AC 2010-2167: INNOVATIVE CONTENT IN A NEW CIRCUIT ANALYSIS COURSE

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Abstract

Recently, a new Circuits I course was created during the development of a degree program. This course consists of three semester hours of lecture accompanied by a one semester hour concurrent lab. Lectures contain numerous formula and concept derivations, in keeping with similar courses. Lecture content of note includes an illustrative "derivation" of the superposition theorem. Lab content reinforces and extends the lecture as it teaches students to: solve and plot analysis equations using MATLAB®, simulate circuit behavior, breadboard circuits, and use standard instrumentation. Lab content of note includes a student derivation during the third week of class to find the load resistance for maximum power transfer, an introductory design exercise during the fourth week of class, and a more challenging two-port network design exercise during the eighth week of class.

This paper describes some of the innovative lecture and lab content that has been included in this new course. It also presents an analysis of a student survey that was used to provide a preliminary assessment of the effectiveness of this content. The results of this analysis reveal that students "tended to agree" that the lecture and lab activities described herein assisted their learning.

Introduction

In 1975, an MIT study published by ASEE¹ noted that "educational experience in design should be promoted as early as possible...and should be available as an integrated part of the engineering curriculum." A subsequent push to "integrate design throughout the curriculum"² led programs to add design content in lower-division (e.g., freshman engineering) courses³,⁴ and augment design activities in upper-division courses.

Instructional laboratories are a natural setting for design⁵, but meaningful exercises in lower-division courses pose a challenge. Several efforts have been reported which involve the addition of electronics topics to an introductory circuit analysis course to improve student engagement, especially in the course's concurrent lab⁶-⁹. This paper describes efforts to embed derivation and design assignments into an "electronics-free" circuits course, primarily in its laboratory component.

Six sections follow: a lecture excerpt, three laboratory exercises, an assessment section, and a conclusion. Bold subtitles are used to identify the start of each of these sections. In the interest of clarity, the laboratory exercises will be referred to as Labs 1, 2, and 3. To improve readability, blanks that would normally be included for student use have been omitted.

The lecture excerpt and laboratory exercise sections include various sub-sections that are presented herein using italicized sub-headings. The Overview sub-section provides interpretive information to the reader, while the Benefits sub-section summarizes the benefits provided by the exercise. The Objectives, Procedure, and Analysis sub-sections comprise the heart of the material used by the student to perform the lab exercises—readers who seek a quick first read of the paper can skip these sections without loss of continuity. A Design Completion sub-section has been appended to the end of Labs 2 and 3 to summarize the design activities that these labs require the student to perform.
Lecture Excerpt: An Illustrative "Derivation" of the Superposition Theorem

Overview

In an effort to make the superposition theorem more meaningful to students, an illustrative "derivation" of the theorem is performed. A simple circuit is analyzed to generate an equation that can be separated into a sum of source-associated terms having easily recognizable forms. These terms are used to synthesize the sub-circuit associated with each source. The sub-circuits are then related back to the original circuit by noting that each source's sub-circuit can be quickly obtained from the original circuit by setting all other sources to zero.

Benefits

81% of the students participating in an end-of-course survey agreed that the "illustrative 'derivation' of superposition" described in the following section "helped [them] to understand the basis for the theorem and how to properly apply it." The technique also provides a "satisfying" derivation early in the course that embeds a motivating review of nodal analysis, voltage division, parallel resistance, and Ohm's Law.

Lecture Excerpt

Objective: Derive an equation for $v_2$ in Figure 1 that clearly separates the contributions of the sources.

Labeling the sources and the nodes, we obtain Figure 2:

Figure 1: Original Circuit

Figure 2: Circuit with Sources and Nodes Labeled
Writing an equation at Node 2, we have:
\[ \frac{v_2 - v_A}{R_1} + \frac{v_2}{R_2} - i_B = 0 \]

Multiplying both sides by \( R_1 R_2 \) yields:
\[ R_2 v_2 - R_2 v_A + R_1 v_2 - R_1 R_2 i_B = 0 \]

Combining like terms and rearranging:
\[ v_2 (R_1 + R_2) = R_2 v_A + R_1 R_2 i_B \]

Simplifying, we have:
\[ v_2 = \left( \frac{R_2}{R_1 + R_2} \right) v_A + \left( \frac{R_1 R_2}{R_1 + R_2} \right) i_B \]

Contribution to \( v_2 \) from Source A

Contribution to \( v_2 \) from Source B

Summarizing:
\[ v_2 = v_{2A} + v_{2B} \quad (1) \]

