# 2006-752: INNOVATIVE EXPERIMENTAL PRACTICES IN VIBRATION MECHANICS

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## INNOVATIVE EXPERIMENTAL PRACTICES IN VIBRATION MECHANICS

#### **Abstract**

This paper presents the laboratory stands and the methodology that are used to provide the laboratory experiments as a supplement to the courses of dynamics and vibration. It is shown in what way the knowledge from the lectures can be used for analyzing the dynamics of mechanical systems or in what way the relations describing vibration can be used to determine some parameters of the system.

**Keywords:** dynamics, vibration, laboratory experiment, identification of parameters, verification of results

#### 1. Introduction

The class-room teaching of mechanics courses, especially of dynamics and vibrations, involves theoretical models and mathematical computations. It is difficult for students to imagine in what way the systems work, what mechanical properties they have, and predict their behavior if a force or moment is applied. It is difficult to evaluate the behavior of the systems if they are governed by non-linear equations. The nonlinearity of the system can be the result of large displacement, friction force, impacts between the elements of the system, and external forces. One option is to make a computer simulation and define the most important parameters as a function of time or the system's parameters. Usually our theoretical models do not consider all the factors that exist in a real world. If we investigate the vibration of the system with excitation it is usually taken as a harmonic one. But the excitation can be very complicated and there can be more sources of excitation; e.g., there are vibrations of the base on which the system is supported or the motor has eccentricity. It gives more degrees of freedom and more complicated mathematical models.

In this paper we are going to present in what way to provide experiments so the theoretical knowledge can be confronted with the results that students obtain in laboratory and what conclusion and observation should be drawn out. For the computer simulation Matlab software is used. Some of the problems we verify in laboratory and for this purpose we built special laboratory stands. They have given us good results and therefore we are going to present what is positive and what should be improved. For some of problems with vibration we develop a new theory that allows explaining much more easily the behavior of the system. The vibration forces (component of inertial force) exist, and they can change the behavior of the system, move the system to a position of equilibrium or change stability of the equilibrium. Using the vibration forces we are able to explain why some systems can balance themselves automatically and why the elements move in vibratory transport. We can also explain an apparent change of dry friction into viscous damping and why the upper position of the pendulum is stable if its pivot vibrates. This theory gives very good results for parametric vibration of the pendulum, automatic balancing, and vibratory transport, and they will be presented in the paper. If we introduce vibration forces then we can demonstrate to the students why the unstable upper position of the pendulum is now stable and why the elements move under the action of vibration <sup>2, 4</sup>.

The study of dynamics and vibrations is based on the theory of differential equations. They can be obtained from Newton's laws or Lagrange's equations. An energy method is preferred for modeling multi-degree-of-freedom systems. We try to find analytical solutions to the differential equations but it is not always possible. For most complicated differential equations (nonlinear) they cannot be solved in this way and therefore we use numerical methods. Now students, researchers, and engineers have some computational software that helps to overcome these difficulties.

Most vibrations in machines and structures are undesirable because of the increased stresses and energy losses. They should therefore be eliminated or reduced as much as possible by appropriate design. It is very important for high-speed machines, lighter structures, and machines that are very close to people e.g., cars, household goods, DVD, tape recorder. Vibration is one of the first courses where students learn to apply the knowledge obtained from mathematics and basic engineering science courses (statics, mechanics of materials, materials, kinematics, and dynamics) to solve practical problems.

## 2. Methodology

At the beginning of the laboratory class a short quiz is introduced to verify student preparation. Instructional lectures on each experimental method are given during a group's laboratory classes. A group consists of 5 students and they have one laboratory class of 2 hours duration per week. The ultimate goal of theses practical exercises is to provide hands-on experience for students in understanding and analyzing properties of vibrating mechanical systems and also the method of measuring and analyzing vibrations. A written report is required to be turned-in before the next laboratory class. There should be a description of the lab stand, method of providing the experiment, the final results, and discussion of the errors. The final grade is the result of the quiz, experiment activities, and the final report <sup>1, 4</sup>.

