120th ASEE Annual Conference & Exposition
FRANKLY, WE DO GIVE A D*MN
June 23-26, 2013

Paper ID #7906

Influence of uncertainties and assessment of significant digits in thermodynamics

Dr. Randall D. Manteufel, University of Texas, San Antonio

Dr. Randall Manteufel is an associate professor in Mechanical Engineering at the University of Texas at San Antonio. He teaches courses in thermodynamics, fluid mechanics and heat transfer. He is the faculty advisor for the student chapter of American Society for Heating Refrigerating and Air-Conditioning Engineers at UTSA.

Dr. Amir Karimi, University of Texas, San Antonio

Influence of Uncertainties and Assessment of Significant Digits in Thermodynamics

Abstract

Thermodynamics calculations are predominantly deterministic and students are expected to solve a problem for quantities which can be calculated to many significant digits using a calculator or software. Students often report final answers to many more significant digits than justifiable. Textbooks often emphasize that final answers are justifiable to a limited number of digits because of the lack of precision of inputs and/or internal coefficients/models. It is often recommended that final results be expressed to three significant digits. Students are encouraged to keep intermediate digits during intermediate calculations and then round the final result to an appropriate number of digits. It has been observed that significant digits continues to be a difficult concept for some students with the common misconception that an answer with five or six significant digits is equivalent to, if not better than, an answer with fewer digits. This paper reports work where students are required to solve thermodynamic problems with uncertain inputs. Typical problems have input values with no specified uncertainty. For example, the inlet temperature may be specified to be 480°C or the pressure to be 2.0 MPa. A number of problems have been developed that include uncertainties, and the student is expected to report a final answer with the propagated uncertainty. Problems are solved using hand calculations and then with the aid of a spreadsheet. Using software, students can access routines to evaluate thermodynamic properties which are tedious if done by hand. The approach is based on the traditional differential method for uncertainty propagation, yet numerical differentiation in used in the spreadsheet program. Examples show that when uncertainties are considered, there can be relatively large uncertainties in final results. By knowing a result's uncertainty, students can report final answers to an appropriate number of digits. A student survey was conducted to gauge the effect of the exercises on student learning and attitudes. Students show an increased aptitude for reporting answers to an appropriate number of significant digits and a positive attitude toward the methods covered. Some comments indicate that the methods are inadequately covered in lower-level engineering classes. This is useful information for continuous improvement of the engineering curriculum. Student feedback indicates they understand how to use these techniques in subsequent classes, indicating students have gained a deeper understanding of the concepts.

Introduction

The propagation of uncertainty and representation of final answers continues to be an area in which students struggle. It is often the case that students represent answers as if only a negligibly small uncertainty exists. There appears to be a contradiction between a reasonable consideration of uncertainties and data reported in the property tables in many thermodynamic textbooks. The data in many tables have more significant digits than typically appropriate for final answers. Answers should be expressed after rounding to an appropriate number of significant digits. When in doubt, textbooks and instructors often suggest that three significant digits are reasonable because most engineering data is known to be this accurate. Reporting computed uncertainties is preferable, but in many cases it is unrealistic because input parameter uncertainties are not given explicitly.

If a number is represented to three significant digits, a reasonable estimate of the uncertainty is plus or minus half the least significant increment. For example, consider the number 4.54. This number is show to three significant digits. As such, the “true” value can be interpreted to be between 4.535 and 4.545 with 95% confidence. This represents an uncertainty of about 0.01/4.54 or ~1/500 or 0.2% uncertainty. This is a fractional uncertainty which is often optimistically small, given the uncertainty in the other inputs and physical parameters. It is often misleading when answers are represented to five or more significant digits. Continuing this example, the final answer may be reported by some students as 4.5455 (to 5 significant digits) which is the same as stating the answer is known to about 0.002% uncertainty. To some students, this appears to be a reasonable if not a preferred representation of the final answer. In an engineering thermodynamics course, this concept is more difficult for students since property values reported in tables often are specified at 6 significant digits, which can be interpreted as 1/500000 or 0.0002% uncertainty. Having property values in thermodynamic tables expressed to 6 significant digits, contributes to the students’ perception that more digits are better.

Students are expected to learn to estimate uncertainties in laboratory measurements and be able to propagate these to final reported measurement values. This is expected in ABET[1] outcome (b) describing the “ability to design and conduct experiments, analyze and interpret data”. The emphasis in the curriculum has been on physical measurements where the reported number is presented with accompanying description and analysis of the underlying uncertainties. This can be stressed in non-laboratory classes such as engineering thermodynamics, because those final answers have inherent assumptions with underlying uncertainties. As a strategy, this paper documents a new approach to presenting uncertainties and significant digits starting with reasonable estimates of uncertainties of more physically-grasped properties such as temperature and pressure. Although a problem may specify the pressure as 2.0 MPa, it would be reasonable that the pressure would have an uncertainty of 0.2% but probably not smaller. As such, the uncertainty of an entropy value may be known to 5 or 6 digits, as reported in a table, but only if the temperature and pressure (if those are two independent intensive properties are sufficient to fix the state) are known to the same level of significance. As stated previously, the temperature or pressure is often known to about 0.2% uncertainty or 3 significant digits. This paper documents the sequence of problems that students solve to develop a deeper understanding and confidence in reporting uncertainties in their answers.

