# Integrated Physics and Math course for engineering students: A first experience 

## Prof. Angeles Dominguez, Tecnologico de Monterrey (ITESM)

Angeles Dominguez is an Associate Professor of the Department of Mathematics within the School of Engineering at the Tecnologico de Monterrey (ITESM), Monterrey, Mexico. She obtained her bachelor degree in Physics Engineering from ITESM and achieved her doctoral degree in Mathematics Education from Syracuse University, New York. She is a member of the National Research System in Mexico (SNI). She is currently the president of the Red de Investigación e Innovación en Educación del Noreste de México (REDIIEN). Professor Dominguez has been a visiting researcher at Syracuse University and at the University of Texas at Austin. She teaches undergraduate courses in Mathematics and graduate courses in Education. Professor Dominguez is a thesis advisor on the master and doctoral programs on education of the Virtual University of Tecnologico de Monterrey. Her main research areas are: a) models and modeling, b) use of technology to improve learning and c) evaluation. In addition, Professor Dominguez is consultant for Texas Instruments (TI), she leads the group conTIgo T3 Latin America, and organizes and moderates webinars on the use of TI technology.

## Prof. Genaro Zavala, Tecnologico de Monterrey

Professor and Chair of the Physics Department at the Tecnologico de Monterrey. He is a member of the National Research System in Mexico, member of the Mexican Council of Educational Research; member of the American Association of Physics Teachers and has been member of the International Committee (2006-2008), president of the committee in 2008, member of the Philanthropy Committee (2011-2013) and member of the Membership and Benefits Committee (2012-2015); founding president of the American Association of Physics Teachers, Mexican section; member of the Consejo Nacional de Ciencia y Tecnología (CONACYT) Network on Information Technology, and coordinator of the Science Education Community of the Corporación Universitaria para el Desarrollo del Internet (CUDI). Professor Zavala teaches and advises master thesis and PhD dissertations in the Graduate School of Education of the Virtual University of the Tecnologico de Monterrey. Professor Zavala's research areas are a) students understanding of science concepts, b) use of technology in science education, and c) evaluation.

## Dr. Juan Antonio Alanis

Dr. Juan Antonio Alanis obtained his PhD in Mathematics Education from the CINESTAV of IPN (Mexico). He works at the Monterrey Institute of Technology, ITESM. His area of research is Didactic of Calculus. He is co-author of "Elements of Calculus: Conceptual Reconstruction for Learning and Teaching". This book was written based on didactic observations from Dr. Alanis doctoral dissertation.

## Integrated Physics and Math course for Engineering Students: A First Experience


#### Abstract

This paper presents the curricular design of an integrated course of Physics and Mathematics for first-year engineering students at a large private university in northern Mexico. The innovation includes the redesign course content, teaching strategies, classroom environment, technology, and evaluation.

Richard Feynman stated "The rules that describe nature seem to be mathematical". This curricular design uses mathematical laws to study physical phenomena so students can make strong predictions. This combination of mathematical and physical content uses mathematics as a powerful tool that offers the concepts and operations needed to analyze and understand physical models. The main pedagogical approach is modeling by instruction, in which students are actively engage in the processes of conjecturing, testing and thinking revision.

The classroom setting consists of round tables that accommodate nine students arranged in groups of three. This setting fosters group interaction, promotes communication, empowers students, and in turn facilitates the development of learning skills such as argumentation and self-regulation. The variety of technological tools and equipment available in the classroom facilitate students' investigation of various models that were constructed based on their own observations and measurements.


This is an ongoing project. This paper compares the grades of freshmen who took the integrated physics-math course and those who enrolled in separate math and physics courses. It will also present the authors' conclusions about engineering students' learning and attitudes towards physics and math, and competencies fostered by the curricular design and classroom setting.

## Background

Integrated math and sciences courses have been a goal for many universities ${ }^{1,2,3,4,5}$. Our attempt focuses on integrating math and physics for first semester engineering students. Traditionally, physics and mathematics content are taught as separate courses. At our university, the first attempt to close that gap was a redesign of the mathematics curriculum. This attempt resulted in a math course that uses physics contexts to trigger the use of math concepts ${ }^{6}$. However, the math and physics courses were still taught separately, and what often occurred was that the physics course needed to apply some math concepts that had not yet been taught. To bridge the gap between the math and physics content and the timing of when those concepts are needed, we decided to design a course for engineering majors that fully integrated the first-semester calculus course with the first-semester physics course and its corresponding lab. This paper provides a description of the course, its teaching strategies, the classroom setting, the characteristics of the participants and the academic results. We also offer our conclusions and proposed steps for the future.