## 3. System with one-degree-of-freedom

Classes of vibration start with the simplest system, i.e., with one degree of freedom (1DOF). The method of analysis (equipment, acquisition of data and its analysis) for this system will be used in the next experiments. The basic relation for the system with 1DOF can be used in different ways. In the first experiment students have to define the natural frequency of the system or the stiffness of the spring. The system consists of a helical spring and mass hanging at the end of the spring. The vibration is generated by moving the mass from its position of equilibrium and release. The natural frequency of free vibration is described as

$$f_o = \frac{1}{2\pi} \sqrt{k/m} \tag{1}$$

The stiffness of the helical spring can be calculated from the formula  $k = Gd^4 / 8nD^3$ , where d is the diameter of the wire, D is coil diameter, the spring has n coils, and G is the shear modulus. The equipment (Vibration Meter 2511 and an accelerometer B-K) allows recording the acceleration, velocity or displacement, and the software SignalCalc ACE gives the spectrum of frequencies. During the experiment a natural frequency  $f_o$  of the mass is obtained. The stiffness of the spring can be calculated from (1) and it gives  $k = (2\pi f_o)^2 m$ . It can be compared with the result obtained from the formula for the helical spring. The third possibility to define the stiffness is to measure the displacement  $x_{st}$  under the gravity mg so the stiffness is  $k = mg/x_{st}$ . If

there is a difference between the results students have to explain possible reasons for it. The vibration of the mass is recorded and the two amplitudes are measured  $A_o$  and  $A_N$  for the instants  $t_o$  and  $t_N$ . The damping of the system can be defined from

$$c = \frac{2m}{t_N - t_o} \ln \frac{A_o}{A_N} \,. \tag{2}$$

The damping ratio is defined as  $\varepsilon = c/c_{cr} = c/4m\pi f_o$  and for small damping it can be calculated as

$$\varepsilon = \frac{1}{2\pi N} \ln(\frac{A_o}{A_N}). \tag{3}$$

For free vibration the laboratory stand allows one to determine the dynamic stiffness and the damping of the system.

In the next step a excited vibration of the system with one degree of freedom is analyzed – Fig.1.

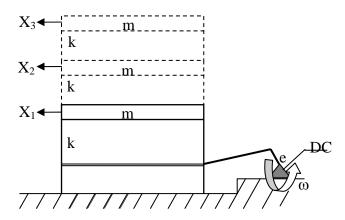


Fig.1. Sketch of building frame structure

The model approximates the behavior of a building (one store building) subjected to an earthquake [4]. The base is excited by DC motor and its speed is changed between 100 and 1600 rpm. The mass m vibrates in a horizontal direction. The stiffness of a column is defined by the relation  $k=12nEI/l^3$ , where E is the Young modulus, I is the moment of inertia of the cross section, l is the free length of the column, and n is the number of columns. Theoretical natural frequency of the floor is  $\omega_o = \sqrt{k/m}$  and it will be compared with the experimental magnitude.

Vibrations are measured by the same equipment as described earlier. The amplitude of excitation  $x_{st}$  can also be changed. When the motor is about 35% of its maximum velocity, the girder will resonate. By changing the motors speed and measuring the amplitude of the displacement or acceleration students can plot the diagram; *Amplitude vs Frequency of Excitation*.

For the system with 1DOF the theoretical relation between the amplitude and frequency is

$$A/x_{st} = 1/\sqrt{(1-s^2)^2 + (2\varepsilon s)^2}$$
  $s = \Omega/\omega_0$ , (4)

where  $\Omega$  is the frequency of excitation,  $\omega_o$  is the natural frequency of the system, and  $\epsilon$  -damping ratio.

The theoretical characteristic can be compared with the diagram from the experiment. By knowing the amplitude of resonance then the damping ratio of the system can be calculated from  $\varepsilon = 2x_{sp}/A_{res}$ .

With two laboratory stands students can learn about systems with 1DOF, their free and forced vibrations, and what properties the systems with 1DOF have; natural frequency, damping, resonance, and in what way each of the parameters influence the amplitude of vibration.

## 4. System with two degrees-of-freedom

A system with two or more degrees of freedom is practiced in next the step of the laboratory class. The laboratory stand from Figure 1 can be developed by adding more girders and columns. In this lab stand all masses and the stiffness of the columns are the same for each floor. Students have to calculate the natural frequencies of the system. Small vibrations of the system with 2DOF are defined by the following equations

$$m\ddot{x}_1 + kx_1 - kx_2 = 0,$$
  

$$m\ddot{x}_2 + 2kx_2 - kx_1 = 0.$$
 (5)

The natural frequencies are defined by the determinate

$$D(\omega) = \omega^4 - \frac{3k}{m}\omega^2 + (\frac{k}{m})^2 = 0.$$
 (6)

It gives

$$\omega_1 = 0.62\sqrt{k/m}$$
 and  $\omega_2 = 1.62\sqrt{k/m}$ .

