AC 2010-36: INDIVIDUALIZED MATLAB PROJECTS IN UNDERGRADUATE ELECTROMAGNETICS

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Individualized MATLAB Projects
In Undergraduate Electromagnetics

Abstract

Four projects are described that require students to compose individualized MATLAB programs to solve a problem in electromagnetics. These projects are: (1) vector electric field from an arbitrary charge distribution, (2) vector magnetic field from an arbitrary current distribution, (3) frequency dependent reflection coefficient looking into impedance matching networks, and (4) beam pattern for an arbitrarily arranged 4 dipole array.

Introduction

MATLAB projects are often assigned in undergraduate electromagnetics courses, in part to satisfy the ABET criteria on use of modern engineering tools. The best projects will enhance understanding of the subject matter while providing a significant programming exercise. A challenge for the instructor is to individualize assignments to make it more likely that students are doing their own work.

Four projects are presented that require students to write a MATLAB program that calculates the project’s objective. First, the vector electric field is determined from an arbitrary charge distribution. Second, the vector magnetic field is determined from an arbitrary current distribution. For these related projects the discrete sum solution of the electrostatic or magnetostatic field are individualized by the charge or current distributions and the configuration of the structure in three dimensions.

In the third project, students are required to find the two fundamental Smith Chart solutions for a stub matching network and realize this network using microstrip transmission line. Variables that are modified to individualize the project include load impedance, operating frequency, stub termination (open or short), and substrate properties. Performance is compared for the two networks over a range of frequencies. The final project requires the student to determine the beam pattern for an array of four dipoles. Each dipole in the array has an individualized current magnitude, phase and orientation that are linked to the student’s ID number. Additionally, an estimate of the array’s beam solid angle and directivity is required. We will discuss how well these projects result in individualized work along with our recommendations for future projects.

1. Fields from Arbitrary Source Distributions

   a. Electrostatics
   The vector electric field from an arbitrary static charge distribution can be calculated by the application of Coulomb’s Law. Utilizing first the conceptually reasonable point charge, then the somewhat implausible infinite line and surface charges densities, closed form integral solutions for the vector electric field are obtained. Solutions using Gauss’ Law for the same charge distributions can simplify the analysis. However, the fundamental aspects of Coulomb’s Law,
expressed as a discrete summation, can provide additional insight for more practical problems in electrostatics.

As part of the first course in electromagnetics, undergraduate students are tasked with the computation of the vector electric field in a region from a unique static charge distribution. The prerequisite for this course includes a course in engineering analysis using MATLAB and they are familiar with both its computational and graphical rendering capabilities.

A typical charge distribution is shown in Figure 1(a) below in which two rectangular conducting plates are offset at an angle \( \theta \) and are specified to have a uniform charge density \(+\rho_S \text{ C/m}^2\) on the upper plate and \(-\rho_S \text{ C/m}^2\) on the lower plate. This plate configuration is introduced as an initial model of electrostatic deflection plates, as shown in Figure 1(b).

![Figure 1](image_url)

Figure 1: (a) Configuration of rectangular conduction plates with a uniform charge distribution (b) actual electrostatic deflection plates

The intentionally vague specification of the task is to calculate the vector electric field at an arbitrary location \( P(x,y,z) \) for a specific uniform charge density \( \rho_S \). The width \( X_1 \) and length \( Z_1 \) of the rectangular plates, the angle \( \theta \) and the charge density \( \rho_S \) are randomly assigned to each student to avoid direct duplication of the results.

The course learning objective is to effect the translation of a problem to an engineering analysis to be solved by discrete summation, rather than integration, and to formulate a reasonable solution. The method utilized is a discussion in the class with groups of students proposing specifications to modify the task, as it becomes readily apparent that the task is “open-ended”. The discussion is quite lively, requires research and takes place over several class meetings. This is a salient object lesson for the students in design and analysis and the points are organized, agreed upon and added to the task:

For example, what is the restriction on the location \( P \)? The region to be analyzed directly affects the computational time and should be limited. What should be the size of the discrete charge \( \Delta x \Delta y \rho_S \) and how does this affect the computational time and the result? Since such plates are discovered by the students to be used to deflect a beam of charged particles, usually electrons by the Lorentz force equation, the salient region is the middle of the plates \( x = X_1/2 \), but what should the \( z \) range of the solution be?
It soon also becomes apparent that specification needs to be further analyzed. For example, the beam of electrons has a dimension and over what region should the vector electric field be uniform? Should the upper plate have a length greater than Z1 because of the orientation angle \( \theta \)? Should the lower plate have an orientation angle of \(-\theta\) to make the vector electric field more uniform? These points provide a natural division of the task which is then parsed to each student to further avoid direct duplication of the results.

b. Magnetostatics
Next in the typical undergraduate course in electromagnetics is a consideration of the vector magnetic field due to an arbitrary distribution of current calculated by the Biot-Savart Law. Utilizing first the somewhat implausible infinite line and ring of current closed form integral solutions for the vector magnetic field are obtained. Solutions using Ampere’s Circuital Law for the same current distributions can simplify the analysis. However, the fundamental aspects of the Biot-Savart Law, expressed as a discrete summation, again provide additional insight for more practical problems in magnetostatics.