Background

A review of engineering thermodynamics textbooks reveals that uncertainty and significant digits are assumed to be prerequisite material and are not explicitly covered[2-4]. This appears unfortunate since many students are confused when confronted with uncertainty and significant digits. Some thermodynamic data is reported in property tables to 5 or 6 significant digits while a problem may be have input parameters specified to only 2 or 3 significant digits. Students should consider the uncertainty of all the inputs and propagate the uncertainty through the calculations to present their answers to an appropriate number of significant digits. Because property values are presented with many digits, student's often feel it necessary to report their answers to the same number of digits. This leads to a misunderstanding of the approximate nature of thermodynamic calculations. As background, a number of papers are discussed here.

Balmer and Spallholz[5] assert that the teaching of engineering thermodynamics hasn't changed significantly in many years, while it continues to be a baffling subject for many students. Confusion stems from two primary areas: (1) definitions and terms which are unique to thermodynamics and have specific meanings and (2) the continued reliance on property tables. It is proposed that active learning techniques be used to master the language with emphasis on the etymology of words. For example, adiabatic is derived from the Greek word *adiabatos*, meaning "not to be passed through." Difficult words include: isothermal, isochoric, isobaric, isenthalpic, isentropic, polytropic, enthalpy, entropy, reversible, quality, saturated, and exergy. In addition, mastering property tables is a challenge since students often err when determining the state: subcooled liquid, saturated liquid, two-phase liquid-vapor, saturated vapor, or superheated vapor. To alleviate this, software can be used for property retrieval. The access to thermodynamic software is increasingly available to students from calculators, laptops, and phones. Overall the scope of thermodynamics applications continues to expand as evidenced by what new textbooks cover, yet students may be missing insight often gleaned by back-of-the-envelope calculations. As such, a more realistic view of the accuracy of property values, either retrieved from tables or a software program, would help improve the conceptual understanding of students.

Wren[6] proposes to actively engage students through human-body thermodynamics activities. This is proposed to counter a prevalent student attitude that thermodynamics is a dry and abstract subject. It is proposed that students spend time in activities requiring them to speak, question, deliberate, propose, plan, execute, collect, analyze, present and explain. These student activities are the hallmark of active learning. In contrast, students often spend more time listening in traditional lecture-dominated classes. One example is to have small teams of students use simple equipment to measure O_2/CO_2 to assess the metabolic rate of another student who is exercising on a stationary bike. The bike is used to measure power and net work. Students are forced to assess uncertainties in measurements from real data in order to make quantitative comparisons with underlying models. It is expected that students struggle with discrepant data as well as the uncertainty of their measurements.

Vigeant[7-9] reports on two inquiry-based activities that address areas of persistent student misconceptions in thermodynamics. Presented with a situation, students are asked to make a prediction before an experiment or simulation is conducted. A brief hands-on experiment or simulation is conducted, and data is collected to compare with student predictions. Students then engage in a set of reflective questions that ask them to explain their initial predictions and observations. The goal is to have students explain the conceptual foundation for their prediction and when necessary use observations to modify or improve the foundational framework to improve understanding and future predictions for similar situations. Inquiry-based activities are effective at improving conceptual learning when activities have (i) unambiguous predictions written by the student, (ii) an action consisting of an experiment or simulation, and (iii) written summary or reconciliation (if needed) conducted by the student. Brief and repeatable experiments are preferred since there is a perception that computer simulations can be "tricked". Experiments can be repeated under varying conditions so that students gain confidence in the observations. Activities are designed to address specific areas from the concept inventory for engineering thermodynamics. Questions are often broad, such as: "Please explain your reasoning for …". When students explain themselves, this instills a firmer understanding of the concept. Students often need to evaluate small variations in observations since there are small differences from observation to observation in real systems. Students develop a more mature perspective on the precision of observations and relative uncertainty in quantitative assessments.

Liberator[10] describes "engineering estimate" problems in a thermodynamics class which begin with a short video. Typical videos involve some unique phenomena, event, or action that exhibits some thermodynamic principle. Students find the video and then pose questions for the class requiring calculations and/or estimates of values such as mass, speed, temperature or pressure. They may also ask the class to sketch a diagram, make predictions, or refute a statement. Students gain critical thinking skills along with improved analysis and estimation skills. In such activities, the uncertainty of data is inherently addressed as students share estimates and often learn the absurdity of reporting estimates with excessive digits. The real world problems are never as clean and simple as textbook problems. For textbook problems, data is accessible in tables often to many significant digits while "real" data is often rough and approximate. It is reported that students have more difficulty in measuring or estimating the important variables rather than mathematical manipulations related to applying the concepts. In the traditional class, students struggle more with mathematical manipulations than with data precision and prediction uncertainties.

Chappell[11], Huguet[12] and Woodbury[13] describe steam tables which have been implemented in Excel. This implementation of the steam properties is appealing since Excel is widely available to students. Instead of requiring students to use specialty software which may come with a textbook, students can download an Excel "Add-in" and avoid the tediousness of property evaluations when solving relatively complex thermodynamic calculations. The accuracy of the implementation depends on the underlying thermodynamic state equations and less on computational error associated with numerical manipulations in the software. For many problems, seeing intermediate and final results in a compact spreadsheet has significant advantages. It allows students to explore parametric studies and plot results. A student can toggle between seeing values or the underlying programming formulation in cells by either selecting the cell or using "ctrl ~" which reveals the equations for each cell. Students find it relatively easy to begin using Excel. As such, this implementation has been adopted for this paper based on an implementation which has steam, air, R134a and R22 tables[13]. These tables are used to construct exercises students complete to improve their understanding of uncertainties and significant digits.