To generate vibration one of the floors is pushed from the initial position, their vibrations are measured by the accelerometers B-K, and their frequency spectrum is given by the software SignalCalc ACE.

With a DC motor and its eccentricity an excited vibration can be generated. The amplitudes of each floor are measured with the same equipment as earlier described. The theoretical amplitudes and magnification factors can be calculated from (7) and compared with the amplitudes obtained from the experiment.

$$(k - m\Omega^{2})A_{1} - kA_{2} = 0,$$
  
-  $kA_{1} + (2k - m\Omega^{2})A_{2} = k\varsigma_{o}.$  (7)

From them one obtain

$$A_1/\varsigma_o = k^2/D(\Omega^2), \quad A_2/\varsigma_o = (k - m\Omega^2)/D(\Omega^2),$$
 (8)

where

$$D(\Omega^2) = m^2 \Omega^4 - 3km\Omega^2 + (km)^2.$$

From the data obtained from the experiment the diagram *Amplitude vs Frequency* can be plotted. When the amplitudes for both resonances are known then the damping for each mode can be calculated. Also the theoretical characteristics can be calculated and compared with the experimental one. Students have to write differential equations of the systems, provide necessary calculation to obtain the theoretical magnitudes, compare the theoretical and practical results, and provide the discussion of the results and the differences between them.

## 5. Systems with more degrees-of-freedom

The system from Fig.1 can be enlarged by adding more stores and columns. For the system with 3DOF or 4DOF similar experiments and calculations are provided.

More detailed description is given for the tape-recorder and the influence of vibration of the tape on linear and non-linear distortion of the sound. The driving system of the tape-recorder is an interesting object for analysis in what way the mechanical properties are connected with an electrical system. There are four rotation elements (Fig. 2) connected by elastic magnetic tape 1. The tape is pushed by the capstan 5 and pinch roller 7. The tape is fed from one reel 2 to another 3. A magnetic universal head 4 is positioned close to the tape.

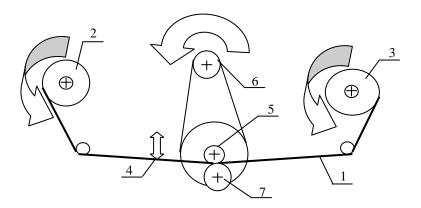


Fig.2. Driving system of the tape recorder 1-magnetic tape, 2-supply reel, 3-take-up reel, 4-magnetic head, 5-capstan, 6-motor, 7-pinch roller

En electric signal is induced in the magnetic head when the tape moves in front of the head 4. As a result of the tape's vibration, there is a coupling of the mechanical system with the electronic system causing modulation of the recorded or reproduced signal. The spectrum of the reproduced signal consists of the carrier spectrum line  $f_o$  and some sidebands displaced with  $f_I$  with respect to  $f_o$ . The results of the analysis of the signal U(t) show that there is linear and nonlinear distortion. The first one causes the amplitude of the carried sound to decrease. And because of the second one in the spectrum of the frequency, there are new sidebands that should not be there. When the recording head is supplied with a harmonic signal  $f_o$  then the reproducing head registers the signals, which have the carried signal  $f_o$  and the side spectral lines in the distance  $+f_I$  and  $-f_I$  from  $f_o$  as shown in Fig. 3. Each component of the tape vibration generates the sidebands in the reproduced signal. The problem is more complicated because we have the same situation first with signal recording and later with sound reproduction. The vibrations of the tape can be caused by an eccentricity of the rotating elements or friction between the tape and guide elements. Usually the vibrations of the tape are complicated and there are more sidebands.

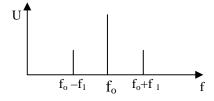


Fig.3. Spectrum of reproducing signal

Mathematical model of the tape recorder is given below

$$\mathbf{I} \cdot \ddot{\mathbf{q}} + \mathbf{c} \cdot \dot{\mathbf{q}} + \mathbf{k} \cdot \mathbf{q} = \mathbf{Q}(\mathbf{t}) \tag{9}$$

where I, c, k are the parameters describing the inertial, damping and elastic properties of the driving system. The generalized coordinates  $\mathbf{q}$  of each rotating element can be presented as a sum of two components; one from steady-rotation and the second one from vibrations. It means that  $q_i = q_{oi}(t) + \alpha_i(t)$ .

