Incorporating Working Model Into The Lab Of An Applied Kinematics Course

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I. Introduction

Mechanical Engineering Technology students take an Applied Kinematics Course in their third year at Purdue University North Central. Kinematics is the study of motion in different machine mechanisms. In this course, students learn techniques necessary to study the motions of machines and perform design concepts to optimize the motion of a machine arrangement. This paper discusses the current methods used and the incorporation of a new motion simulation software package.

The topics of this course are the analysis and design of cams, gears, and linkages. Students analyze position, velocity, and acceleration of these different mechanisms. Some examples of the mechanisms analyzed are four bar linkage, slider crank linkage, scotch yoke, crank shaper mechanism, cam follower, spur gear train, and planetary gear train. Analytical and graphical are the two current methods used to analyze the mechanisms. The analytical method requires the derivation of the equations of motion to analyze the mechanism. These equations can be solved by hand or put into a spreadsheet to solve for multiple positions of the mechanism. This method is more cumbersome and does not allow the student to see the motion of the mechanism. The graphical method has the students use CAD software to analyze a mechanism. While students find this method easier, they can only evaluate one position at a time. Working Model has been introduced into the curriculum of the kinematics course. Working Model is a motion simulation package that allows students to build and analyze the motion of different mechanisms. Specific examples of the mechanisms analyzed are discussed in this paper.

II. Cam Design and Analysis

A cam is a mechanism that drives a mating component called a follower. Students design cams and followers, and analyze the mechanism's motion. The rotational speed of the cam and the time for a full cycle are calculated for the cam mechanism. The motion diagrams of the cam follower are graphically plotted. Prescribed equations are used to plot the displacement, velocity, and acceleration diagrams of the cam follower. The follower motion can have a constant velocity, constant acceleration, harmonic, or cycloidal motion. There are different types of cam mechanisms and followers that are covered in this course. This paper will cover an example of the in-line knife-edge follower used with a disk cam. The students are asked to a design and analyze a disk cam that will provide a follower the displacement shown if figure 1. The general procedure used to graphically construct the profile is covered in the course outline. AutoCAD is used to graphically design the disk cam shown in figure 2 that would give the follower the motion in figure 1.