## Course description

Our integrated physics and mathematics course for first-year engineering majors (Fis-Mat) uses the physics curriculum as its backbone, with mathematics giving support for idea-building and operations. In developing this course, we considered the findings of previous research research ${ }^{7,8,9,10}$ and added modeling as a principal teaching strategy, along with an innovative classroom that could double as a physics lab.

The primary goals of the Fis-Mat Project are: a) to improve students' abilities to make connections between physics and mathematics, b) to increase students' motivation to advance in their engineering studies, and c) to develop diverse competencies, such as critical thinking and the ability to do collaborative work.

There were four faculty members involved in the development of the Fis-Mat course. All four faculty member were from the School of Engineering, two each from the Physics and Mathematics Departments. Joint teaching began in the fall of 2012 with one physics and one math faculty member. Another faculty member observed the class, and all four met regularly to discuss the experience and decide on actions to be implemented. The major topics covered in the Fis-Mat course are presented in Table 1.

Table 1.
Topics covered in Fis-Mat corresponding to Physics I, Calculus I, and some from Calculus II (courses for engineering majors).

| Physics content | Calculus content |
| :---: | :---: |
| - Vectors <br> - Motion at constant speed <br> - Motion with constant acceleration <br> - Constant acceleration, quantitative <br> - Motion in two dimensions <br> - Energy <br> - Work <br> - Forces <br> - Forces of friction <br> - Momentum <br> - Forces of spring and circular motion <br> - Rotational and harmonic motion | - Linear model <br> - Quadratic model <br> - Derivatives <br> - Euler's method <br> - Noncontinuous functions <br> - Integral <br> - Line integral <br> - Applications of derivatives and integrals <br> - Applications of mathematical models |

The implementation strategy of using projects in engineering education has proven successful, as it fosters individual responsibility in students and creates an environment that is similar to that of professional engineering. Based on that, this integrated course of physics and mathematics for freshman engineering students proposes a final project for students to design, implement, document, and (orally) present. This project was designed to serve as a complex problem that would allow students to implement what they have learned, use knowledge from their field and broaden their understanding of these concepts. There were a number of criteria that students had to meet while working in the project. First, the students needed to make connections between physical and mathematical concepts and procedures taught in the Fis-Mat course in order to
solve the problem. Second, the students needed to present the project design before beginning setting the experiment and collecting the data. Last, the project had to be documented and presented to the entire class on the last day of classes. The report and the presentation were graded was grade on the results and creativity of the design project and the oral presentation.

## Teaching strategies

Educational research in the disciplines has shown that teaching based on active learning is more successful than teaching based on the teacher's presentation ${ }^{10}$. Richard Hake showed in a study of 6,000 students that, regardless of their level of knowledge or year in school at the start of the course, students learn more in active learning classes than in traditional ones ${ }^{10}$. Meyers and Jones ${ }^{11}$ argue that active learning encourages the student to participate in activities that promote cognitive modification or acquisition of knowledge.

For the Fis-Mat course, the main active learning strategy implemented was modeling instruction ${ }^{12,13,14,15}$. Students worked in formal groups of three to solve the given problems; they recorded their analysis on portable whiteboards. Then the entire class would sit in a big circle to discuss their findings. All students were able to see every other group's boards and were encouraged to ask questions of their peers (see Fig.1). Most of the physics worksheets were adopted from the material designed by the Physics Education Research Group lead by Eric Brewe at the Florida International University.


Figure 1. Students working in groups and whole class discussion.
Other teaching strategies implemented in the Fis-Mat Project are based on educational research in the disciplines, for example, models ${ }^{16}$ and tutorials for introductory physics, ${ }^{17}$ among others. These strategies have been designed by researchers of the discipline (physical or mathematical) working in academic departments at universities, and are based on rigorous research that has documented improved student learning through the use of these strategies. One strategy that has been very successful in classrooms with spaces for laboratory type work is presented by Thornton and Sokoloff ${ }^{18}$. This strategy, which requires investment from the beginning of the course, has been successful in student learning. At the beginning, students make predictions
about a physical situation, and then they use sensors to take measurements and verify whether or not their predictions were correct, thus often fostering cognitive conflict which drives learning. This strategy is implemented in a classroom designed to both foster communication and facilitate the use of equipment, thus promoting exploration and the group discussion. The room is described in the following section.