For this work, uncertainty analysis methods are introduced in a Thermodynamics II class in the the second half of the semester following the coverage of thermodynamic relations and psychrometrics. A take-home exam was assigned which accounted for 50% of a mid-term exam. The in-class portion of the exam consisted of traditional questions covering thermodynamic relations and psychrometrics. The take-home portion included uncertainty analysis applied to property evaluations and moist air calculations. The next section of this paper describes the problems students solved and the results of a student survey.

Uncertainty of Ideal Gas Properties for Air

Students were asked to consider how a small uncertainty in temperature affects the uncertainty in the properties of air. In many thermodynamic textbooks, an appendix is provided with columns for temperature (T), enthalpy (h), internal energy (u), low pressure entropy (s^o), pressure ratio (p_r), and volume ratio (v_r). The last two are for isentropic processes[2]. A portion of a typical thermodynamic property table is given in Table 1.

Many of the property values are represented to 5 and 6 significant digits. The temperature is often represented to two significant digits. The temperature is often considered an "input" for the table and students lookup the other properties. If a modest uncertainty is assumed for temperature, then one can calculate the uncertainty in each of the properties. For example, the uncertainty in enthalpy is calculated as

$$\Delta h = \left(\frac{\partial h}{\partial T}\right)\Delta T$$

Table 1. Ideal Gas Properties for Air (from Ref. 2).

T, K	h, kJ/kg	u, kJ/kg	s^o, kJ/kgK	p_r	v_r
200	199.97	142.56	1.29559	0.3363	1707
300	300.19	214.07	1.70203	1.386	621.2
500	503.02	359.49	2.21952	8.411	170.6
1000	1046.04	758.94	2.96770	114.0	25.17
1500	1635.97	1205.41	3.44516	601.9	7.152
2000	2252.1	1678.7	3.7994	2068	2.776

The differentiation can be completed numerically. Assuming a 0.2% uncertainty in the temperature, the uncertainties in the other properties are calculated as shown in Table 2.

Table 2. Uncertainties of Air Properties.

T, K	h, kJ/kg	u, kJ/kg	s^o, kJ/kgK	p_r	v_r
200 ± 0.4	199.97 ± 0.40	142.56 ± 0.29	1.29559 ± 0.0020	0.3363 ± 0.0025	1707 ± 7.8
300 ± 0.6	300.19 ± 0.60	214.07 ± 0.43	1.70203 ± 0.0020	1.386 ± 0.0099	621.2 ± 3.0
500 ± 1	503.02 ± 1.0	359.49 ± 0.74	2.21952 ± 0.0020	8.411 ± 0.62	170.6 ± 0.85
1000 ± 2	1046.04 ± 2.3	758.94 ± 1.7	2.96770 ± 0.0023	114.0 ± 0.94	25.17 ± 0.15
1500 ± 3	1635.97 ± 3.6	1205.41 ± 2.8	3.44516 ± 0.0024	601.9 ± 5.2	7.152 ± 0.045
2000 ± 4	2252.1 ± 5.0	1678.7 ± 3.8	3.7994 ± 0.0025	2068 ± 19	2.776 ± 0.018

Table 3 shows the same data as uncertainties in a fractional format with the property value rounded to an appropriate number of significant digits. Most properties have about the same fractional uncertainty as the input temperature uncertainty of 0.2%. It is less for s^o, which is about half the value for the input (0.1%). It is greater for p_r which is about four times the input value (0.80%). The fractional uncertainty does depend on temperature, as shown in Figure 1.

Table 3. Fractional Representation of Air Property Uncertainties.

T, K	h, kJ/kg	u, kJ/kg	s^o, kJ/kgK	p_r	v_r
$200 \pm 0.20\%$	$200 \pm 0.20\%$	$142.6 \pm 0.20\%$	$1.296 \pm 0.15\%$	$0.336 \pm 0.74\%$	$1707 \pm 0.46\%$
$300 \pm 0.20\%$	$300 \pm 0.20\%$	$214.1 \pm 0.20\%$	$1.702 \pm 0.12\%$	$1.39 \pm 0.72\%$	$621 \pm 0.49\%$
$500 \pm 0.20\%$	$503 \pm 0.20\%$	$359.5 \pm 0.21\%$	$2.220 \pm 0.09\%$	$8.4 \pm 0.74\%$	$170.6 \pm 0.50\%$
$1000 \pm 0.20\%$	$1046 \pm 0.22\%$	$759 \pm 0.23\%$	$2.968 \pm 0.08\%$	$114 \pm 0.82\%$	$25.2 \pm 0.58\%$
$1500 \pm 0.20\%$	$1636 \pm 0.22\%$	$1205 \pm 0.23\%$	$3.445 \pm 0.07\%$	$602 \pm 0.86\%$	$7.15 \pm 0.62\%$
$2000 \pm 0.20\%$	$2252 \pm 0.22\%$	$1679 \pm 0.23\%$	$3.799 \pm 0.07\%$	$2070 \pm 0.91\%$	$2.78 \pm 0.64\%$