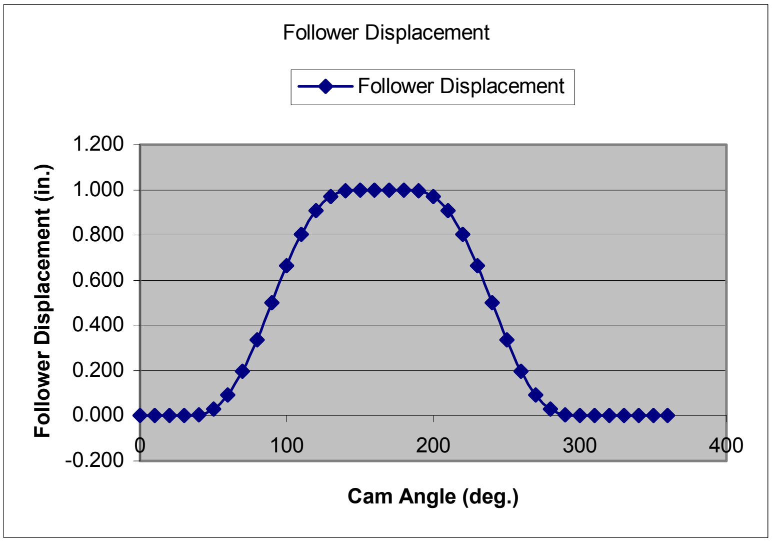


Figure 1

CAM FOLLOWER DISPLACEMENT

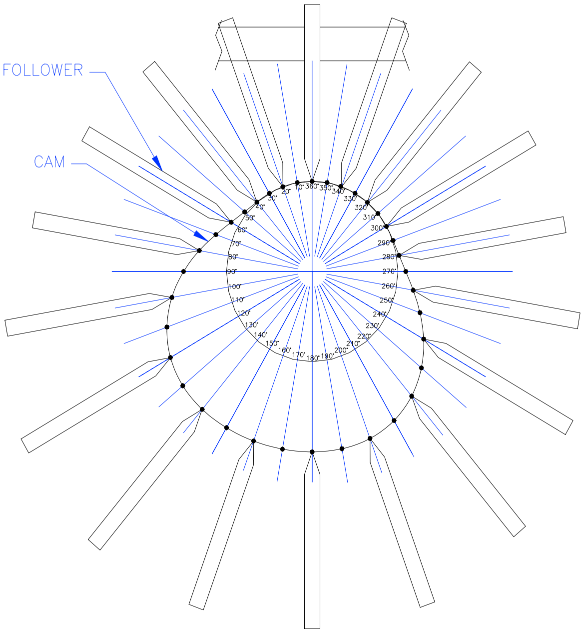
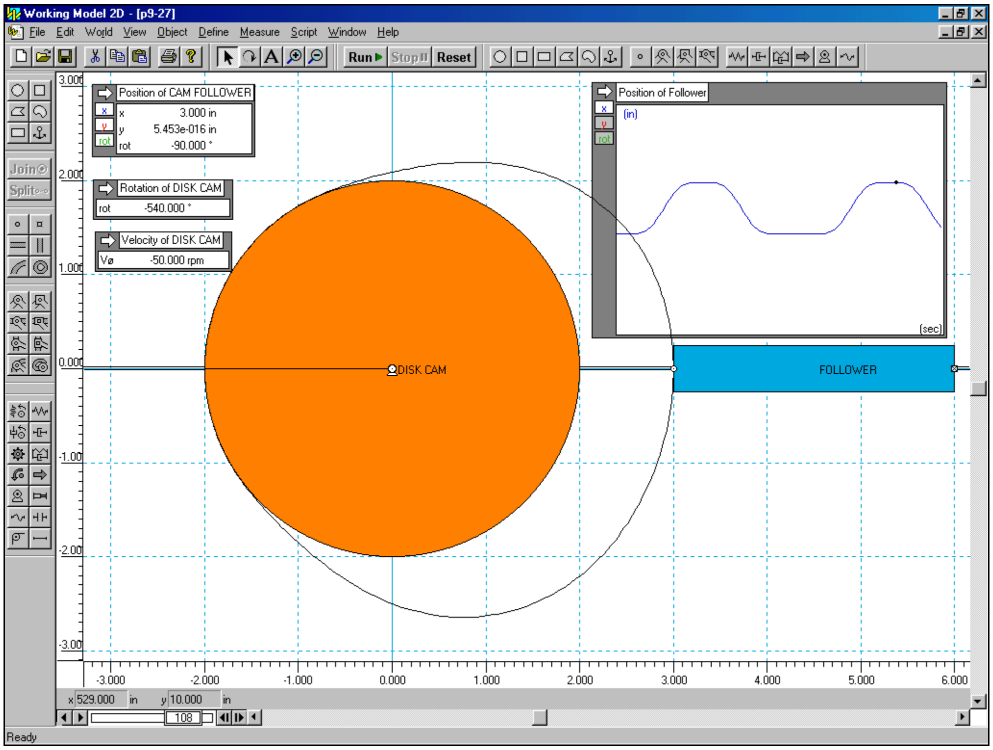


Figure 2

CAM AND FOLLOWER DESIGN

The students use Working Model to analyze the cam and follower motion shown in figure 3. The follower displacement is input into the program in order to design the shape of the cam. The light curved line represents the cam shape. A follower is attached to the cam and analyzed to show the motion of the follower mechanism.



Position of CAM FOLLOWER
x 3.000 in
y 5.453e-016 in
rot -90.000 °

Rotation of DISK CAM
rot -540.000 °

Velocity of DISK CAM
Vø -50.000 rpm

x 529.000 in y 10.000 in

108

Figure 3

CAM DESIGN USING WORKING MODEL

The position of the follower can be tracked as a line graph, bar graph, or as a text value. The text value shows the x-position of the follower as the cam rotates. The line graph plots the x-position of the follower overtime. It can be noted the shape of the x-position curve is the same as shown in figure 1. Working Model creates an easy to build cam profile that can be used in analysis of cam mechanisms. Another nice feature of the program is that it can be stopped, restarted, reversed, or stepped through. This allows the user to view and analyze the mechanism at various positions. The input speed can be varied to show how the cam speed affects the follower motion. Working Model allows the user to easily input the follower position to determine the cam profile. This gives the student a cam and follower that they can actually visualize and analyze.

III. Gear Kinematic Analysis

The kinematic analysis is done on simple gear trains, compound gear trains, and planetary gear trains. A simple gear train is one where there is only one gear on each shaft. A compound gear rain has multiple gears on one shaft. The kinematic analysis of gears includes calculations of ratios and speeds of the different types of gears for various applications. Working Model allows the user to model all the different applications discussed above. The user can use Working Model to layout a simple gear train or a compound gear train and track the angular speeds of all the gears based on the speed of the input gear. Although it is not difficult to analyze the gear trains using a standard set of equations, Working Model has some advantages over this method.

Working Model allows the user to easily change the input and to see the effect of the change on the motion of the mechanism. Also Working Model shows the motion of the mechanism which aides in visualizing complex mechanisms. Students are asked to analyze the motion of the piston in the gear mechanism shown below in figure 4. While it may look obvious that the piston moves back and forth, the motion would not be truly understood unless the user sees the mechanism in motion. The layout of the mechanism in Working Model reveals the motion of the piston to be a quick return motion and slow forward motion shown in figure 5. This means the

