AC 2007-1922: INNOVATIVE MODEL ROCKET PROJECT FOR SOPHOMORE AEROSPACE ENGINEERING STUDENTS

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A Sophomore Model Rocket Project

Abstract:

In this paper, a model rocket project suitable for sophomore aerospace engineering students is described. This project encompasses elements of drag estimation, thrust determination and analysis using digital data acquisition, statistical analysis of data, computer aided drafting, programming, team work and written communication skills. The project is cost effective and provides good outcome measures.

Nomenclature:

C_D = drag coefficient
C_{DBT} = drag coefficient for body and tube
C_{DB} = base drag coefficient
C_{DF} = drag coefficient of fins
C_f = skin friction coefficient
d = body tube diameter
d_B = base diameter of rocket
g_0 = acceleration due to gravity at sea level
Isp = specific impulse
λ = length of the body
m_e = empty mass of rocket
m_{prop} = propellant mass
m_{pl} = payload mass
m_i = initial mass of rocket
m_f = final mass of the rocket
n = number of fins
S_{BT} = maximum frontal area of the body
S_F = wetted area of fins
S_W = wetted area of the body
s_b = distance covered during burn
s_c = total distance covered during coasting
Δs = distance covered during one iteration of coasting portion
$\Sigma\Delta s$ = total distance covered during coasting
t/c = thickness ratio of fins
V = velocity
ρ = air density, sea level

Introduction:

Most aerospace engineering curriculums contain an introductory course that introduces a sophomore student to the world of aerospace. Generally this course tends to be a broad introduction to terminology, basic aerodynamics, performance, propulsion and structures. In some programs, a hands-on project is assigned to the students to make the course more interesting and provide an opportunity to the students to use the fundamental knowledge they have gained in mathematics and physics in their freshman year. At Parks College the projects assigned may include the design, build and test of a glider for a specified set of constraints, or the assembly and test of a model rocket (Figure 1) for a specified set of constraints. In this paper, the details of the model rocket project are provided. The constraint is that the payload mass must be determined such that the rocket altitude is 100 feet. This allows the test firing of the rocket in a relatively small area such as a baseball field.

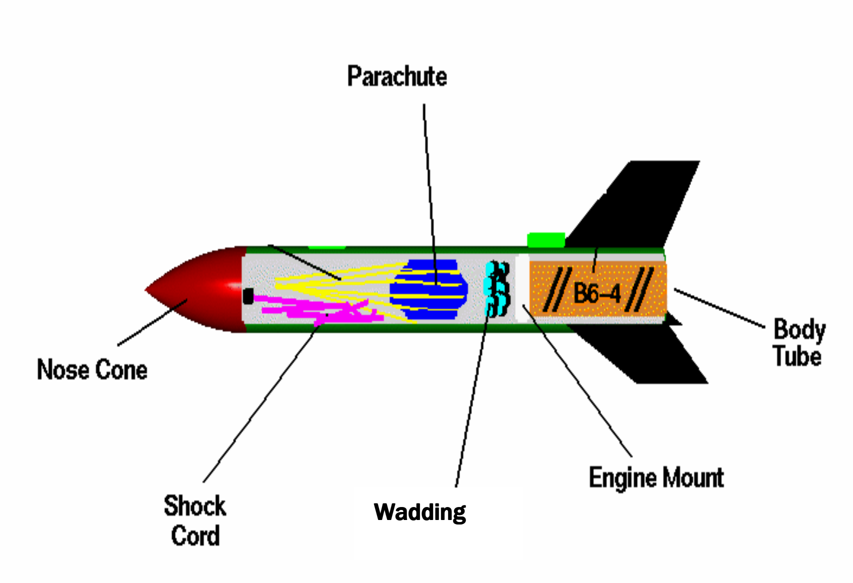

Figure 1 Diagram of a typical model rocket

It is noted that model rocket projects have been assigned as student projects since the 70's. [1]. At Parks, modern tools of data acquisition and analysis such as MATLAB and LabVIEW provide an opportunity for more sophisticated methods to be employed than was possible in earlier years.

Project statement:

For a given model rocket (such as Estes Skywriter No.2) and a given rocket motor (such as Estes B6-4) estimate the payload that the rocket can carry to reach an altitude of 100 feet. Assemble the rocket, including the payload, conduct a test and submit a report in AIAA format.