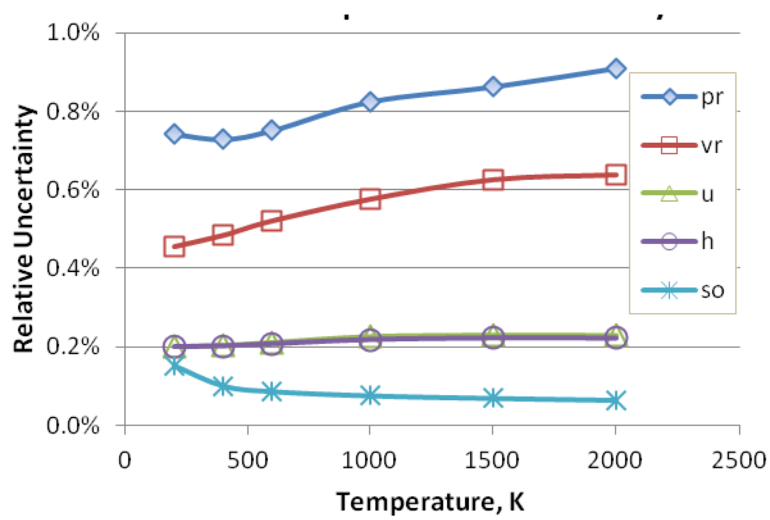

Figure 1. Uncertainty in Air Properties with 0.2% Uncertainty in Temperature.

From this simple analysis, students learn that the properties shown in the air table should only be used in intermediate calculations with the full 5 or 6 significant digits, yet because of a modest uncertainty in an input to the table, such as temperature, the properties are less certain than the 5 or 6 digits. When a modest uncertainty of 0.2% for temperature is assumed, the properties are certain to at most 3 to 4 digits, but not more. The same can be shown for steam.

Uncertainty of Steam Properties

In the superheated steam tables, property data (v, u, h and s) is often given in pressure "blocks." Students find the correct pressure block, then the line for the appropriate temperature to look up the other properties. Although two other intensive properties can be used to fix the state, the most common are pressure and temperature; hence, tables are organized this way. The property data in the steam tables are often given to 5 significant digits. The uncertainty of steam properties is investigated assuming some uncertainty in pressure and temperature, and propagating the uncertainty to the retrieved property value. For example, the uncertainty in the enthalpy is calculated as:

$$\Delta h = \left\{ \left(\left(\frac{\partial h}{\partial T} \right)_p \Delta T \right)^2 + \left(\left(\frac{\partial h}{\partial P} \right)_T \Delta P \right)^2 \right\}^{1/2}$$

The partial derivatives are calculated numerically. This is included in a sequential-perturbation technique which is readily implemented in Excel.[14] Figure 2 shows an Excel spreadsheet with the formula displayed in each cell.

	A	B	C	D	E	F
1					T, C	P, kPa
2	Input			uncertainty	1	2
3	1	T, C	320	0.002	=IF($A3=E$2,$C3*(1+$D3),$C3)	=IF($A3=F$2,$C3*(1+$D3),$C3)
4	2	P, kPa	500	0.002	=IF($A4=E$2,$C4*(1+$D4),$C4)	=IF($A4=F$2,$C4*(1+$D4),$C4)
5	Output					
6		T, K	=C3+273	=SQRT(D7)/C6	=E3+273	=F3+273
7				=SUMPRODUCT(E6:F6-C6,E6:F6-C6)	=($C6-E6)^2/$D7	=($C6-F6)^2/$D7
8		v, m^3/kg	=v_pT_H2O(C4,C3)	=SQRT(D9)/C8	=v_pT_H2O(E4,E3)	=v_pT_H2O(F4,F3)
9				=SUMPRODUCT(E8:F8-C8,E8:F8-C8)	=($C8-E8)^2/$D9	=($C8-F8)^2/$D9
10		u, kJ/kg	=u_pT_H2O(C4,C3)	=SQRT(D11)/C10	=u_pT_H2O(E4,E3)	=u_pT_H2O(F4,F3)
11				=SUMPRODUCT(E10:F10-C10,E10:F10-C10)	=($C10-E10)^2/$D11	=($C10-F10)^2/$D11
12		h, kJ/kg	=h_pT_H2O(C4,C3)	=SQRT(D13)/C12	=h_pT_H2O(E4,E3)	=h_pT_H2O(F4,F3)
13				=SUMPRODUCT(E12:F12-C12,E12:F12-C12)	=($C12-E12)^2/$D13	=($C12-F12)^2/$D13
14		s, kJ/kgK	=s_pT_H2O(C4,C3)	=SQRT(D15)/C14	=s_pT_H2O(E4,E3)	=s_pT_H2O(F4,F3)
15				=SUMPRODUCT(E14:F14-C14,E14:F14-C14)	=($C14-E14)^2/$D15	=($C14-F14)^2/$D15

Figure 2. Implementation of Sequential-Perturbation Uncertainty Calculations for Steam.

The top-left section shows the inputs, the top-right shows the sequential perturbation matrix, the bottom-right, the computation of outputs for the perturbed values, and the bottom-left, the outputs with uncertainty estimates. The two inputs are T and P, shown in Cells C3 and C4. Below are outputs evaluated using the steam tables in cells C8, C20, C12, and C14. The uncertainty for each input is in cell D3 and D4. A two-by-two matrix in E3 to F4 contains a matrix of inputs with the diagonal element perturbed by adding the absolute uncertainty. The formula used to calculate the outputs from the inputs is copied from column C to columns E and F. In column D are calculations of the fractional uncertainty of the outputs. It is completed in two steps because the relative contribution of each input is also calculated in the second line for each output. For example, the relative importance of temperature variations in changing enthalpy is:

$$I_{h,T} = \frac{\left(\left(\frac{\partial h}{\partial T}\right)_p \Delta T\right)^2}{\left(\left(\frac{\partial h}{\partial T}\right)_p \Delta T\right)^2 + \left(\left(\frac{\partial h}{\partial P}\right)_T \Delta P\right)^2}$$

Likewise the importance of pressure variations in changing enthalpy is:

$$I_{h,p} = \frac{\left(\left(\frac{\partial h}{\partial T}\right)_T \Delta P\right)^2}{\left(\left(\frac{\partial h}{\partial T}\right)_p \Delta T\right)^2 + \left(\left(\frac{\partial h}{\partial P}\right)_T \Delta P\right)^2}$$

The importance depends on the response being sensitive to the input and the existence of some uncertainty in the input. The sum of all importance equals 100%.