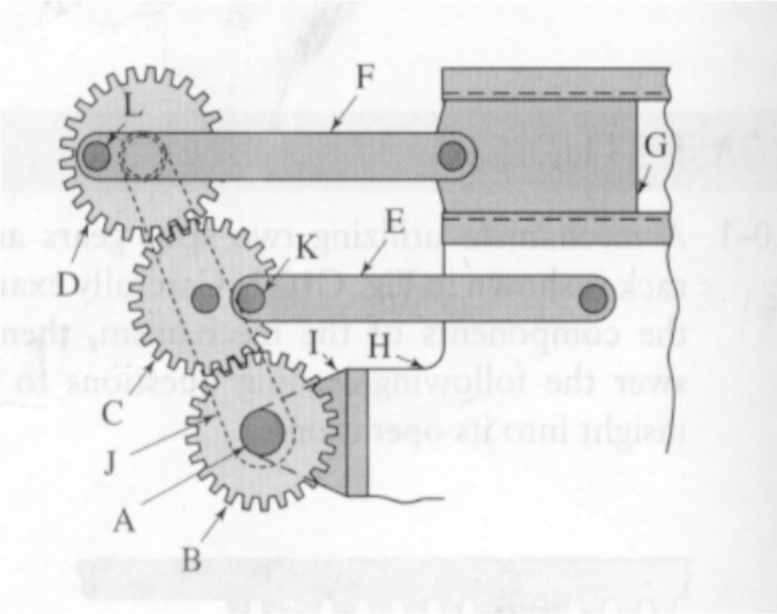


Figure 4

PISTON MECHANISM

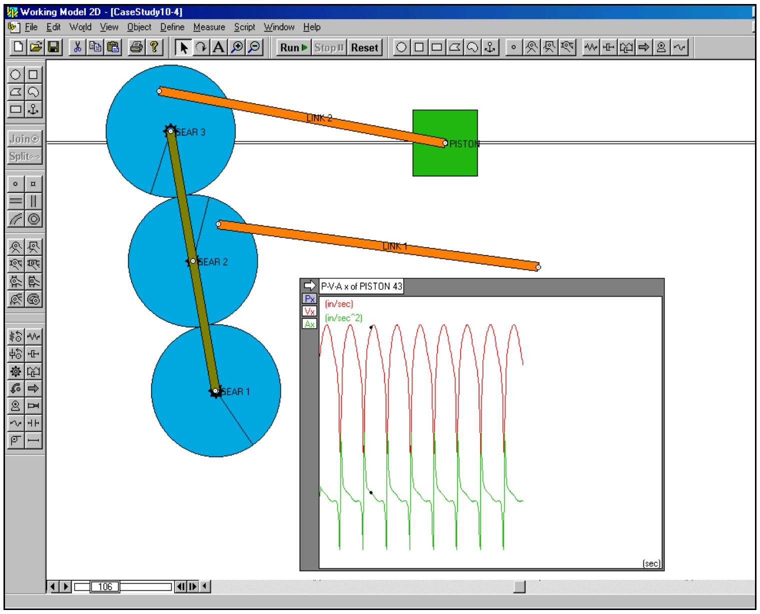


Figure 5

PISTON MECHANISM USING WORKING MODEL

piston moves quickly to the left and moves slowly to the right. The use of Working Model allows the analysis of this type of problem to be possible. Where in the past, discussion of the problem would have been done but would not have had the same impact as having the students actually see the mechanism moving. Working Model allows the students to watch the mechanism in motion. This can be very important if the motion of the mechanism is complicated. This makes for a student who can better understand the material. The planetary gear train is another type of gear design that is covered in this course. The planetary gear train shown in figure 6 has one or more gears that move relative to the frame. The gear in the center is called the sun and the gears whose axes move with the carrier are called planets. The internal gear is termed the ring gear. The motion is complex and is not easily visualized. The method of superposition is used to analyze the planetary gear train motion. In one example of a planetary gear train, the students solve for the output speed of the planetary gear train based on the information shown in Table 1.