The students are provided some steps for the project as outlined below.

The Determination of Specific Impulse:

A simple static test rig is built consisting of a cantilever beam. The rocket motor is attached to the tip of the cantilever beam. A strain gage is attached near the fixed end. To calibrate the device three to four readings of the strain gage are recorded corresponding to static weights hung from the tip of the cantilever beam. It is noted here that most sophomores may not have had exposure to strain gage techniques but have had enough knowledge from physics (Wheatstone bridge circuit) to comprehend the experiment. A sample calibration curve is shown in

Figure 2 below. A regression analysis is done to obtain the relation between load and voltage. Students are required to show error bars as well.

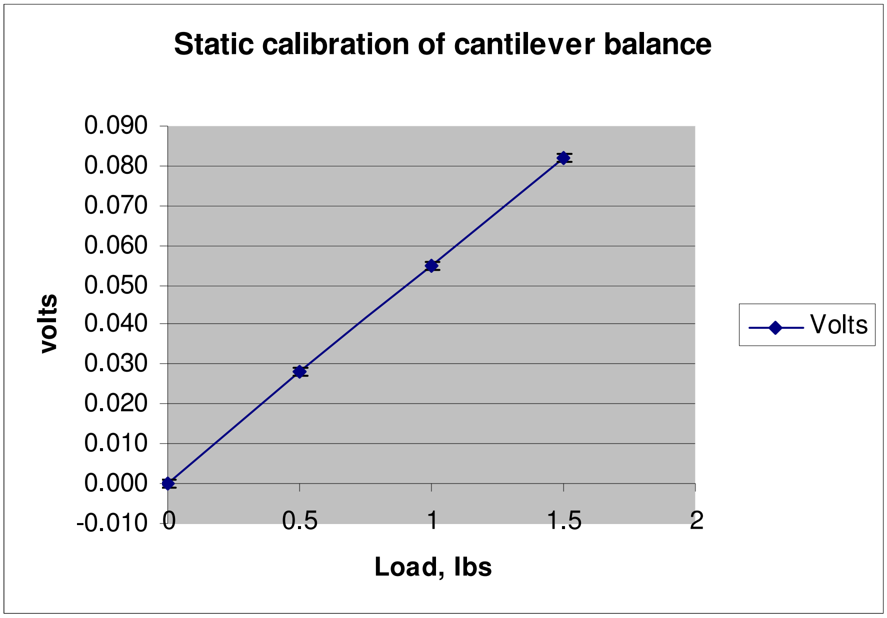

Figure 2 Static calibration of cantilever "load cell".

Then the rocket assembly is attached to the cantilever beam load cell. The rocket motor is fired and the output of the strain gage is fed to a data acquisition board to collect time history of thrust from the rocket motor using LabVIEW. A photograph of the rocket motor attached to the cantilever test rig is shown in Figure 3 below.

Figure 3 Rocket motor on test rig

A wiring diagram for acquiring time history of thrust is shown in Figure 4 below.

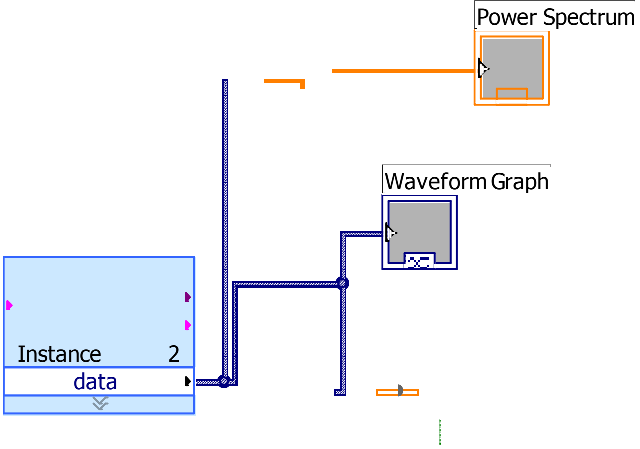

Figure 4 Wiring diagram in LabVIEW for thrust time history

A sample test run of the thrust data obtained from the above test is shown below, in Figure 5.