Thus, \( v_2 \) can be thought of as being established by components produced by the individual sources, where:

\[ v_{2A} = \left( \frac{R_2}{R_1 + R_2} \right) v_A \quad \text{and} \quad v_{2B} = \left( \frac{R_1 R_2}{R_1 + R_2} \right) i_B \]

Recognizing the \( v_{2A} \) equation as a voltage divider equation, we can postulate a representative subcircuit for the "Source A Only" case:

Figure 3: Source A Subcircuit

Recognizing the \( v_{2B} \) equation as an Ohm's Law equation for parallel resistors, we can postulate a representative subcircuit for the "Source B Only" case:

Figure 4: Source B Subcircuit

Note that Figure 3 can be quickly obtained from Figure 1 by setting \( i_B \) equal to zero, which "kills" Source B (a zero amp current source is equivalent to an open circuit).

Note that Figure 4 can be quickly obtained from Figure 1 by setting \( v_A \) equal to zero, which "kills" Source A (a zero volt voltage source is equivalent to a short circuit).
Thus, Equation (1) reveals that we can add the responses from sources acting alone to get the response produced by sources acting together. This principle, known as the superposition theorem, is applicable to all linear networks. It is also applicable to networks whose operation can be approximated as being linear within a limited range.

To determine the response to a particular source, kill all other sources by replacing voltage sources with short circuits and replacing current sources with open circuits.

**Lab 1: Maximum Power Delivered to a Resistive Load**

*Overview*

The *Procedure* portion of this exercise is relatively routine—its basic tasks include performing the measurements and calculations needed to: plot the power delivered to a load as a function of its resistance, determine the maximum power dissipated by the load, and compute the value of load resistance that extracts maximum power from the rest of the circuit. The noteworthy aspect of this lab is the *Analysis* section, which guides the student to perform an applied derivation of the maximum power transfer theorem before the theorem is introduced in lecture. Thus, the exercise engages *Circuits I* students in a meaningful application of calculus during about the third week of class.

*Benefits*

79% of the students who participated in an end-of-course survey agreed that Lab 1 "helped to improve [their] ability to perform derivations."

*Objectives*

1. Use a decade resistance box to implement a variable resistor.
2. Compute resistor power given voltage and resistance.
3. Plot load power versus resistance; determine the maximum power and the resistance that produces it.
4. Derive the expression for the load resistance that absorbs maximum power from a particular circuit.

*Procedure*

1. Construct the circuit in Figure 5, using a decade resistance box as $R_L$.

2. Temporarily detach $R_L$ from the rest of the circuit, measure its resistance for each of the settings given in Table 1, and then record your values in the table. Reinsert $R_L$ into the circuit.

3. Measure $V_L$ for each of the values of $R_L$ in Table 1.

![Figure 5: Series DC Circuit](image-url)
4. Measure $V_s$, and then detach $R$ from the circuit and measure it.

5. Use the value of $R_L$ and $V_L$ in each row of Table 1 to compute the corresponding value of $P_L$; complete Table 1.

**Analysis**

1. Plot load power versus load resistance. Use the plot to determine the maximum power absorbed ($P_L^{\text{max}}$) and the load resistance associated with maximum power absorption ($R_L$ for $P_L^{\text{max}}$).

2. Apply voltage division to the circuit in Figure 5 to write an equation for $V_L$ in terms of $R_L$, $R$, and $V_s$.

3. Use the equation from Step 2 to write an equation for $P_L$ in terms of $R_L$, $R$, and $V_s$. Keep in mind that $R$ and $V_s$ are merely symbols that represent known (i.e., constant) values.

4. Differentiate the $P_L$ equation with respect to $R_L$, set the result equal to zero, and solve the resulting expression for the load resistance that dissipates maximum power. Substitute the measured value of $R$ (Procedure 4) into your expression to obtain a calculated value for the $R_L$ associated with $P_L^{\text{max}}$.

5. Calculate the maximum power absorbed by the load resistor by substituting the value of $R_L$ from Step 4 into the equation for $P_L$ derived in Step 3.

6. Compute the relative difference between the calculated and measured values of $R_L$ associated with $P_L^{\text{max}}$ (Steps 4 and 1, respectively). Repeat this computation for $P_L^{\text{max}}$ (calculated in Step 5 and measured in Step 1).