Figure 3 shows the results of the uncertainty propagation analysis for superheated steam at three pressures: 500 kPa, 5 MP, and 10 MPa. For each calculation the fractional uncertainty of the pressure and temperature are 0.2% for all cases. The fractional uncertainties for specific volume, enthalpy and entropy are plotted for each of each pressure level from nearly the saturation temperature up to 500°C. The fractional uncertainty is above 0.2% for the specific volume, indicating that the specific volume can be calculated to no more than 3 significant digits. The fractional uncertainty of specific volume is greater near when the steam temperature is near the saturation temperature. The fractional uncertainties for enthalpy and entropy are greater than 0.02% indicating they can be calculated to no more than 4 significant digits given the assumed uncertainties in temperature and pressure. The uncertainty in *u* is similar to *h*, so it is not plotted in Figure 3. In summary, the uncertainty in *v* can be expressed to three significant digits while for *u*, *h* and *s* can be expressed to four significant digits at the most. In thermodynamic tables, the property values for *v* are often expressed to 4 significant digits, while *u*, *h* and *s* are expressed to 5 significant digits.

The spreadsheet uncertainty analysis also highlights which input has the strongest contribution to each output. For the specific volume at 320°C and 500kPa, the uncertainty in temperature contributes 24% and pressure76% to the uncertainty in *v*. For internal energy and enthalpy, nearly 100% of their uncertainties are due to temperature and none due to pressure. At this condition, the superheated vapor is behaving as an ideal gas, where one would expect both u and h to be functions of temperature only. The entropy has a relatively small uncertainty of 0.3%, of which 85% is due to temperature and 15% to pressure. Similar to the earlier discussion about expressing the property values of air to an appropriate number of significant digits, this type of analysis helps students understand that small uncertainties in input properties such as T and P result in significant uncertainties in properties that are often “looked-up” from either tables or calculated using software routines. When the properties are used in analyses, the uncertainties in final results account for uncertainties of multiple inputs.

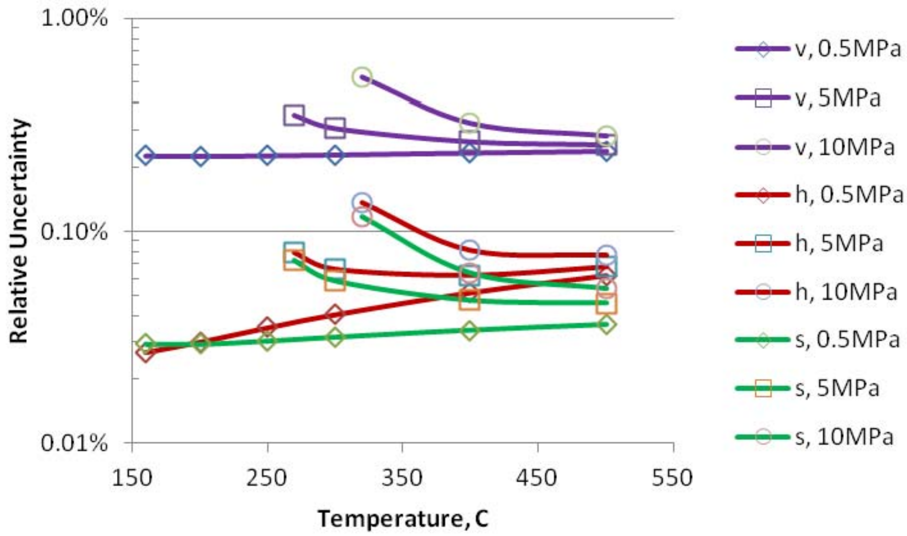

Figure 3. Uncertainty in Steam Properties with 0.2% Uncertainty in both Temperature and Pressure.

Quenching a Hot Metal Bar

The propagation of uncertainty through a multi-stepped thermodynamics problem can be a significant challenge. With the aid of a spreadsheet, the calculations are more organized and the results are more straightforward to interpret. For example, students were asked to resolve an example problem from the textbook[2] involving the quenching of a hot metal bar (T_{mi} = 1900°R, m_m =0.8 lb, c_m = 0.1 Btu/lb°R) in a closed tank of liquid water (T_{wi} = 530°R, m_w = 20 lb, c_w = 1 Btu/lb°R) and calculate the final equilibrium temperature and the amount of entropy produced. Each of the six inputs is specified to be uncertain with a 0.2% fractional uncertainty. The results are presented in Figure 4. The mean result is the same as can be confirmed from the example problem in the textbook. The students observe that the final temperature is dominated by the initial water temperature. The uncertainty in the entropy generation is split between the initial temperatures (40% each) and the mass and specific heat of the metal (10% each). The fractional uncertainty of the entropy generation is three time that of the inputs so that this output is sensitive to the input uncertainties. If T_{wi} uncertainty were to double, then the T_f uncertainty would double. Students are encouraged to explore changing the input values and the uncertainties. For example, only the uncertainties are changed between the results shown in the top and bottom of Figure 4. The bottom values were suggested as being reasonably large. When calculating the final temperature, the initial water temperature remains dominant. When calculating the entropy generation, there is a change in the most important inputs. For the first case, the change is in the initial metal temperature and initial water temperature. Both are about 40% responsible for the uncertainty in the entropy generation. For the new uncertainties, the specific heat of the metal is dominant and accounts for about 68% of the uncertainty in Sgen. This example shows that the uncertainties in the inputs have a significant impact on the uncertainty of the outputs.