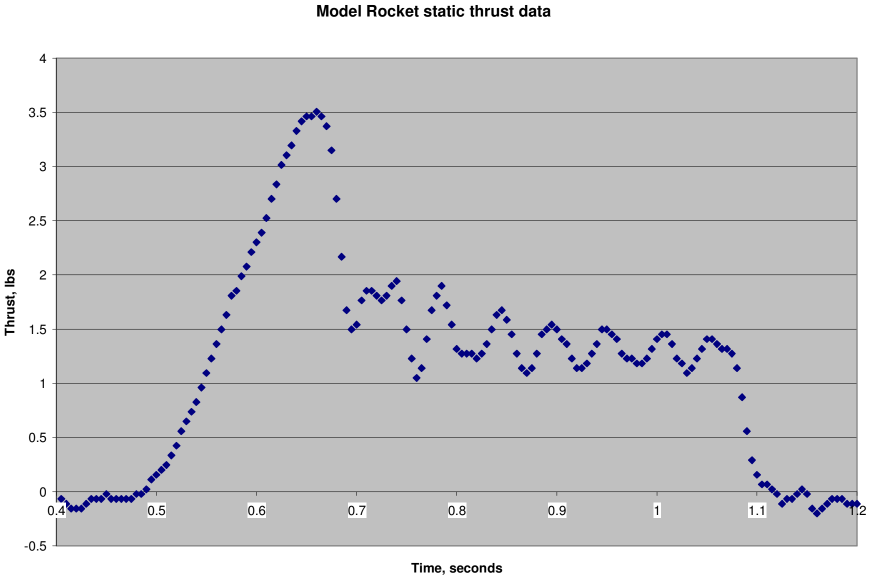

Figure 5 Thrust vs. time for rocket motor Estes B6-4.

Using numerical data of Figure 5, obtained in Lab View, students use either Simpson’s rule or trapezoidal rule to calculate the total impulse by calculating the area under the thrust vs. time curve. Then the specific impulse is obtained knowing the mass of the propellant (=mass of rocket motor before burn-mass of casing after burn) and duration of burn. An interesting data not relevant to the present study is the calculation of the first few resonant frequencies of the cantilever beam by a power spectral analysis of the thrust data shown in Figure 5 using LabVIEW.

Drag Coefficient Estimation:

By methods outlined in summary equations provided by Gregorek [1] or DATCOM [2] students estimate the drag coefficient for the rocket model. The summary equations outlined by Gregorek are repeated here for reference. The friction coefficient varies with Reynolds number and hence varies with velocity of the rocket. An average estimation of C_f must be determined by the student.

$$C_D = 1.05(C_{DBT} + C_{DB} + C_{DF}) \qquad (1)$$

$$C_{DBT} = 1.02\mathrm{C_f}\left[1 + 1.5/(\lambda/d)^{3/2}\right]\frac{\mathrm{S_W}}{\mathrm{S_{BT}}} \qquad (2)$$

$$C_{DB} = \frac{0.029}{\sqrt{C_{DBT}}}\left[\frac{d_B}{d}\right]^3 \qquad (3)$$

$$C_{DF} = 2\,\mathrm{C_f}\left[1 + 2\left(\frac{t}{c}\right)\right]\frac{\mathrm{S_F}}{\mathrm{S_{BT}}}n \qquad (4)$$

The determination of payload – method #1:

Error! Reference source not found., below, shows a typical mission profile for the flight of model rocket.

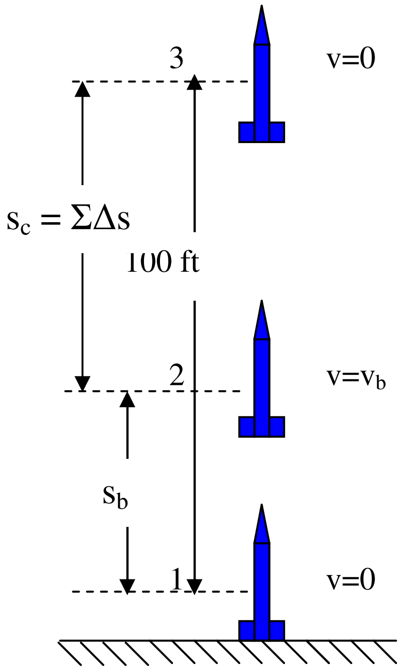

Figure 6 Mission profile for the model rocket

The height gained by the rocket during burnout, s_b is calculated by the simple Newton's laws of motion. Knowing the average thrust during the burn, and average mass of the rocket between points 1 and 2, one can calculate the average acceleration between 1 and 2 and thus the distance

s_b as well as velocity at burn, v_b. One can then use the rocket equation (see for example, Anderson [3]) given in Equation (5) to estimate the payload mass.