**Lab 2: Introduction to Circuit Design**

**Overview**

This exercise provides the student with an introductory circuit design exercise during about the fourth week of class: a voltage and two currents are given in a simple series-parallel circuit—the student completes the design by writing simple equations to solve for three resistor values. Performance of the design is then verified via analysis, simulation, and construction/measurements.
Benefits

81% of the students who participated in an end-of-course survey agreed that Lab 2 provides "a useful introduction to circuit design." The exercise introduces the design-analyze-simulate-test process as a means for validating a design. The exercise also provides students with interesting applications of Kirchhoff’s Laws, Ohm's Law, voltage division, current division, and resistor combination.

Objectives

1. Apply Kirchhoff’s Laws and Ohm's Law to select resistor values for a simple series-parallel circuit given voltage and current specifications.

2. Apply voltage division, current division, and resistor combination techniques to analyze a simple series-parallel circuit.

3. Use PSpice to determine voltages and currents in a simple series-parallel circuit.

4. Measure voltages and currents in a simple series-parallel circuit.

Procedure

1. Given the circuit in Figure 6, apply Ohm's Law, Kirchhoff’s Voltage Law, and Kirchhoff’s Current Law to compute the resistor values needed to make $I_s = 11.8$ mA, $I_2 = 2.74$ mA, and $V_1 = 4.59$ V.

2. Analyze the circuit in Figure 6 using the resistor values computed in Step 1: apply resistor combination and Ohm's Law to find $I_s$, employ current division to find $I_2$, and use voltage division to find $V_1$. Do not use current or voltage values from Step 1 until they have been computed in Step 2.

3. Use PSpice to determine $I_s$, $I_2$, and $V_1$ for the circuit in Figure 6.

4. Construct the circuit in Figure 6. Measure $I_s$, $I_2$, and $V_1$.

Analysis

1. Tabulate relative differences to compare the values of $I_s$, $I_2$, and $V_1$ in Procedure 1 with the respective values determined in Procedures 2, 3, and 4. Use the Procedure 1 values as the standards of comparison.
Design Completion

First, Figure 6 is embellished by labeling junction a and identifying $I_3$ and $V_3$ to create Figure 7:

![Figure 7: Series-Parallel Circuit](image)

The design proceeds as follows:

Ohm's Law ($R_1$): \[ R_1 = \frac{V_1}{I_s} = \frac{4.59 \text{ V}}{11.8 \text{ mA}} = 389 \text{ Ω} \]

Kirchhoff's Voltage Law (Outer Loop): \[ V_3 = 12 \text{ V} - V_1 = 12 \text{ V} - 4.59 \text{ V} = 7.41 \text{ V} \]

Ohm's Law ($R_2$): \[ R_2 = \frac{V_3}{I_2} = \frac{7.41 \text{ V}}{2.74 \text{ mA}} = 2.70 \text{ kΩ} \]

Kirchhoff's Current Law (Junction a): \[ I_3 = I_s - I_2 = 11.8 \text{ mA} - 2.74 \text{ mA} = 9.06 \text{ mA} \]

Ohm's Law ($R_3$): \[ R_3 = \frac{V_3}{I_3} = \frac{7.41 \text{ V}}{9.06 \text{ mA}} = 818 \text{ Ω} \]

Thus, the appropriate standard values are: $R_1 = 390 \text{ Ω}$, $R_2 = 2.7 \text{ kΩ}$, and $R_3 = 820 \text{ Ω}$

Lab 3: The Resistive T-Network from a Two-Port Network Perspective

Overview

This exercise presents the student with a more challenging design problem during about the eighth week of class: simultaneous equations must be written and solved to select resistor values, and the circuit and some of the terminology associated with it is unfamiliar. Design verification is performed using both simulation and construction/measurements. A realistic application of dependent sources is employed, and dependent source modeling in PSpice is introduced. Interesting applications of Thévenin's Theorem are included—two while deriving formulas for input and output resistance, and two while using PSpice to verify input and output resistance values. A "bonus" introduction to two-port networks is also provided.

Benefits

80% of the students who participated in an end-of-course survey agreed that Lab 3 "helped [them] to learn how to design a circuit to meet...specifications." The exercise draws together
applications of Thévenin’s Theorem and dependent sources, and it introduces T-networks and two-port networks to first-semester circuits students.

Objectives

1. Design a resistive T-network to meet specifications.
2. Use PSpice to verify the T-network design.
3. Construct the T-network to verify its operation.
4. Use a dependent source to develop a two-port network model for a T-network.
5. Use PSpice to verify the two-port network model.

Terminology

**Input port**: the pair of terminals to which an input is applied, usually located on the left side of a network such as Figure 8.