The undergraduate students are now tasked with the computation of the vector magnetic field in a region from a unique constant current distribution. The usual two circular coils of the Helmholtz configuration are modified and presented as five rectangular coils as shown in Figure 2(a). The rectangular coils have a width X1, a height Y1, an arbitrary number of turns and spaced along the Z axis as shown.

![Configuration of rectangular coils with an arbitrary number of turns](image1)

Figure 2: (a) Configuration of rectangular coils with an arbitrary number of turns
(b) an actual Helmholtz coil

Following the precepts of the course learning objective, the students are familiar with the process and begin the lively, iterative discussion to modify the task. Such coils are discovered by the students to be used to provide a uniform magnetic field. It soon becomes apparent to them that the coils should be spaced symmetrically.

However, should the coils be rectangular or square? What is the optimum distribution of the relative number of turns in each coil? Over what region and what is the specification for the
uniformity of the vector magnetic field? What should be the size of the discrete current element \(I \Delta L\)? As before these points provide a natural division of the task which is then parsed to each student to further avoid direct duplication of the results.

The results of the calculation of the vector electric and magnetic fields are provided as written reports from each student with tabulated graphical renderings of the fields using MATLAB. The written reports provide practice in the formatting of an engineering analysis with summary, introduction, discussion and conclusion. These reports are to satisfy the additional course learning objective of the application of technical writing. The results are summarized and discussed in class to further illustrate the concepts of vector electric and magnetic fields.

The course learning object of the translation of a problem in electrostatics and magnetostatics to an engineering analysis to be solved by discrete summation is directly assessed by focused questions on examinations that propose a charge and current distribution and require that the student provide the MATLAB code for the solution. These questions are graded by the choice of the limit size \(\Delta x \Delta y\) and \(\Delta L\) of the discrete summation and the dot and cross product vector manipulations required.

An indirect assessment of the course learning objective is obtained by the survey at the end of the course. Student comments on the survey have favorably noted the proportion (30%) of the final grade assigned to and the insight provided by the tasks.

2. Impedance Matching Networks

A MATLAB project is assigned at the end of a two semester electromagnetics sequence. This sequence begins with transmission lines, where students learn such things as how to calculate input impedance looking into a terminated transmission line and how to use a Smith Chart to design an impedance matching network using sections of transmission line (a shunt stub matching network). In the second course, students study applied electromagnetics culminating in microwave engineering topics including microstrip. Students therefore have covered all the necessary topics to perform a MATLAB project involving microstrip impedance matching networks. The project handout follows, with the “Given” information varied for each student:

ELEC 3320 MATLAB Project Name:________________________________________

You are expected to develop your own MATLAB code for this project. Teamwork is unacceptable.

If a constant \(|\Gamma|\) circle for transmission line terminated in a mismatched load is drawn on a Smith chart, it will intersect the \(1 \pm jx\) circle at two points. Thus, there are two fundamental solutions to a stub matching problem. The magnitude of the reflection coefficient \(|\Gamma|\) looking into the matching network will ideally be zero at the design frequency. Your task is to plot and compare \(|\Gamma|\) vs. frequency for the two fundamental matching networks realized in microstrip. Your microstrip substrate has perfect conductors sandwiching a lossless dielectric.
Given:
Substrate relative permittivity $\varepsilon_r = \text{________________________}$

Substrate height $h = \text{________________________}$

Characteristic impedance $Z_o = \text{________________________}$

Load impedance $Z_L = \text{________________________}$

Type of shunt stub: \text{________________________}

Design frequency: \text{________________________}

Frequency range for plot: \text{________________________}

Note that you can solve for the matching network in terms of wavelength, but to find the actual lengths you must design 50 $\Omega$ microstrip for your given circuit board material and determine guide wavelength at your design frequency.

This project covers several interesting concepts. Students must find the two fundamentals stub matching networks using a Smith Chart. The stub lengths from these solutions are in terms of wavelengths, so to find the physical lengths students must design their microstrip transmission lines and determine a guide wavelength at the design frequency. Finally, students compare the reflection coefficient for the two solutions over a frequency range about the design frequency. The project therefore ties together concepts from both semesters of electromagnetics.

A project grading rubric (Table 1) is provided to the students to guide their effort. This detailed rubric both simplifies and standardizes the project grading. Student results were verified by a MATLAB code developed that accepts the “given” information in the order it is displayed in the project handout. The routine solves for the line lengths (in both wavelengths and in physical lengths) needed for the two fundamental solutions and then plots the reflection coefficient magnitudes as a function of frequency. In this way, it was straightforward to assess the individual projects.