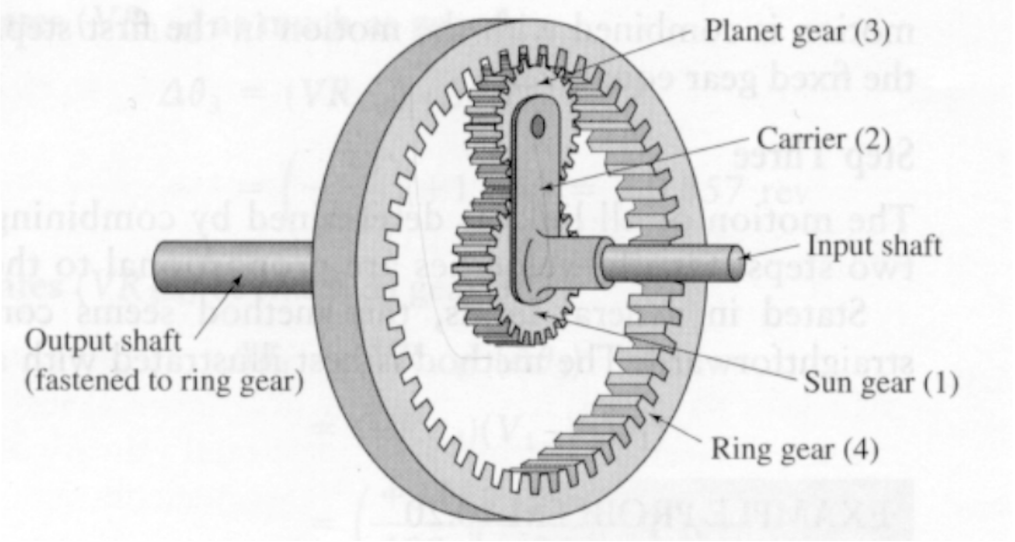


Figure 6

PLANETARY GEAR SYSTEM

Table 1

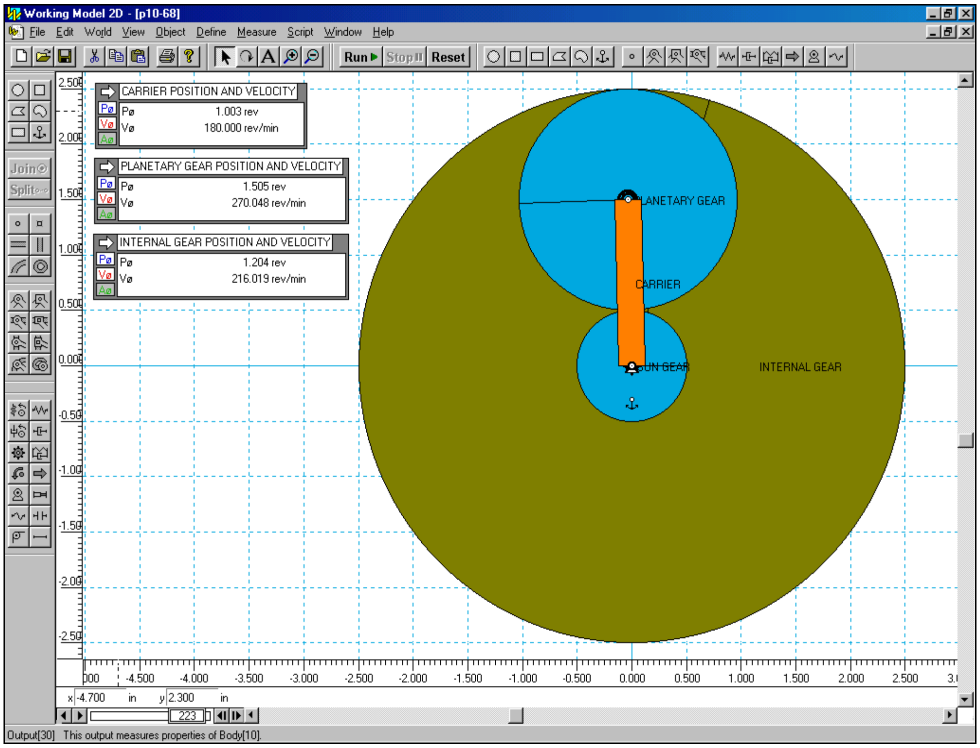
INPUT	
Carrier (link 2) is the input.	$\omega = 180$ rpm
Sun Gear (gear 1) is fixed.	N = 16
Planet Gear (gear 3).	N = 32
Ring Gear (gear 4) is the output gear.	N = 80

The carrier (link 2) is turning at 180 rpm clockwise. The number of teeth on the gears is given in table 1. The output of the method of superposition is shown in table 2. The first step is lock the carrier and to rotate the gear that was constrained which is the sun gear in this example. The sun gear is rotated one revolution and the angular displacements for each of the gears is calculated. The second step is to rotate all gears back one revolution in order to get the sun gear, which is fixed, back to a zero rotation. The ratio of the carrier can now be compared to the planet and ring gear. The output speed of planet and ring gears shown in table 2 can be calculated when the carrier is running at 180 rpm. The planet gear turns 1.5 times faster than the carrier and the ring gear turns 1.2 times faster than the carrier. Both gears turn the same direction as the carrier.

Table 2

Tabulated Planetary Gear Analysis					OUTPUT	
Link	Sun	Planet	Ring	Carrier	Sun Gear (gear 1)	ω = 0 rpm
Rotate with fixed carrier	1	-0.5	-0.2	0	Carrier (link 2)	ω = 180 rpm cw
Rotate all links	-1	-1	-1	-1	Planet Gear (gear 3)	ω = 270 rpm cw
Total rotations	0	-1.5	-1.2	-1	Ring Gear (gear 4)	ω = 216 rpm cw

A planetary gear train can also be modeled in Working Model. The above example is shown below in figure 7. Working Model can be stopped to view the output at a specific moment in time. The planetary gear train is stopped when the carrier has rotated 1.0 revolution. The planetary gear has rotated 1.5 revolutions and the ring gear has rotated 1.2 revolutions to the 1.0 revolution rotation of the carrier. This confirms the output calculated by hand. The output speed for the planetary gear is 270 rpm and the output speed of the internal gear is 216 rpm when the carrier input speed is 180 rpm.



CARRIER POSITION AND VELOCITY
Pø 1.003 rev
Vø 180.000 rev/min

PLANETARY GEAR POSITION AND VELOCITY
Pø 1.505 rev
Vø 270.048 rev/min

INTERNAL GEAR POSITION AND VELOCITY
Pø 1.204 rev
Vø 216.019 rev/min

Output[30] This output measures properties of Body[10].

Figure 7

PLANETARY GEAR MODELED IN WORKING MODEL

A good teaching technique to incorporate Working Model is to first have the students solve the problem graphically or analytically. This allows them to become comfortable with the procedure to solve the problem. Then have the students use Working Model to verify the hand calculations. This allows the student to understand how to set up a specific problem using Working Model. It also gives the student confidence in the results to see the outputs match in both hand calculations and Working Model. Once the student understands the material and knows how to apply Working Model, a more difficult problem can be illustrated using Working Model. A difficult problem that may take to much time to solve by hand may be more practically analyzed using Working Model.