**Output port**: the pair of terminals from which an output is extracted, usually located on the right side of a network such as Figure 8.

**Two-port network**: a network having an input port and an output port.

Procedure

1. Using the circuit in Figure 8 as a prototype, derive equations as needed to select the resistor values that will produce the following behavior:
   a. The input resistance (i.e., the resistance looking into the input port with the output port open-circuited) is about 600 Ω.
   b. The output resistance (i.e., the resistance looking into the output port with the input port short-circuited) is about 50 Ω.
   c. The voltage gain (i.e., the ratio of output voltage to input voltage) is about 0.05.

2. Use PSpice to verify your design:
   a. Place a 999 MΩ [999 MEG] across the output port and apply a 1A current source to the input port. Use the resulting voltage across the input port to verify that your design offers the proper input resistance.
   b. Replace the 1A current source across the input port with a short, and replace the 999 MΩ across the output port with a 1A current source. Use the resulting voltage across the output port to verify that your design offers the proper output resistance.
c. Replace the short across the input port with a 1V voltage source, and replace the 1A current source across the output port with a 999 MΩ. Use the voltage across the output port to compute the voltage gain of your network.

3. Construct the circuit in Figure 8. Use the DMM to measure the resistance across the input port (R₁) with the output port open-circuited.

4. Use the DMM to measure the resistance across the output port (R₂) with the input port short-circuited.

5. Remove the short from the input port. Adjust the power supply to produce an output of about 1V. Apply this voltage to the input port of the circuit, and then use the DMM to set its value to "exactly" 1V. Use the DMM to measure the voltage on the output port, and compute the voltage gain (Aᵥ) of your network.

6. Figure 9 is an application circuit for the previously-designed T-network. Construct this circuit, carefully adjust Vₛ to the value specified, and then measure the voltage at the input and output of your T-network. Compute the voltage gain produced by your T-network in the presence of realistic source and load resistances.

![Figure 9: Application Circuit for the Resistive T-Network](image)

7. Figure 10 shows the model for a generic two-port network. Use the measured values obtained in Steps 3, 4, and 5 to develop a generic model for your T-network.

8. Figure 11 shows the application circuit of Figure 9 with its T-network replaced by the generic model given in Figure 10. Apply the E source in PSpice to model the dependent source in Figure 11, and then use PSpice to determine the port voltages and the voltage gain of the circuit.

![Figure 10: Generic Model for Two-Port Network](image)
Analysis

1. Compute the relative differences to compare your measured results in Procedure Steps 3, 4, and 5 with the design specifications in Procedure 1.

2. Compute the relative difference to compare the voltage gain of the application circuit in Procedure 6 to the design specification in Procedure 1c.

3. Compute the relative difference to compare your measured voltage gain in Procedure Step 6 with the simulated voltage gain in Procedure Step 8.

Design Completion

First, markings for input and output resistance are added to Figure 8 to create Figure 12:

\[ R_i = R_1 + R_3 \]  \hspace{1cm} \text{(2)}

\[ R_1 + R_3 = 600 \, \Omega \]  \hspace{1cm} \text{(3)}

Output resistance is calculated based on a short-circuited input, so \( R_1 \) is in parallel with \( R_3 \).
and their combination is in series with $R_2$:

Resistance combination:  \[ R_o = R_2 + R_1 \parallel R_3 = R_2 + \frac{R_1 R_3}{R_1 + R_3} \]  (4)

Equating (4) to its specification:  \[ R_2 + \frac{R_1 R_3}{R_1 + R_3} = 50 \, \Omega \]  (5)

$R_2$ is open-circuited for the purpose of calculating voltage gain, so $V_2$ appears across $R_3$ and voltage gain is simply the familiar voltage division relationship:

Voltage division:  \[ A_v = \frac{V_2}{V_1} = \frac{R_3}{R_1 + R_3} \]  (6)

Equating (6) to its specification:  \[ \frac{R_3}{R_1 + R_3} = 0.05 \]  (7)

Multiplying both sides by $R_1 + R_3$:  \[ R_3 = 0.05 R_1 + 0.05 R_3 \]

Simplifying:  \[ 0.95 R_3 = 0.05 R_1 \]

Multiplying both sides by 20:  \[ 19 R_3 = R_1 \]  (8)

(8) $\rightarrow$ (3):  \[ 19 R_3 + R_3 = 600 \, \Omega \implies R_3 = 30 \, \Omega \]