Vibration of the elements adjacent to the head gives a variable component of the tape velocity. Velocity of the tape in the front of the head from the angular velocity of two neighboring elements is defined by the relation

$$v = [\dot{\alpha}_4 r_4 (l_{24} - x_4) + \dot{\alpha}_2 r_2 x_4] / l_{24}, \tag{10}$$

where  $\dot{\alpha}_4$ ,  $\dot{\alpha}_2$  are the angular oscillation of the elements 4 and 2,  $x_4$  is the distance of the head from the capstan 5, and  $l_{24}$  is the length of the tape between the supply real and the capstan. The radii  $r_2$ ,  $r_3$  of the tape on both reels changes slowly in time. The tape with the signal 3150 Hz is reproduced by the tape recorder. The meter of signal fluctuation ND-960 is used to measure the tape oscillations. The meter ND-960 gives the average difference between the real velocity of the tape and the standard velocity of 4.76 (or 9.53 cm/s), and the spectrum of frequencies of the vibrating tape. By knowing the bends in the frequency spectrum the reasons for the tape's vibration can be established.

In the program of laboratory classes a car with more degrees of freedom is also analyzed. The car can be taken as a car body and two wheels without the tires' elasticity (4DOF) or with elasticity of the tires (8DOF). The excitation is caused by road roughness. The amplitude of vibration is a function of the car velocity, the amplitude of the roughness, and the length of the roughness wave. The analysis of car vibration shows that for some velocities of the car there are resonances.

## 6. Vibration of the pendulum

The analysis of parametric vibration of the pendulum is mathematically complicated and its properties can not be easily explained. If we introduce a vibration force that is generated by vibration then the problem is simpler for students. Under the action of vibration an additional force appears and it changes the behavior of the system. Under the action of vibration the lower position of the pendulum can be dynamically unstable and in the opposite upper position, which is statically unstable, it can be dynamically stable <sup>4,5</sup>.

The pivot O vibrates in a vertical direction in a harmonic way  $x_o = A\sin(\Omega t) - \text{Fig.4}$ . As the result of vibration there is an inertial force  $I_x = mA\Omega^2\sin(\Omega t)$ , where m, A, and  $\Omega$  are the mass of the pendulum, amplitude, and frequency of vibration of the pivot.

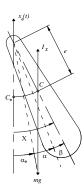


Fig.4. Pendulum with vibrating axis of rotation

Vibration of the pendulum with respect to its lower position of equilibrium is defined by

$$J\ddot{\psi} + c\dot{\psi} = mge\sin\psi + I_{r}e\sin\psi = F(\psi) + P'(\psi, \Omega t), \qquad (11)$$

where  $F(\psi) = -mge \sin \psi$ ,  $P'(\psi, \Omega t) = meA\Omega^2 \sin \psi \sin \Omega t$ , J is the mass moment of inertia and e defines the position of the mass center.

The forces  $F(\psi)$  and  $P'(\psi, \Omega t)$  have a different effect on the behavior on the pendulum. The first one changes slowly and the other fast in time. The solution of eq.(11) can be written as

$$\psi = \alpha(t) + \beta(\Omega t). \tag{12}$$

Fast vibration is defined by

$$J\ddot{\beta} + c\beta \cong P'(\alpha, \beta, \Omega t). \tag{13}$$

For small angles  $\beta$  the solution of (13) has a form

$$\beta(\Omega t) = \beta_0 \sin(\alpha - \beta)\sin(\Omega t - \varphi) = \beta_1 \sin(\Omega t - \varphi), \qquad (14)$$

where  $\beta_1 = \beta_o \sin(\alpha - \beta)$  is an amplitude of fast vibration and  $\varphi$  is a shift angle. For the frequency  $\Omega$  higher than the natural frequency of the pendulum  $\omega_o$  it can be taken as;  $\beta_o \cong meA/J$ ,  $\varphi \cong \pi$ .