	A	B	C	D	E	F	G	H	I	J	K
1	Example 6.5, Moran et.al. (2011)					Tmi, R	cm, Btu/lbF	mm, lb	Twi, R	cw, Btu/lbF	mw, lb
2	Inputs			abs. unc.	rel. unc.	1	2	3	4	5	6
3	1	Tmi, R	1900	3.8	0.20%	1903.8	1900	1900	1900	1900	1900
4	2	cm, Btu/lbR	0.1000	0.0002	0.20%	0.1	0.1002	0.1	0.1	0.1	0.1
5	3	mm, lb	0.800	0.0016	0.20%	0.8	0.8	0.8016	0.8	0.8	0.8
6	4	Twi, R	530	1.06	0.20%	530	530	530	531.06	530	530
7	5	cw, Btu/lbR	1.000	0.002	0.20%	1	1	1	1	1.002	1
8	6	mw, lb	20.00	0.04	0.20%	20	20	20	20	20	20.04
9	Outputs					Tmi, R	cm, Btu/lb	mm, lb	Twi, R	cw, Btu/lb	mw, lb
10	1	Tf, R	535.5	1.1	0.20%	535.47331	535.46904	535.46904	536.51394	535.44732	535.44732
11					1.115366	0%	0%	0%	**100%**	0%	0%
12	2	Sgen, Btu/R	0.10360	0.00064	0.62%	0.1040042	0.1038014	0.1038014	0.1031895	0.1035984	0.1035984
13					4.16E-07	**40%**	10%	10%	**40%**	0%	0%

	A	B	C	D	E	F	G	H	I	J	K
1	Example 6.5, Moran et.al. (2011)					Tmi, R	cm, Btu/lbR	mm, lb	Twi, R	cw, Btu/lbR	mw, lb
2	Inputs			abs. unc.	rel. unc.	1	2	3	4	5	6
3	1	Tmi, R	1900	20	1.05%	1920	1900	1900	1900	1900	1900
4	2	cm, Btu/lt	0.1	0.01	10.00%	0.1	0.11	0.1	0.1	0.1	0.1
5	3	mm, lb	0.8	0.05	6.25%	0.8	0.8	0.85	0.8	0.8	0.8
6	4	Twi, R	530	5	0.94%	530	530	530	535	530	530
7	5	cw, Btu/lt	1	0.03	3.00%	1	1	1	1	1.03	1
8	6	mw, lb	20	1	5.00%	20	20	20	20	20	21
9	Outputs					Tmi, R	cm, Btu/lbR	mm, lb	Twi, R	cw, Btu/lbR	mw, lb
10	1	Tf, R	535.5	5.0	0.94%	535.5378	536.0016	535.7979	540.4382	535.2998	535.1992
11					25.31037	0%	1%	0%	**98%**	0%	0%
12	2	Sgen, Btu/	0.10360	0.01242	11.99%	0.10575	0.11384	0.11000	0.10169	0.10363	0.10365
13					0.000154	3%	**68%**	**27%**	2%	0%	0%

Figure 4. Quenching Uncertainty Analysis Results for Smaller (top) and Larger (bottom) Uncertainties.

Assessment on Student Learning

The impact on student learning is assessed using two mechanisms: results from the mid-term exam and a follow-up student survey. The results of the exam showed that students were able to correctly apply uncertainty analysis methods to propagate uncertainties through calculations to final results. As a result, students were able to correctly round final answers to an appropriate number of significant digits. Overall, there were high scores on the exam with the average being 43 out of 50 possible points. Missed problems were often in the following areas: (1) failure to consider plus/minus two standard deviations to capture 95% of the data so that some students reported answers to plus/minus one standard deviation, (2) confusion in the interpretation of the difference between fractional and absolute representation of uncertainty, (3) errors in underlying thermodynamic calculations which must be correct before the added complexity of uncertainty analysis. Concerning the first point, this highlights a deficiency in the prerequisite courses where applied statistics are covered. Without a firm understanding of the concepts of mean, standard deviation, and normal distribution, it is difficult for students to understand the basis for expressing final answers to appropriate number of significant digits. Those students who missed problems testing foundational statistical concepts did poorly on the exam. Overall, the scores on the take-home exam were high for the class with a number of students earning perfect scores of 50 out of 50.

A questionnaire was provided to students after they finished the take-home exam. It consisted of ten Likert scale questions with the following responses:
1 = strongly disagree; 2 = disagree; 3 = neutral; 4 = agree; 5 = strongly agree

The ten survey questions are:

- Q1 Excel is easy to use.
- Q2 Excel is readily available for you to use.
- Q3 Before this class, you were familiar with uncertainty analysis methods.
- Q4 Before this class, you used uncertainty analysis methods in the treatment of experimental data.
- Q5 Before this class, you used uncertainty analysis methods in the evaluation of non-experimental problems.
- Q6 You can report final answers to an appropriate number of significant digits.
- Q7 You have a positive attitude toward the *sequential perturbation* method used to perform an uncertainty analysis.
- Q8 You can envision using *sequential perturbation* in other engineering courses, such as Senior Design.
- Q9 You have a better perspective on uncertainties and significant digits in final results.
- Q10 You can propagate uncertainties from inputs to final results for a thermodynamics problem.