$$v_b = g_0 I_{sp} \ln\left[\frac{m_i}{m_f}\right] \qquad (5).$$

Here $m_i = m_e + m_{prop} + m_{pl}$ and $m_f = m_e + m_{pl}$.

Then, writing the equation of motion for the rocket for the coasting portion of the flight (free flight) including the effects of drag, one gets,

$$m_f \frac{dv}{dt} = m_f g_0 + \frac{1}{2}\rho V^2 S_{BT}\, C_D \qquad (6), \text{ from which}$$

$$\frac{dv}{dt} = g_0 + \frac{1}{2m_f}\rho V^2 S_{BT}\, C_D \qquad (7).$$

From Equation (7), one can write

$$\Delta v = \Delta t\left[g_0 + \frac{1}{2m_f}\rho V^2 S_{BT}\, C_D\right] \qquad (8).$$

Equation (8) is used successively after burnout to determine the decrease in velocity Δv in each time interval Δt and thus the incremental distance covered, $\Delta s = V\,\Delta t$ (where $V_1 = V_b - \Delta v$ for the first step and $V_n = V_{n-1} - \Delta v$ for subsequent steps) until the final velocity is zero. If the total distance covered by the rocket ($= \Sigma\Delta s + s_b$) is equal to 100 feet, the payload estimated from equation (5) is the appropriate payload. If not a revised payload is used and the steps repeated until the payload yields the desired height of 100 ft. From these calculations, students also observe that the drag force is not significant in the present study. Nonetheless, this exercise provides them with an appreciation for all the interdependent parameters.

The determination of payload – method #2 - numerical simulation solution:

Students also use MATLAB 7.0 to run a time domain simulation of the rocket (see Figure 7). Gravity, weight reduction due to fuel usage, thrust profiles and drag forces were all incorporated into the simulation. This method is preferred to hand calculations due to the time-savings, and over a closed-form solution due to the increased accuracy due to compensation for time-dependent variables such as drag, thrust and weight. A sample m-file is found in the appendix.

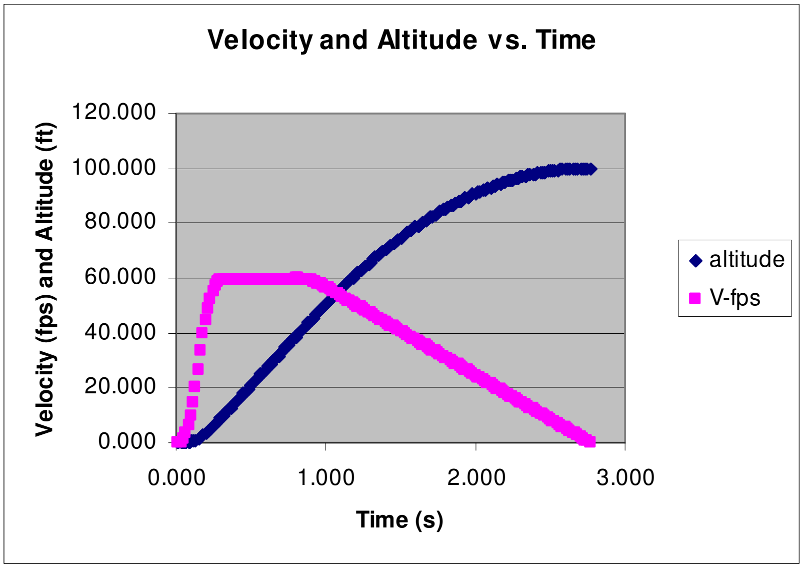

Figure 7 Sample Numerical Simulation Time History

The amount of payload to be carried is decided by running several iterations of the m-file flight simulation program. The maximum altitude achieved with each payload is recorded and arranged into a chart showing payload vs. maximum altitude (see Figure 8). From this chart, the proper payload to reach a maximum altitude of 100 feet is decided upon.