The average force from fast vibration

$$P = \frac{1}{T} \int_{0}^{T} P'(\alpha, \Omega t) dt = -0.25 meA \Omega^{2} \sin 2\alpha.$$
 (15)

The slow vibration  $\alpha(t)$  depends on the moment given by the gravity and the average moment P

$$J\alpha = F(\alpha) + P(\alpha) = W(\alpha) = -meg(\sin\alpha + 0.25 \frac{A\Omega^2}{g} \sin 2\alpha).$$
 (16)

For a small vibration of the pendulum with respect to the position of a equilibrium  $\alpha_o$ 

$$\alpha + 2h\alpha + \omega_o^2 \alpha = -\frac{mge}{J} (\sin \alpha_o + 0.25 \frac{A\Omega^2}{g} \sin 2\alpha_o). \tag{17}$$

The position of the equilibrium can be determined from (17) if one takes W( $\alpha_o$ )=0 – Fig.5. It gives  $\sin \alpha_o = 0$  and from it  $\alpha_o = 0$  or  $\alpha_o = \pi$ .

In the position of the equilibrium derivative  $\frac{\partial W}{\partial \alpha}$  should be negative. It shows that the upper position of the pendulum is dynamically stable for  $\Omega > \frac{1}{A} \sqrt{\frac{2gJ}{me}}$ . If the pivot vibrates in any direction then the position of the equilibrium is different than zero or  $\pi$ .

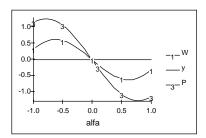


Fig.5. Vibration moment vs. the position of the pendulum for  $\alpha_o = \pi$  and  $\Omega = 200 \text{ rad/s}$  Fig.6 presents the vibration of the pendulum with respect to the upper position (a, b) and the vibration about the lower position which is unstable (c).

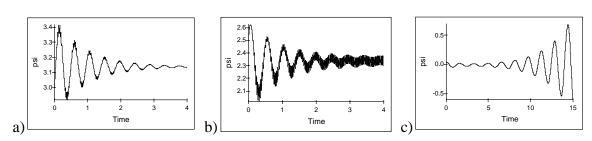


Fig.6. Vibration of the pendulum with respect to the upper (a, b) or lower positon (c) of the pendulum

a) for vertical vibration of the pivot b) for inclined vibration of the pivot, c) when lower position is unstable

## 7. Self-balancing system

Unbalanced rotors generate dynamic forces and vibrations that are dangerous for them-they can decrease the life span of the machine and produce noise that is harmful for people. The rotor with free elements inside it can balance itself automatically <sup>4</sup>. The balancing in one plan is shown in Fig.7. The vibration of an unbalanced rotor produces vibration forces that act on free elements (balls or rollers) and change their position with respect to the rotor. When free elements occupy the positions that balance the rotor then the vibration and also vibration forces vanish and these positions are the positions of the ball's equilibrium. For two balls these positions are defined by  $\alpha_{1f} = -\alpha_{2f} = \arccos(-Me/2mR)$ . The behavior of the rotor and the balls are governed by the following equations;

$$M\ddot{x} + c_x \dot{x} + k_x x = Me\omega^2 \cos\varphi + \sum_{i=1}^{N} m_i R_i (\omega + \dot{\alpha}_i)^2 \cos(\omega t + \alpha_i), \qquad (18)$$

$$m_i R_i \ddot{\alpha}_i = m_i \ddot{x} \sin(\omega t + \alpha_i) - F_i = I_i - F_i, \qquad i=1,2.$$
 (19)

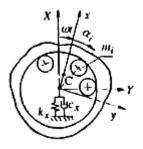
The vibration of the rotor can be taken as

$$x(t) = A_{xe} \cos(\omega t - \gamma) + \sum_{j=1}^{N} A_{xj} \cos(\omega t + \alpha_{j}).$$
 (20)

The vibration force acting on the ball decides in what way and what direction the ball will move and where is its position of equilibrium. The average value of the inertial force is

$$I_{i} = \int_{0}^{T} m_{i} \ddot{x} \sin(\omega t + \alpha_{i}) \cdot dt / T = -0.5 m_{i} \omega^{2} [A_{xe} \sin(\alpha_{i} + \gamma) + \sum_{j=1}^{N} A_{xj} \sin(\alpha_{i} - \alpha_{j} + \gamma)] . \tag{21}$$

If the rotor is balanced by one ball then the diagram of the vibration force as a function of the position of the ball is shown in Fig.8. One ball is able to compensate for the rotor unbalance if its static moment is equal to the rotor unbalance mR=Me.