In total, 72 students completed the survey. The entire class enrollment was 78, but some students had withdrawn or were planning to withdraw from the class so that there was not 100% participation in the survey. Likewise, some students left one or more responses blank. Overall there was a high level of participation in the survey. Table 1 shows a numerical summary of the student responses, with the last column containing the class average.

Table 1. Frequency of response to questions and class average.

	Response					
Question	**1**	**2**	**3**	**4**	**5**	**Average**
1	3	5	12	**34**	17	**3.8**
2	3	7	8	19	**34**	**4.0**
3	**26**	16	12	12	5	**2.4**
4	**32**	12	8	14	6	**2.3**
5	**38**	13	15	4	2	**1.9**
6	2	3	13	**33**	21	**3.9**
7	4	5	19	**27**	17	**3.7**
8	3	3	14	**27**	25	**3.9**
9	3	2	10	**32**	25	**4.0**
10	3	1	13	**36**	19	**3.9**

Results from the surveys are also shown in Fig. 5 for the average class response. The responses with the strongest class response are for question #2 and #9. Question #2 is reassuring and was included in the survey to determine if the implementation of the uncertainty analysis method in excel should be changed if this is repeated in the future. Students were also given the opportunity to provide written comments on the survey. Of the 72 respondents, 31 students provided written comments. A surprising number of comments addressed practical issues associated with Excel. A number are included here:

> Excel is quite easy to use but for people who never used Excel it can be quite a burden since they have to learn so many new things at a time. That is, it would be better if Excel is taught in another class or as a different subject.
>
> At first I was reluctant to learn excel but I know I will need it in the future so I'm glad we were forced to learn it. I found the material was quite interesting to learn about.
>
> Using excel is a real tedious, time consuming process.
>
> The incorporations of excel during this course will definitely assist me in the future for solving problems using excel. The only thing I wish we had done would have been to go

to the computer lab as a class once and been given a “hands-on” crash course for using excel and it's commands. This was my 1st time using excel and I feel it would have been an easier process had I had more of a "how to" demonstration in which I could participate in.

Understood what to do from lecture but not necessarily lingo, such as what the $ were used for.

Overall it was surprising the lack of exposure many upper-division students have to Excel and the difficulty some have in using it. In the future, a “hands-on” crash course in Excel in a computer lab will be given for students who haven’t used it before. It was a false perception that all students have used Excel before this class.

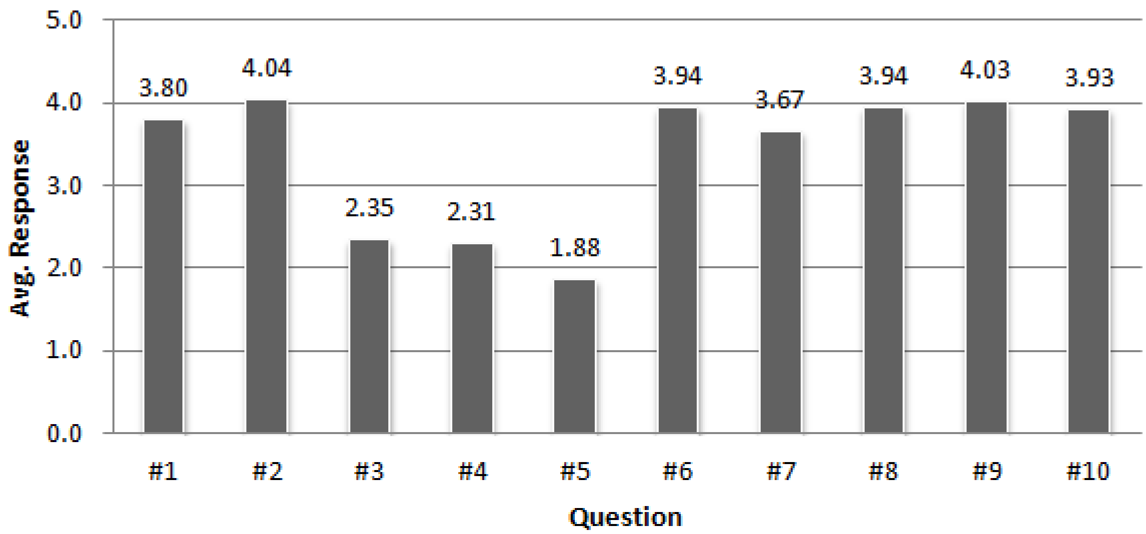

Figure 5. Results from student surveys in Thermodynamics class.

The positive response on Question #9 is significant given that one of the educational goals is to strengthen the students understanding of uncertainties and significant digits. On the exam, students were more careful to avoid excessive significant digits and reported final results to three significant digits. It is believed that students didn’t just memorize a rule that final answers need to be expressed to no more than three significant digits. Comments from the survey also support the idea that student learning was improved. Questions #8 and #10 suggest that students have an improved perspective on how to apply uncertainty analysis through other problems and in other classes. Some comments supported the idea that these methods could be used to analyze almost any previous cycle problem, such as Rankine, Otto, Diesel, Brayton and vapor-compression refrigeration. This offers insight into what parameters most strongly drive the system. Knowing this, students made the connection that one can evaluate different design alternatives to improve an overall design.