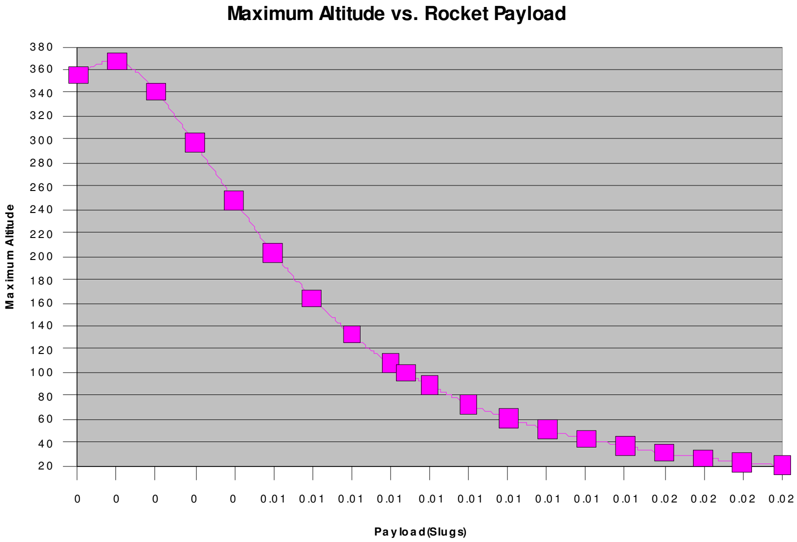

Figure 8 Sample Payload Determination Chart

Calculation of center of mass and mass moment of inertia:

Although these quantities are not used in the present project, this exercise is an interesting one for the students to calculate the center of mass knowing the mass of the components and the mass moment of inertia of the model rocket along the longitudinal axis. Students then verify the value of the location of center of mass by balancing the assembled rocket on a knife edge. They also verify the moment of inertia value by suspending the rocket by a long string and measuring the period of torsional oscillations. It is noted here that while students are introduced to the concept of center of mass, moment of inertia in calculus, physics and statics, simple exercises mentioned above reinforce these fundamental concepts.

Computer Aided Drawings:

An integral part of this project is for students to produce the component and assembled drawings using CAD software. The intent here is two fold. This exercise keeps the students current on their CAD skills and provides an opportunity to export the CAD drawings into a professional report. A sample CAD drawing is shown in Figure 9 and Figure 10 below.