IV. Linkage Kinematic Analysis

The kinematic analysis of different linkages can be performed using Working Model. This course covers the analysis of position, velocity, and acceleration of the different linkage mechanisms. The four bar linkage, slider crank linkage, scotch yoke, crank shaper mechanism, and crank and rocker mechanism are some of the mechanisms analyzed. Analytical and graphical techniques are two methods used to solve a mechanism. The student uses prescribed equations given from the textbook[1] to analytically solve for the motion of the mechanism. This method only covers a few mechanisms such as a four bar linkage or a sliding crank mechanism[2]. The student also uses graphical methods analyze the mechanism's motion. There are two different graphical methods used to analyze the velocity and the acceleration of these mechanisms. The first method is called the relative velocity method. An example of this method is shown below in figure 8. This method is used to solve a slider crank mechanism[3].

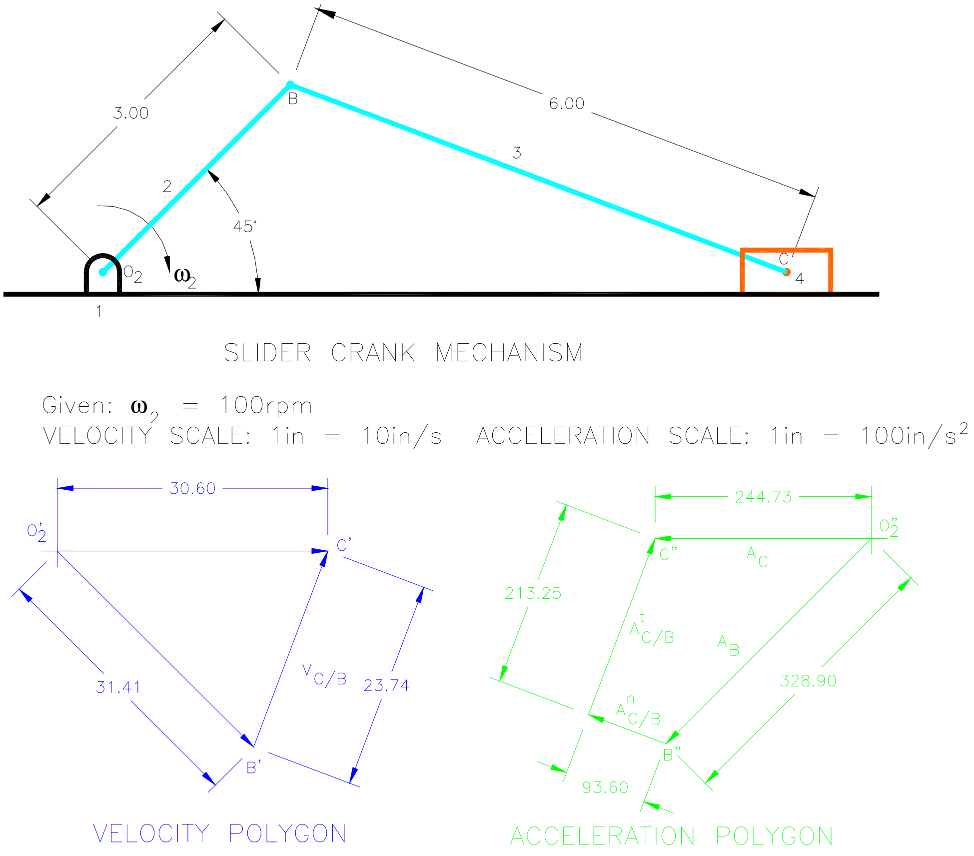


Figure 8

VELOCITY AND ACCELERATION OF A SLIDER

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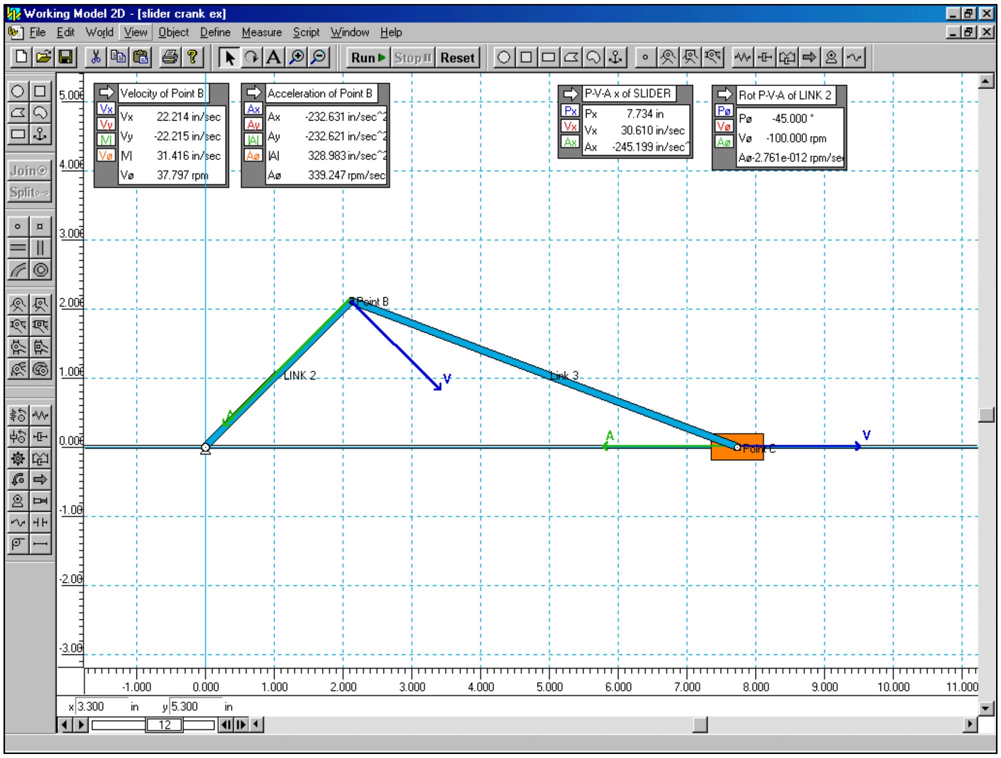
The velocity of the slider can be found by solving the velocity polygon. The velocity vector equation is used to construct the velocity polygon. The velocity of point C is equal to the velocity of point B plus the velocity vector of point C relative to B.