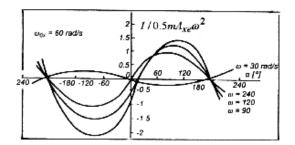


Fig.7. Unbalanced rotor with free elements

Fig.8. Vibration force

When the rotor speed is smaller than its natural frequency then the vibration force generated by the unbalance of the rotor tries to move the ball to the position close to the unbalance. The ball increases the total unbalance of the system. For the rotor velocity bigger than its natural velocity the vibration force pushes the ball to the positions opposite to the unbalance and these positions are stable. If the rotor has more degrees of freedom then each component of vibration generates its component force. The vibration force is a sum of forces from all components of the rotor's vibration. The principle of the automatic balancing can presents as it is shown in Fig.9.

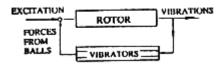


Fig.9. Principle of synchronous eliminator of vibrations

There is also rolling resistance, which makes it impossible to fully balance the rotor. In our laboratory we have two lab stands for balancing the rotor in one and two planes. Students have to do some experiments with different initial rotor's unbalance and determine the final positions of the balls. For the given initial unbalance they calculate the theoretical final positions of the balls for which the system is balanced. There is a difference between the theoretical and practical positions of the balls. The difference in positioning of the balls causes the system not to be balanced 100%. The resultant unbalance of the system can be defined from the relation (22). The residual unbalance is

$$\Delta Me = \sqrt{\left(\Delta Me_x\right)^2 + \left(\Delta Me_y\right)^2} , \qquad (22)$$

where the components of the unbalance are

$$\Delta Me_x \cong mR\cos\alpha_t(\Delta\alpha_1 + \Delta\alpha_2), \quad \Delta Me_y \cong mR\sin\alpha_t(\Delta\alpha_2 - \Delta\alpha_1).$$
 (23)

This is an example of the system that can reorganize itself in such a way that the balls eliminate the vibration of the system.

### 8. Vibratory transport

Vibratory forces exist for vibratory transport. These forces are responsible for the motion of a particle on a vibrating plane. In Fig. 10a there is an element on the plane that vibrates in a horizontal direction but a coefficient of friction between two bodies is different in right and left directions. The motion of the particle is defined with respect to the vibrating plane and therefore an inertial force is introduced. It helps to overcome the force of friction as shown in Fig. 10b. As a result of the inertial force and the friction there is a resultant force that pushes the element and it starts to move <sup>3</sup>. For the velocity different from zero the friction and resultant forces change in a more complicated way as shown Fig. 11.

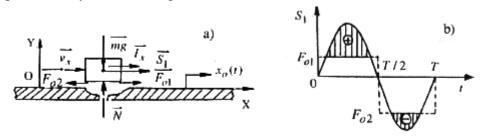


Fig.10. Element on vibration plane
a) forces acting on the element b) diagram of the resultant force

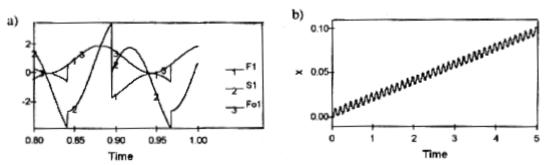


Fig.11. Forces acting on the particle and its motion a) forces b) displacement in time

Normally the coefficient of friction is the same in both directions and therefore to obtain different forces of friction in two directions the plane has two components of vibration- vertical and horizontal. In other applications the plane is inclined and its vibration is also inclined to the plane – Fig.12, 13.

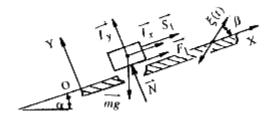


Fig.12. Element on the inclined plane and the component forces

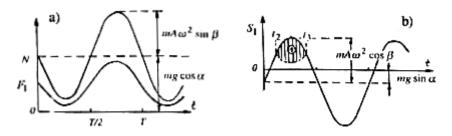


Fig. 13. Forces acting on the element on the inclined plane when the velocity is zero

Students have to define the mathematical model of the particle in the vibratory feeder from Fig.14. In the next step they have to write a computer program, do a simulation of the behavior of the element, and calculate its average velocity. They do an experiment with the feeder and determine the velocity of transportation.