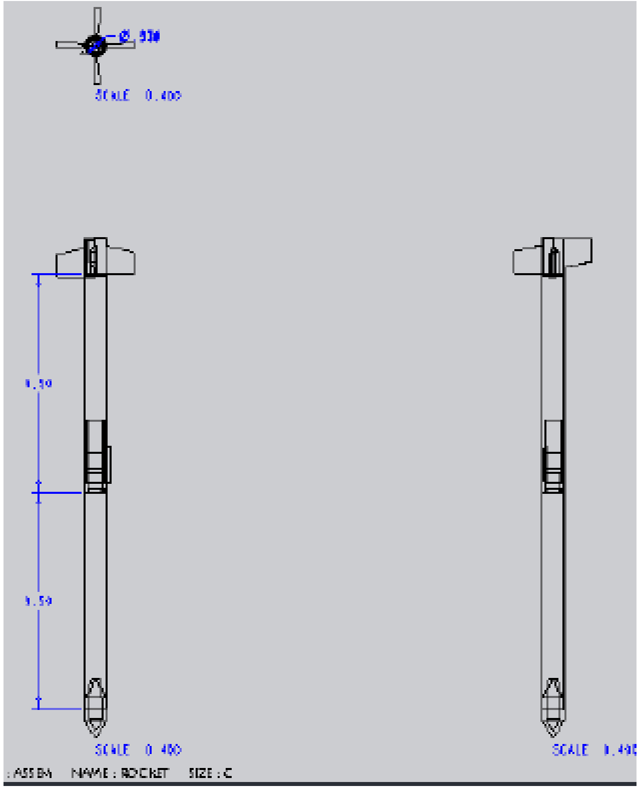

Figure 9 Pro Engineer three-view of model rocket

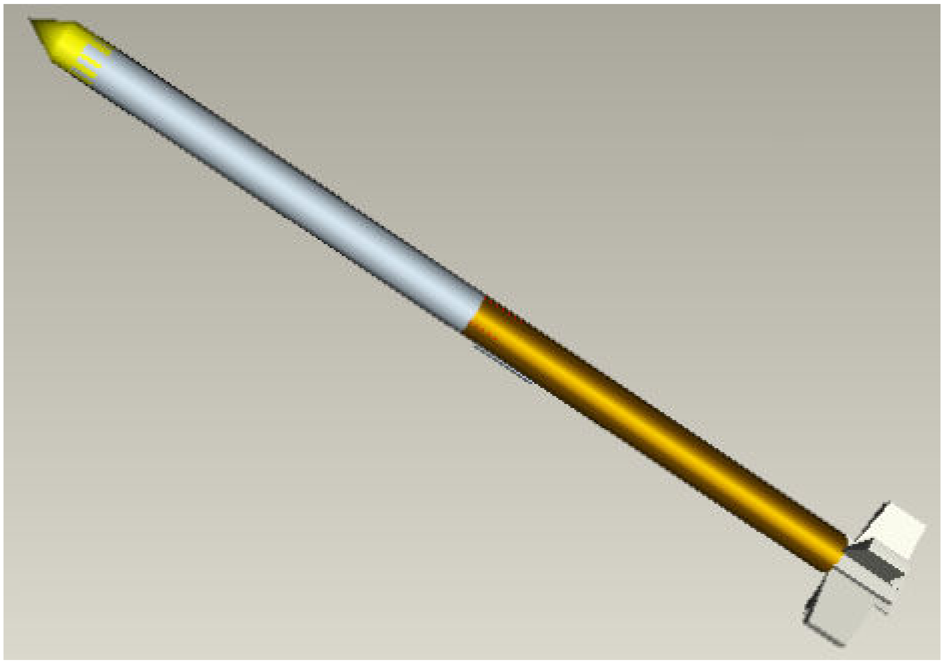

Figure 10 ProE Solid Model of model rocket

Flight test:

Students conduct a flight test with the payload (usually either sand or lead pellets) in the rocket model. Two sextants located orthogonally 100 feet apart from the launch pad measure the maximum altitude reached by the model rocket. In the report, students are asked to assess the errors in the analysis as well as flight test results. If the same model is used by all student groups, the students use the data to conduct a statistical analysis (usually using student-t distribution) to determine the confidence level of the data scatter.