$$\overline{V_C} = \overline{V_B} + \overline{V_{C/B}} \tag{1}$$

The acceleration vector equation is used to construct the acceleration polygon. The acceleration of point C is the sum of the acceleration of point B and the acceleration of point C relative to point B. The acceleration vector of each point is resolved into its tangential and normal components. The velocity and acceleration of point C can be scaled from the velocity and acceleration polygons.

$$\overline{A_C^n} + \overline{A_C^t} = \overline{A_B^n} + \overline{A_B^t} + \overline{A_{C/R}^n} + \overline{A_{C/B}^t} \tag{2}$$

The draw back to this method is that the velocity and acceleration of the mechanism can only be found for one location. If there are any other points of interest, the mechanism needs to be redrawn and the velocity and acceleration polygons need to be reconstructed. The second method used to solve velocity and acceleration is instant centers and Kennedy's Theorem. While this paper will not go into detail of this method, both of these methods are taught so students understand the procedures used in solving the motion of a mechanism. A model was set up to analyze the motion of the slider crank mechanism using Working Model shown in figure 9.



```
Velocity of Point B
Vx     22.214 in/sec
Vy    -22.215 in/sec
|V|    31.416 in/sec
Vø     37.797 rpm

Acceleration of Point B
Ax    -232.631 in/sec^2
Ay    -232.621 in/sec^2
|A|    328.983 in/sec^2
Aø     339.247 rpm/sec

P-V-A x of SLIDER
Px      7.734 in
Vx     30.610 in/sec
Ax   -245.199 in/sec^

Rot P-V-A of LINK 2
Pø      -45.000 °
Vø     -100.000 rpm
Aø-2.761e-012 rpm/se

x 3.300   in    y 5.300   in
```

Figure 9

SLIDER MECHANISM MODEL USING WORKING MODEL

The four bar sliding mechanism can be easily constructed in Working Model. All components can be labeled and velocity and acceleration vectors can be turn on to view while the mechanism is running. Looking at figure 9, the velocity of point B is perpendicular to link 2 while the normal acceleration of point B is along link 2. There is no tangential acceleration of point B since link 2 rotates at a constant angular speed. The orange slider shows a velocity to the right and the acceleration in the opposite direction. The lengths of the velocity and acceleration can be change to accommodate the size of the view screen. The outputs can be measured and shown in the output dialog boxes. The outputs confirm the graphical solution solved at 45 degrees. One of the features of the software is the ability to step through the motion of the mechanism. This allows the student to view the motion of the mechanism at different positions. It illustrates and reinforces the kinematic concepts learned by allowing the student to actually visualize the velocity and the acceleration of selected links or points of the mechanism. As the student steps through the motion of the slider crank mechanism, they see the velocity is to the right and the acceleration is opposite. This states the velocity is decreasing or decelerating until it stops and starts moving in the opposite direction. This is a point that can be comprehended more easily if the student can see the velocity and acceleration vectors on the mechanism while it is moving. Compare this to a static mechanism and an instructor explaining the concepts and motion. The use of Working Model has improved the student's ability to understand the topics of kinematics. Another example is the Scotch-yoke mechanism shown in figure 10. The working model example is shown in figure 11. The Scotch-yoke mechanism is a variation of the slider crank mechanism with an infinitely long connecting rod. The scotch-yoke mechanism has simple harmonic motion. A simple harmonic motion has an acceleration that is proportional to the displacement but has an opposite direction. This concept was presented in class in a traditional format of a lecture and a sketch on the blackboard. The introduction of Working Model along with the lecture was an improved way to present this topic.