Questions #3 through 5 have the lowest average responses and indicated that many students have not been exposed to uncertainty analysis methods. This is a surprising result since it is clearly described in the course objectives of a lower-level class called "Measurements and Instrumentation". A few comments referenced this lower-level class and one student wrote that "these two lectures cleared up some ambiguity in dealing with propagation of error." Yet it remains a surprising result that so many students claim they were unfamiliar with uncertainty analysis methods.

All comments were not positive and one is clearly negative: "As I progressed through the take-home, my confidence with significant figures decreased, unsure concerning uncertainties." This comment is difficult to interpret. It probably stems from a weak understanding of the underlying methodology and purpose of an uncertainty analysis. Some students didn't finish the take-home exam, and the student who provided this comment may be expressing frustration at not completing the assignment.

Another negative comment received was: "Super difficult Test! I now hate my class mates ("friends")!". Students were allowed to talk with other students, but each had to do their own individual work. In this way, students didn't work in teams. After talking with a few students after the exam, some shared that weaker students expected others to teach them how to solve the problems. If they would have been in a team, the stronger student would have given a cursory explanation and the weaker student readily accept it, because the team would turn in one solution which would have been the primary work of the stronger student. Having individual exams, each student had unique sets of inputs. Sharing code was prohibited. Each student needed to code their own work. In order for the stronger student to help the weaker, it took more time to talk through the solution and answer more questions. Some students didn't have the time to devote to weaker students, especially if there was the perception of waiting until the last minute to start the assignment.

Conclusions

This work is based on an observation that too many upper-division engineering students have conceptual problems with significant digits. Many students don't understand why or how to round final answers to an appropriate number of significant digits. Students are especially confused when thermodynamic property tables express values to 5 and 6 significant digits. Having struggled with re-teaching this foundational concept which students should have mastered in lower-level classes, a review of uncertainty analysis was introduced in a thermodynamics class and a portion of a take-home exam was used to measure student mastery of significant digits.

From this work it is concluded that:

1) Student understanding of significant digits and uncertainty can be reinforced by reviewing uncertainty analysis concepts in thermodynamics using problems that students have already solved in the class. One needs to specify uncertainties on input parameters and propagate the uncertainties through the calculations.
2) Students show an increased perception of the significance of properly reporting final results to an appropriate number of significant digits and a positive attitude toward the methods covered. Some comments indicate that the methods are inadequately covered in lower-level engineering classes. This is useful information for continuous improvement of the engineering curriculum.
3) Student comments indicate a deeper understanding of the methods used and an ability to apply uncertainty analysis methods to other areas of engineering, especially to design-oriented problems which are typically encountered in upper-division classes in the curriculum.

In summary, this paper helps ensure the mechanical engineering educational objectives summarized in ABET criteria “b” where students are expected to have the ability to design and conduct experiments, as well as to analyze and interpret data, and criteria “g” where students are expected to have the ability to communicate effectively. Clear communication of results requires that they be expressed to an appropriate number of significant digits.

References

1. ABET, General Criteria 3. Student Outcomes, www.abet.org, 2012.
2. Moran, M.J., H.N. Shapiro, D.D. Boettner, and M.B. Bailey, *Fundamentals of Engineering Thermodynamics*, 7th Edition, Wiley, 2011.
3. Cengel, Y. and M.A. Boles, *Thermodynamics, An Engineering Approach*, 7th edition, McGraw-Hill, 2011.
4. Borgnakke, C. and R.E. Sonntag, *Fundamentals of Thermodynamics*, 7th edition, Wiley, 2009.
5. Balmer, R. and L. Spallholz, “21st Century Thermodynamics,” Proceedings of ASEE Annual Conference, 2006.
6. Wren, J.S., “Engineering Thermodynamics in the 21st Century: Active Learning by Human Body Thermodynamics”, Proceedings of ASEE Annual Conference, 2012.
7. Vigeant, M.A., and M.J. Prince, and K.E.K. Nottis, “Making their Brains Hurt: Quick and Effective Activities for Thermodynamics”, Proceedings of ASEE Annual Conference, 2012
8. Prince, M.J, M.A. Vigeant, K.E.K. Nottis, “The use of Inquiry-Based Activities to Repair Student Misconceptions Related to Heat, Energy, and Temperature”, Proceeding of ASEE Annual Conference, 2011.
9. Prince, M.J, M.A. Vigeant, K.E.K. Nottis, “Development of the heat and energy concept inventory: Preliminary results on the prevalence and persistence of engineering students’ misconceptions”, *Journal of Engineering Education*, v101, n3, p412-438, 2012.
10. Liberator, M.W., C.R. Vestal, and A.M. Herring, YouTube Fridays: Student led development of engineering estimate problems, *Advances in Engineering Education*, v3, n1, p1-16, 2012.
11. Chappell, J. R. Taylor, and K. Woodbury, “Introducing Excel Based Steam Table Calculations into Thermodynamics Curriculum”, Proceeding of ASEE Annual Conference, 2008.

12. Huguet, J., K. Woodbury, R. Taylor, “Development of Excel Add-In Modules for Use in Thermodynamics Curriculum: Steam and Ideal Gas Properties”, Proceeding of ASEE Annual Conference, 2008
13. Woodbury K. and B. Taylor, *Excel in M. E*, University of Alabama, http://www.me.ua.edu/excel, 2012.
14. Manteufel, R.D., “Sequential Perturbation Uncertainty Propagation in Thermal-Fluid Applications”, Proceedings of ASME International Mechanical Engineering Congress and Exposition, IMECE2012-88316, 2012.