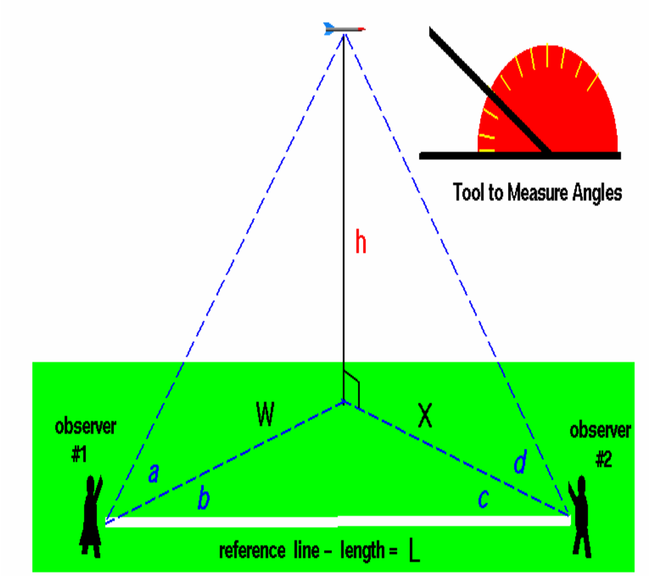

Figure 11 Experimental Determination of Rocket Height

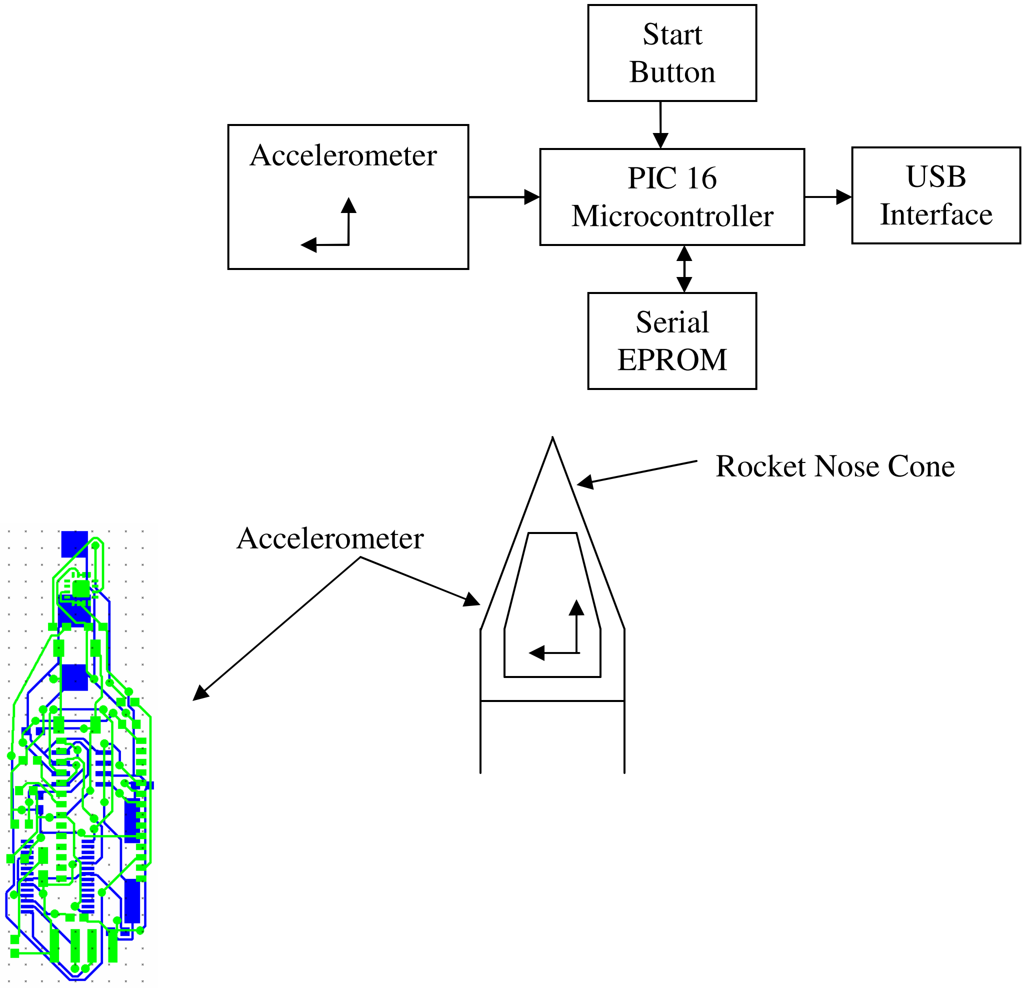

Figure 12 Accelerometer Data Recorder Package

During the flight test an onboard micro computer is capturing the output of two accelerometers. The accelerometer package chosen is the ADXL321 two axis gravity sensitive accelerometer. The accelerometer package is mounted so one axis measures acceleration in line with the rocket. The axis of the second accelerometer is orientated to measure a change in the gravity vector if the rocket diverges from a vertical trajectory. It is important to know if the rocket is flying vertical in order to know if gravity is affecting the measurement of acceleration in line with the rocket. The device has a memory for over 100 seconds of flight between readings and a battery life of a few hours. It has a start/reset button that resets the position the microcontroller is writing into the Serial EPROM to the beginning. The USB interface is used to read the recorded flight data into a windows PC. Once downloaded to the PC students will be able evaluate the flight of their rocket using onboard measurements.

Summary and future plans:

The simple project described in this paper introduces the student to some fundamental concepts such as drag coefficient, estimation of drag coefficient, specific impulse, data acquisition using modern tools, use of CAD for professional reports, and strain gages while reinforcing fundamental concepts such as the rocket equation, center of mass, moment of inertia, numerical integration, statistical analysis and report writing skills. The cost of the project is low enough that one or two students can be assigned to each group even in a relatively large class. Currently efforts are under way to outfit the rocket with an onboard accelerometer/real-time data collection package to record the acceleration time history. At the time of this writing the accelerometer hardware has been prototyped in the lab and a flight version is being constructed. When this is completed, students can use this data as an alternate method of estimating the height achieved by the rocket.