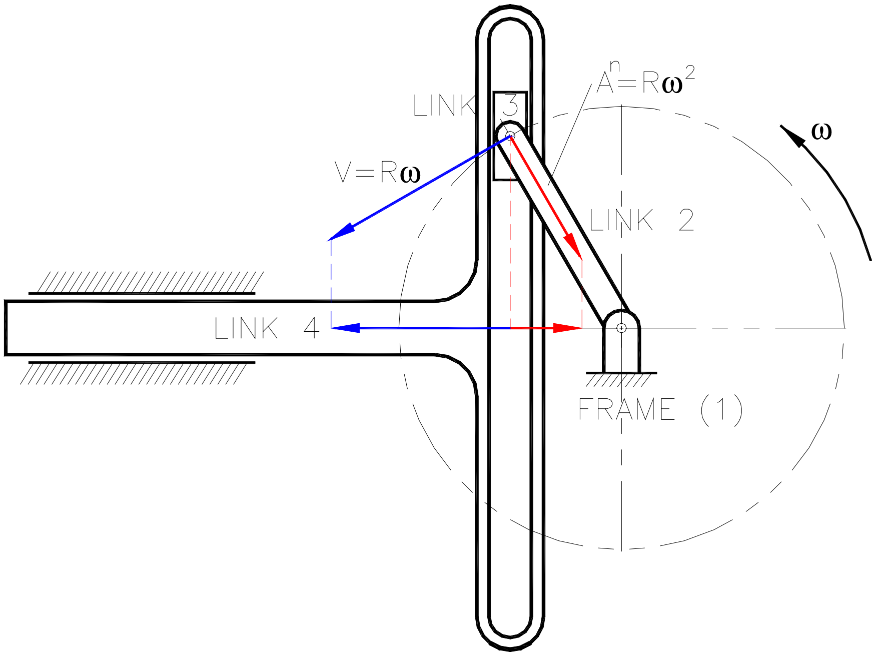


Figure 10

SCOTCH-YOKE MECHANISM

It allowed students not only to receive information on the topic, but it also allowed the student to see the motion of the mechanism. The displacement, velocity, and acceleration[4] of link 4 are

$$
\begin{aligned}
x &= R \cdot \cos \omega t \\
v &= \frac{dx}{dt} = -R \cdot \sin \omega t \\
a &= \frac{d^2 x}{dt^2} = -R \cdot \omega^2 \cdot \cos \omega t
\end{aligned}
\tag{3}
$$

The students solve for a Scotch-yoke problem, where link 2 rotates counterclockwise at 100 rpm and is 203 mm long. The position, velocity, and acceleration were calculated and are shown in table 2. The motion of the Scotch-yoke is shown in figure 11.

Table 3

θ	Position [mm]	Velocity [mm/s] x 10^2	Acceleration [mm/s^2] x 10^3
0	203	0	-223
30	176	-106	-193
60	102	-184	-111
90	0	-213	0
120	-101	-184	111
150	-176	-106	193
180	-203	0	223
210	-176	106	193
240	-102	184	111
270	0	213	0
300	101	184	-111
330	176	106	-193
360	203	0	-223

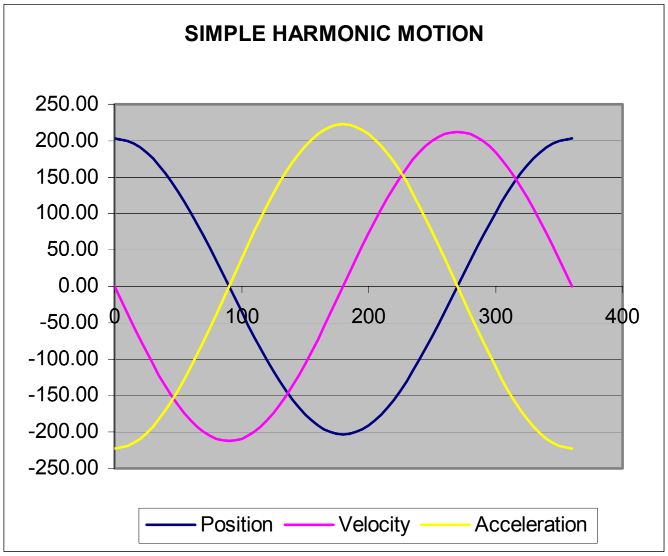
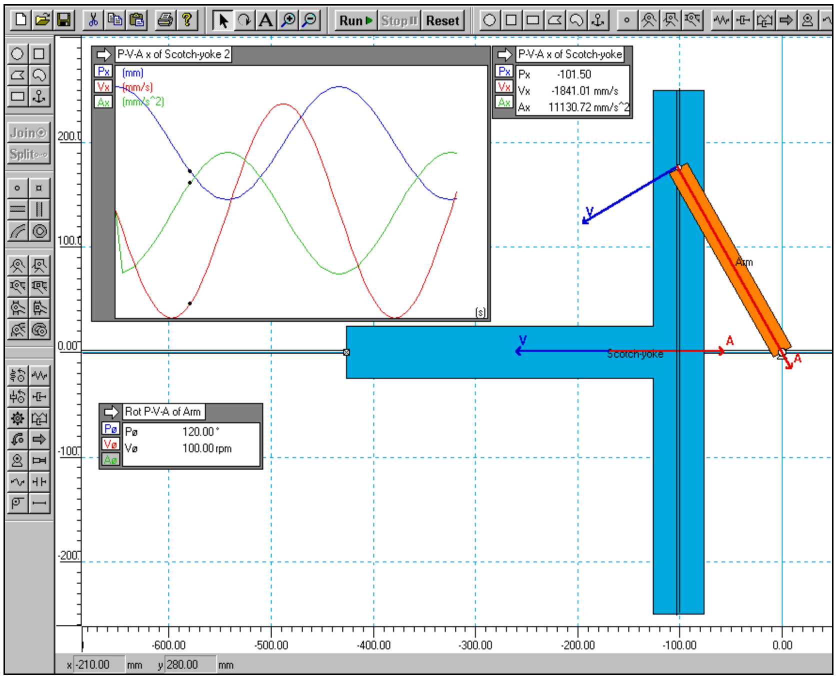


Figure 11

SIMPLE HARMONIC MOTION

The Scotch-yoke mechanism was modeled in Working Model shown in figure 12. The output of Working Model can be set to text or a line chart as shown. The program also allows the user to display the velocity and acceleration vectors of any link or point. The size of the vectors changes as the value increases or decreases.