Substituting $R_3 = 30 \, \Omega$ into (3):  \[ R_1 + 30 \, \Omega = 600 \, \Omega \implies R_1 = 570 \, \Omega \]

Substituting $R_1=570 \, \Omega$, $R_3=30 \, \Omega$ into (5):  \[ R_2 + \frac{(570 \, \Omega)(30 \, \Omega)}{570 \, \Omega + 30 \, \Omega} = 50 \, \Omega \implies R_2 = 21.5 \, \Omega \]

Thus, the appropriate standard values are:  \[ R_1 = 560 \, \Omega, \, R_2 = 22 \, \Omega, \, \text{and} \, R_3 = 30 \, \Omega \]

**Assessment**

During the Fall 2009 semester, the students in *Circuits I* were surveyed to evaluate the effectiveness of some of the innovative lecture and lab content used in the course. The statements that composed the survey are summarized in Figure 13. A five-point Likert scale was used to construct the possible responses: 1-Strongly disagree, 2-Disagree, 3-Not sure, 4-Agree, and 5-Strongly agree. Participants were asked to select the response "that best describes your level of agreement" with each statement. A summary of the responses is tabulated in Figure 14 and the distribution of responses by statement is illustrated in Figure 15.

As shown in Figure 14, the sample means ranged from 3.86 to 4.48, which strongly suggests that most of the students benefited from the lecture and lab content under evaluation. A brief study of the solid bars in Figure 15 provides additional support for the same conclusion: the combined responses of “Agree” and “Strongly Agree” exceeded 70% for eight of the nine statements.
After defining a mean response of 3.5 or higher as a “tendency toward agreement,” the t-distribution was used to compute, for each statement, the p-value associated with the hypothesis that the population mean of the responses to a particular statement exceeds 3.5. The results of these computations are recorded in the rightmost column of Figure 14. The responses to eight of the nine statements generated p-values less than 0.05, indicating that the students "tended to agree" with those statements. The statement whose p-value exceeded 0.05 will be discussed in the next paragraph.

The p-value of 0.078 for Statement 6 suggests less agreement with this statement than with the others. Statement 6 dealt with the usefulness of Lab 3 in helping students to become more proficient at deriving formulas. Statement 5, which related to Lab 1, posed a similar statement and attained a p-value of 0.043. Evidently, the students viewed Lab 1 as being more useful than Lab 3 at improving their derivation skills. It is worth noting, however, that students agreed with Statement 7 (p = 0.027), which stated that Lab 3 was useful for developing their design skills. Thus, despite the high p-value for Statement 6, all three labs were viewed by the students as having merit.

1. The illustrative "derivation" of superposition helped me to understand the basis for the theorem and how to properly apply it.

2. The example in which a practical voltage source was converted to a current source prior to erroneously interpreting the resistor current in the converted source to be the current supplied by the original source helped me to be able to avoid the pitfall illustrated by the example.

3. The previously mentioned example helped me to know how to properly apply the source conversion principle.

4. The lab exercise in which I selected the values of three resistors in a series-parallel circuit given specifications for a voltage and two currents provided me with a useful introduction to circuit design.

5. The lab exercise which required me to derive equations for the load resistance that draws maximum power from a practical voltage source and the power thus received helped to improve my ability to perform derivations.

6. The resistive T-network design exercise in which I derived the input resistance, output resistance, and $V_2/V_1$ equations helped me to become more proficient at deriving formulas.

7. The resistive T-network design exercise in which I derived the input resistance, output resistance, and $V_2/V_1$ equations prior to using them to select resistor values for the T-Network helped me to learn how to design a circuit to meet a set of specifications.

8. The usage of PSpice in the lab exercises helped me to better appreciate the role that simulation plays to verify that a circuit design meets specifications.

9. The lab exercises in which MATLAB was used to solve equations or plot data helped me to gain proficiency at using MATLAB for these basic tasks.

**Figure 13: Statements Used in the Lecture Techniques and Lab Activities Effectiveness Survey**
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<th>Statement</th>
<th>1 Strongly disagree</th>
<th>2 Disagree</th>
<th>3 Not sure</th>
<th>4 Agree</th>
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Figure 14: Survey Responses and Statistics

Figure 15: Response Percentages by Survey Statement
Conclusion

An excerpt from a circuits lecture has been presented as an aid to help students gain a better understanding of the superposition theorem. Three exercises from a circuits lab have been offered as tools for helping students to develop derivation and design skills. When surveyed, students "tended to agree" that these lecture and lab activities assisted their learning.

Bibliography