. However, if a trivial weight is added on the longer arm, the crane may remain in equilibrium. The trivial load is a small disturbance. A physical, chemical, or ecological system in critical equilibrium may or may not remain in equilibrium when it experiences a small disturbance. The point is that a small disturbance may result in completely different consequences under certain conditions. Give students several examples of critical equilibrium and small disturbance. Ask students to give examples, too, and record their examples on the worksheet.
4. Ask students to remove the load on the longer arm of the jib and adjust, if necessary, the weight and location of the counterweight to balance the crane. Tell them this is another critical state of the crane.
5. Ask students to determine the minimum weight and distance of the counterweight such that the crane can maintain equilibrium in both critical states.
6. Ask students to try some middle states which lie between the two critical states. The crane should not topple in these middle states. Guide students to draw the conclusion that a system will keep in equilibrium in any other states as long as it can keep its equilibrium in any critical states.
7. Ask students to determine the weight and location of the counterweight mathematically by a symbolic approach, then compare the mathematical results with the experimental results and calculate the errors, if any. The errors should not exceed 5%. Ask them to write their solutions on the worksheet. Tell students that acceptable errors are problem-dependent. For some engineering problems, the error may be as large as 15% and still be considered good due to the complexity of the problem and related limitations.
8. Tell students how a tower crane is fixed on the ground in reality. Tell students there may be one or more mobile counterweights in addition to a fixed counterweight for a large capacity tower crane, and the movement of mobile counterweights is computer-controlled.

Discussion
Hughes High School is an urban school. Most of its students are African Americans, and so also are the students in the physics class. Through the implementation of the module, it revealed that the participating students were weak in several skills. At the time the module was implemented, they had taken pre-calculus and were taking calculus; and they had just finished the study of a chapter which covered moment. However, although they had been exposed to the subjects such as derivative, integration, and series, they were not learned in the real sense. Therefore, they might be able to solve simple problems involving derivatives, but were not prepared to attack a real-world problem to find the derivative of a variable. In addition, they were too dependent on the use of calculators. So when they were asked to solve a symbolic equation by hand, most
students considered it too difficult to proceed. Among the few students who tried, nobody was successful because they had never been trained to do so. As for their skills in mechanics, it was found that they did not know how to isolate an object properly and draw a correct free body diagram of it, a fundamental skill in mechanics.

The module exposed the above problems by addressing them and set an example to address those problems in the future. It brought real-world design problems into the classroom. It provided an opportunity for students to review and apply knowledge essential for a career in science and engineering. It showed clearly that mathematics played a very important role in an engineering scenario. Quite obviously, integrating math and physics together in an engineering circumstance is an effective teaching approach. Its advantage can be seen more clearly by comparing it with a routine teaching approach. The author once observed a class of teaching Coulomb’s law in which the teacher asked students to solve problems one after another using the law. In each problem, three or four electric charges with different quantities were placed at arbitrary locations. Although variations were made as to each charge’s quantity and location, students soon became tired because they were just repeating a tedious work without real fresh ingredients.

Professors in the STEP project had a high evaluation of this module so it was selected to be presented at the NSF GK-12 annual meeting in 2004 in Washington, DC. Students’ reflections on the module were encouraging according to a survey with 17 questions. Of all 31 replies received, 8 rated the module to be “excellent”, 9 rated it “very good”, the remaining results included 10 “good”, 4 “fair”, 0 “poor”, and 0 “very poor”. Among the comments received from the survey, one student said, “These activities are very good but we could have added more things to it.” Another student said, “I love it! Very informational.” Considering that the instructor was not a native speaker and several students in the class did not take pre-calculus and calculus, so they felt frustrated with regard to derivatives, these results should be regarded as very positive. In addition, every student felt the activities were challenging, and they were found to be quite engaged in carrying out their investigations.

Although students’ reflections should be heeded, they need to be heeded with analysis. To investigate students’ attitudes about learning, a third activity was implemented following the two
ones described above. The third one involved neither math nor physics. It taught students the art of origami --- making paper cranes (birds). In the same survey, they were asked to evaluate the third activity, too. Surprisingly, only 20% of students gave the third activity a lower rating; the others either considered it better, or just as good as the other two activities. This result illustrates a fact that some students would rather play around for fun than learn something important to their future careers. It is a good example to show that, without analysis, reflections may be interpreted incorrectly.

As mentioned above, the students have been taught quite a bit of math. But they were slow in solving problems. Moreover, they usually chose to give up when they had difficulty in solving a problem. While there may be many ways to address this issue, one solution is to provide students with more opportunities to practice math in physics (and other appropriate) classes and embed the teaching in an engineering background. The module presented in this paper embodies such an approach. This way, math will appear more realistic, and students are more likely to feel interested in learning and practicing it. Although many students consider math a very challenging course, the challenge comes from lack of practice and interest. While more practice may be regarded as a vehicle for students to succeed in math study, more engagement serves as the power of the vehicle. By incorporating math with engineering, students will see the connection between math and technology, between their current study and future career, and consequently, will become more motivated. In addition, such a strategy will benefit not only math teaching, but also physics teaching, since students with better mathematical concepts and skills will generally perform better in physics.

Generally, students are more interested in doing hands-on activities rather than those that are not hands-on. However, this does not necessarily mean that they will learn more and better by just doing hands-on activities. In fact, most students tend to play around for fun, without much thinking and reasoning, if they are not directed. Hence, well-prepared guidance should be provided so that students will do the activities in a well-organized manner, finish all requirements in time, and grasp the principles embedded in the activities. Moreover, good activities should be attractive and challenging. Attractiveness is the key to keeping students busy; a proper difficulty level is the key to making students think in higher orders and better understand the subject matter.

Obviously, designing creative hands-on activities is time-consuming, and teachers usually do not have enough time to design innovative activities. Even if a teacher can find some time to develop a few good activities, it is hard to do so for a whole course. So, if a teacher developed just one activity and put it in a database available to all teachers free of charge, it would be a valuable resource. Since a teacher just needs to make one high quality lesson plan, many teachers would be able to find the time to finish the job. If teachers nationwide all contributed to the database, the database would have thousands or hundreds of thousands of good lesson plans. The author hopes that professional organizations such as the National Council of Teachers of Mathematics (NCTM), and the National Science Teachers Association (NSTA) will sponsor such a campaign and maintain the database. This will benefit teachers, students, education, and society.
Finally, it is always interesting to compare academic performances of students from different countries and find the reasons for the difference. This is especially true in a time of global competition. In fact, there must be some good things for U.S. teachers, students, and the educational authorities to learn from other countries. For example, a group of professors from the University of Dayton presenting at the 2003 annual conference of the National Council of Teachers of Mathematics concluded that U.S. textbooks are the heaviest in the world, while Chinese textbooks are just one tenth as heavy. Although few people may consider this an important issue, most will admit that students benefit from lighter textbooks. While it may be hard for the U.S. to adopt any practice from China, it is feasible to learn from other countries such as Canada since these two countries bear so much similarity, and Canadian students performed better in both math and science than their U.S. counterparts, according to surveys conducted by TIMSS and PISA.

Bibliography

1. International Study Center, Trends in International Mathematics and Science Study (TIMSS), 2003, <http://timss.bc.edu>

Biographical Information

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