P-V-A x of Scotch-yoke
Px -101.50
Vx -1841.01 mm/s
Ax 11130.72 mm/s^2

Rot P-V-A of Arm
Pø 120.00 °
Vø 100.00 rpm

Figure 12

SCOTCH YOKE MECHANISM MODELED USING WORKING MODEL

Other examples of problems solved in class are quick return mechanisms. The mechanism shown in figure 13 and 14 is a Crank-Shaper mechanism. Link 2 rotates at a constant angular velocity. The motion creates an oscillation in link 4. Link 4 has a slow motion to the right and a quick return motion to the left. The time ratio is equal to θ_1/θ_2. Working Model makes the

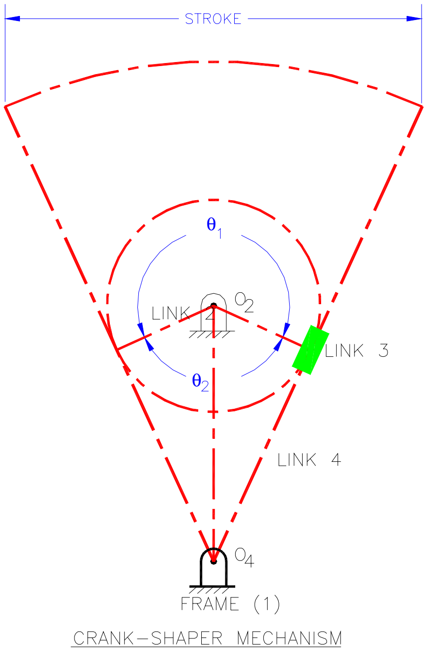
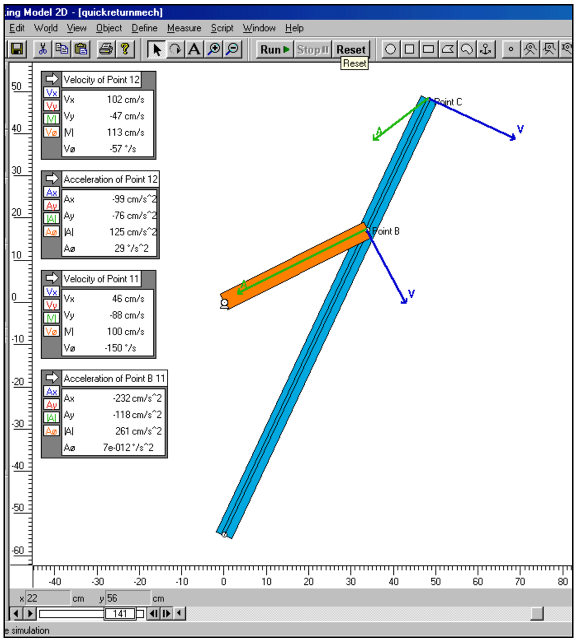


Figure 13

CRANK-SHAPER MECHANISM



Velocity of Point 12
Vx 102 cm/s
Vy -47 cm/s
|V| 113 cm/s
Vø -57 °/s

Acceleration of Point 12
Ax -99 cm/s^2
Ay -76 cm/s^2
|A| 125 cm/s^2
Aø 29 °/s^2

Velocity of Point 11
Vx 46 cm/s
Vy -88 cm/s
|V| 100 cm/s
Vø -150 °/s

Acceleration of Point B 11
Ax -232 cm/s^2
Ay -118 cm/s^2
|A| 261 cm/s^2
Aø 7e-012 °/s^2

Figure 14

CRANK-SHAPER MECHANISM USING WORKING MODEL the mechanism more realistic. This course is kinematics, the study of motion, by using the software program we are closer to the analyzing a moving mechanism.

V. Conclusion

The introduction of Working Model has had a positive result. Modeling a mechanism using Working Model allows the student to visually see the motion of the mechanism. This is a tremendous advantage to students trying to understand complex movements of a mechanism. The software allows the student to make minor modifications to the mechanism easily. Also, more than one position of a mechanism can be analyzed. None of the content of the course was deleted to include Working Model. Instead of only analyzing the mechanisms using AutoCAD and a graphical approach, some of the mechanisms were analyzed using Working Model. Most students found the software to be easy to learn and use. If problems occurred, the mechanism would become unstable and would come apart. Since it was not difficult to build the model, the students could rebuild the model if they could not get theirs to work properly the first time. Over the course of the semester, I spoke to all the students about their thoughts on the software. Most students liked the software and felt they could understand the mechanisms better with the software. I think most of the students found the class more interesting and the software fun to use. There was no formal assessment performed to judge whether the software improved the course or the students performed at a higher level. It is my opinion that the students did have a better understanding of the material and were able to analyze more complicated problems.

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