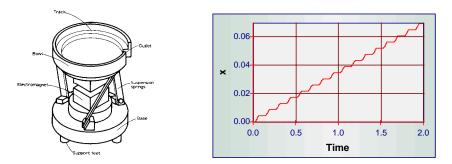


Fig.14. Vibratory feeder and the displacement of the element in time

From the results of the experiment and simulation they should define the parameters that are the most important and explain the difference between the theoretical and experimental results.

## 10. Vibration of string

In our laboratory we have four lab stands where the vibration of the systems with distributed mass and elasticity are analyzed. The first one is the electric guitar. Electromagnetic transducers change the vibration of the strings into electric signals which can be analyzed in the domain of frequency or time. The frequency spectrum of the string gives information about the harmonics that are generated by the string – Fig.15. If we know the fundamental frequency of the string then the tension can be calculated. By changing the tension the precise spectrum of harmonics for each string can be obtained. During the experiment students have to calculate the tension of

the string and determine the ratio of the mode amplitudes  $A_1/A_2$ ,  $A_1/A_3$ ,  $A_1/A_4$  if the string is plucked at different points.

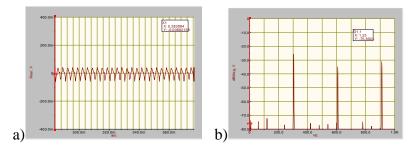


Fig. 15. Vibration of the string and its frequency spectrum

### 10. Bending vibration

In this experiment students have to determine the modes of vibration and their frequency for a leaf spring made of ferromagnetic material (Fig. 16) and later use this information to determine the Young modulus of the sample <sup>4</sup>.

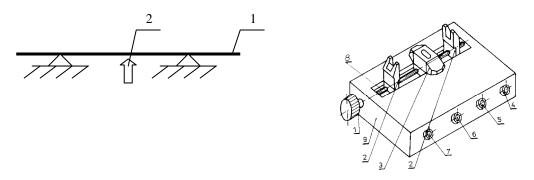


Fig.16. Lab stand for bending vibration 1-leaf spring, 2-electromagnetic exciter

The frequency for the first mode is  $f_1 = \frac{11.18}{\pi l^2} \sqrt{\frac{EI}{\rho F}}$ . Harmonic voltage of different frequency

is supplied to the exciter. When its frequency is equal to the natural frequency then the resonance occurs and the beam generates noise. Knowing the natural frequency of the sample then the Young modulus can be determined. For the leaf spring with the mass m, section  $b \times h$  and longitude l the Young modulus can be calculated from the relation

$$E = 0.9465ml^3 f^2 / (bh^3). (24)$$

Students have to provide experiments for 3 or 4 samples, calculate their Young modulus, define the errors of each component, and calculate the error of Young modulus  $\Delta E/E$  (%).

## 11. Summary

Cooperative learning, discussion model approach, lecture quiz approach and other relevant classroom teaching methods are considered very effective for the teaching of and learning

vibration topics. Specifically designed laboratory experiments enable students to appreciate the basic and practical concepts of dynamics and vibrations. Students have the opportunity to verify the knowledge taken from the lectures and compare the theoretical model with a practical one. At the end of the laboratory class students are able to see the difference or similarity between different mechanical systems, are able to recognize the most important parameters, provide the experiments, and define the errors of the experiment.

## **Biography**

- 1. D.W.Johnson, R.T.Johnson, E.J.Holubec, Cooperation in the Classroom. Edina 1993, MN:Interaction Book Company
- T.Majewski, Vibration forces in Physical Systems, 14th US Congress of Theoretical and Applied Mechanics, Blacksburg 2002, USA
- T.Majewski, O.Paleta, Modelado de transportes vibratorios, X Congreso Annual SOMIM, Queretaro 2004, Mexico
- R.Sokolowska, T.Majewski, Handbook for Laboratory of Vibrations, Publishing House of Warsaw University of Technology 2004
- 5. T.Majewski, R.Sokolowska, Vibration mechanics for the pendulum with oscillating pivot, 5<sup>th</sup> International Conference MECHATRONICS2004, Warsaw 2004, Poland
- 6. T.Majewski, V.Sudhakar, Experimental verification of properties of vibrating systems, Global Congress on Engineering and Technology Education 2005, Santos, Brazil