There were 30 students in the class and their MATLAB project scores ranged from a low score of 8$\%$ to a high score of 100$\%$, with an average score of 79$\%$. Figure 1 shows the results from one student’s efforts. This particular student did an excellent job on all aspects of the project (see appendix for student code). Figure 2 shows a plot from a student who did a very poor job not only in the basic design but in the coding to generate a plot as well. The goal of the individualized work was mostly met, though it was apparent that some collaboration did take place in the code development.
<table>
<thead>
<tr>
<th>Table 1: Stub Matching Project Grading Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>score</strong></td>
</tr>
<tr>
<td>1. Problem Solution</td>
</tr>
<tr>
<td>a. Neat/organized</td>
</tr>
<tr>
<td>b. approach</td>
</tr>
<tr>
<td>c. execution</td>
</tr>
<tr>
<td>d. Smith Charts</td>
</tr>
<tr>
<td>e. Circuit sketch (top-down view of microstrip circuit)</td>
</tr>
<tr>
<td>2. Program</td>
</tr>
<tr>
<td>a. Task definition</td>
</tr>
<tr>
<td>b. organization</td>
</tr>
<tr>
<td>c. comments</td>
</tr>
<tr>
<td>d. code quality</td>
</tr>
<tr>
<td>3. Results</td>
</tr>
<tr>
<td>a. Performance</td>
</tr>
<tr>
<td>b. Discussion</td>
</tr>
<tr>
<td>c.</td>
</tr>
</tbody>
</table>
Figure 3: One student’s microstrip stub matching network, top-down view and plot.

Figure 1 - Fundamental Solution 1 using Smith Chart (Note: units are wavelengths)

Figure 5 - Microstrip Realization for fundamental solution 1
The radiation pattern for both dipole antennas and antenna arrays was covered in the second electromagnetic course of a two course sequence in electromagnetics. There are a large number of possible projects related to antennas and antenna arrays the students could be assigned. The true difficulty is in determining a project that is achievable in a reasonable amount of time, individualized, theoretically and computationally demanding and yet can be graded efficiently.

The project required the students to determine the radiation pattern in two planes for an array of four dipoles located on the z-axis. Each dipole was separated by a quarter wavelength (\(\lambda/4\)) and a twentieth of a wavelength (\(\lambda/20\)) long. The projects were individualized based on the last four digits of a student’s identification number (ID). This was done in the following manner:

Assuming the last four digits of a student’s ID number are \(d_1 \ d_2 \ d_3 \ d_4\):

**Magnitude and location of currents of dipole currents:**

<table>
<thead>
<tr>
<th>Current number</th>
<th>Current location</th>
<th>Current magnitude</th>
<th>Current phase</th>
<th>Current orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element 1</td>
<td>(z = 0)</td>
<td>(I_1 =</td>
<td>d_1</td>
<td>)</td>
</tr>
<tr>
<td>Element 2</td>
<td>(z = \lambda/4)</td>
<td>(I_2 =</td>
<td>d_2</td>
<td>)</td>
</tr>
</tbody>
</table>

Figure 4: Poor and incorrect plot for the impedance matching project
All students were asked to develop a MATLAB program that accurately does the following:

1) Plots the normalized power pattern at $\theta = 90$ degrees for all angles $0 < \varphi < 360$ degrees in polar form.

2) Plots the normalized power pattern at $\varphi = 0$ degrees for $0 < \theta < 180$ degrees in polar form.

3) Describes the code and specifically discusses how they developed the formulas used in MATLAB. Also, a flowchart of the code is required along with sufficient comments in the MATLAB file to ensure the code can be understood. Students are also asked to include a printout of their code along with a CD in their project report.

The final projects showed a remarkable amount of diversity with one individual researching the advanced technique of using Euler angles for determining three dimensional surface plots of the radiation pattern. What became clear from reading the student reports and programs was that students could basically be separated into four groups: a) Students that understood the antenna and array theory who were good MATLAB programmers b) Students that understood the antenna and array theory who were poor MATLAB programmers, c) Students that had little understanding of antenna and array theory who were good MATLAB programmers and c) Students that had little understanding of antenna and array theory who were also poor MATLAB programmers.

Grading of the plots for correctness was relatively simple since a code that only required the entry of the last four digits of the persons ID number had already been created. It was likely that some collaboration occurred in a small number of cases. However, most of the codes were very different with several people creating completely vectorized codes. The same basic grading rubric of Table 1 modified for this project was used. A correct radiation plot from a student is shown in Figure 5.

Conclusions

Several individualized MATLAB projects have been presented where students use MATLAB to study fundamental topics in electromagnetics. These projects help satisfy the ABET criteria on use of modern tools by requiring a significant degree of MATLAB programming that improves student understanding of the course material.

Typical project reports and code for parts 1a and 1b are available at:
http://www.astro.temple.edu/~silage/archive.htm

Project and code for parts 2 and 3 are available at:
http://www.eng.auburn.edu/~baginme/matlab_emag_projects/
Figure 5: Polar Plot of Normalized Power Pattern from Variation in $\theta$

Bibliography