References:

1. Gerald M. Gregorek., Simplified Model Rocket Drag Analysis, IAA Student Journal, December 1973.
2. Ellison and Malthan, USAF DATCOM.
3. Anderson, *Introduction to Flight*, 5th Ed, McGraw Hill, 2005, p 688.

Appendix – **The determination of payload – numerical simulation solution:**

M-file

```
%  Rocket Simulation
%  March 2006
%  Ashley / Peter / Stephanie

%********  Constants  ************************
CD = 0.00;                        %(pounds) coefficient of drag
rweight = 0.06;                   %(pounds) total rocket weight without engine
eweight0 = 0.04;                  %(pounds) initial engine weight
eweight1 = 0.0213848394;          %(pounds) final engine weight
eweightd = (eweight0-eweight1)/0.860; %(pounds) change/slope in engine weight per time
pweight = 0.2455;                 %(pounds) payload weight
weight = rweight+pweight+eweight0;  %(pounds) total weight
mass = weight / 32.17;            %(slugs)
S = 0.008013;                     %(ft^2)
rho = .0023769;                   %(slugs/ft^3)

%********  Initial Values  ************************
q_bar = 0.0;            %initial dynamic pressure
Vdot = 0.0;             %initial acceleration
altitude = 0.0;         %initial altitude
time = 0.0;             %initial time
delta_t = 1.0/60.0;     %runs simulation at 60 Hertz
ascending = true;       %sets the rocket to ascending
Moutput = [0.0 0.0 0.0]; %sets the original output line
row = 1;                %sets the original output row number

%MatLAB Announcer
disp (' ')
disp ('Beginning Rocket Altitude Computations')
disp ('Please be patient...')

%  Thrust Data Table for B6-4 engine
Thrust_Y =    [0.00 00.688 02.457 04.816 07.274 09.929 12.140 11.695 10.719 09.240 07.667 06.488 05.505 04.816 04.620 04.620 04.521 04.226 04.325 03.145 01.572 00.000 00.000];
Thrust_time = [0.00 0.023 0.057 0.089 0.116 0.148 0.171 0.191 0.200 0.209 0.230 0.255 0.305 0.375 0.477 0.580 0.671 0.746 0.786 0.802 0.825 0.860 10000.000];

%  Run in a loop to simulate rocket flight from time = zero
while (ascending)
```

```
%  Compute Drag Force
Drag = CD*q_bar*S;

%  Determine Thrust Force
Thrust = interp1(Thrust_time, Thrust_Y, time);

%Convert thrust from Newtons to pounds
Thrust_lbs = Thrust * 0.2248;

%  Compute Acceleration
Vdot = (Thrust_lbs - Drag - weight) / mass ;

%  Integrate acceleration to update velocity
if (altitude < 0.01 & Vdot < 0.0)
  V_fps = 0.0;
else
  V_fps = V_fps + Vdot*delta_t;
end

%  Integrate velocity to update altitude
altitude = altitude + V_fps*delta_t;

%  Compute Dynamic Pressure
q_bar = 0.5 * rho * V_fps^2;

%  Update time
time = time + delta_t;

%Update Weight (assuming constant burn)
if (time < 0.86)
weight = weight - eweightd * delta_t;
end

%  Collect time history data
col = 1;
Moutput(row , col) = time;

col = col + 1;
Moutput(row , col) = altitude;

col = col + 1;
Moutput(row , col) = V_fps;

col = col + 1;
Moutput(row , col) = Drag;
```

```
 col = col + 1;
 Moutput(row , col) = weight;

 row = row + 1;

 %  Check for doneness
 if (altitude > 1.0 & V_fps < 0.0)
  ascending = false;
 else
  ascending = true;
 end

end

% output data point to file in EXCEL format
d = {'Time (s)' , 'Altitude (ft)', 'Velocity (ft/s)', 'Drag', 'Weight (lbs)'};
s = xlswrite('APSRocketData', d, 1, 'A1');
s = xlswrite('APSRocketData', Moutput, 1, 'A2');

%MatLAB Announcer
disp ('Calculations completed')
```