## Classroom setting

For several years Professor Robert Beichner, from North Carolina State University, researched and experimented with different classroom designs to enhance learning ${ }^{19,20}$. The SCALE-UP classroom is the result of this extensive investigation. Beichner and other creators of SCALE-UP classroom type have shown that active learning is generated in that environment very successful ${ }^{21}$. Among the results that have been documented is an increase in learning, and a decrease in the failure rate, mainly of women and minorities in the United States ${ }^{20}$.

The Fis-Mat course was held in a SCALE-UP type classroom that we named the "ACE" classroom (acronym for Student Centered Learning, or in Spanish, Aprendizaje Centrado en el Estudiante). The design is based on the research of Dr. Beichner in the SCALE-UP project ${ }^{19,20}$. There are many other SCALE-UP type classrooms in the United States and throughout the world. In general, all of these sites share the basic elements proposed in the original SCALE-UP, differing only in the number of tables (class size) and the technology they have. Due to their characteristics, these rooms are ideal for teaching the science such as physics, mathematics, chemistry and biology.


Figure 2. Classroom arrangement in diagram view and in real view.
For many years, mathematics has been taught from an exact science perspective, with a strong emphasis on memorizing theorems and proofs and repetition of exercises. However, in practice, what students are required to learn is how to analyze, summarize and transfer knowledge. Therefore, a new vision of teaching presents this exact science as a tool that uses various contexts and situations to trigger a deeper understanding of mathematical concepts and their applications. The ACE arrangement facilitates data collection of physical phenomena for analysis and interpretation. This allows for the full cycle ${ }^{16}$ of problem solving, going from a real problem to mathematical modeling to the interpreting of results within the context of what was being studied.

A very important precedent is that two of the faculty members involved in the Fis-Mat project have had experience in the implementation of active learning strategies in the ACE classroom and were familiar with the equipment (computers, graphing calculators, whiteboards, network connectivity, etc.). That experience gave them the confidence to perform with ease and professionalism in an innovative environment.

## Participants

For this first experience, participants in the Fis-Mat course were 20 freshman enrolled in three engineering programs. Twelve of the participants were enrolled in the Innovation and Development Engineering Program, six were Mechatronics Engineering Program and two were in the Industrial Engineering Program. All these programs have a common core of physics and math courses for the first semester.

## Curriculum assessment

## Academic achievement in Physics

The Physics Department in our University has a departmental final exam that consists of 20 multiple-choice questions in which all the topics of the course are covered (left column of Table 1). Half of the questions are conceptual and half are procedural. The department faculty has worked for several years to refine and modify the questions in order to create four equivalent versions of the test. The questions are designed so that the incorrect choices represent misconceptions or main difficulties students have. The sources used for designing the questions are found in much of the physics education research literature.

The final test is administered to all students at the same time with the versions of the test administered randomly among students. Students are limited to using a pencil, an eraser and a simple scientific calculator (no graphing calculator or CAS calculator allowed). The test is graded by an automatic optical system that sends results to instructors that same day.

Table 2 shows the results of the final tests for the Fall 2012 semester. Three groups are being compared. The first column is the results for students who are either in the honors class or who are physics majors. The second column is the results for students' scores in the experimental group (Fis-Mat). The third column is the results for students' scores in the remaining groups.

Table 2.

| Mean and standard deviation of the final test scores for three different groups. |  |  |  |
| :--- | :---: | :---: | :---: |
|  | Honors/Physics <br> Majors | Fis-Mat | Other <br> groups |
| Mean | 77.2 | 65.5 | 59.8 |
| Std Dev | 17.2 | 18.8 | 19.9 |
| Number | 120 | 20 | 595 |

Students in the experimental group scored higher than those in the rest of the groups; however, they scored less than those in the honors/physics majors groups. We believe that these results are encouraging since this was the first implementation of the course.

## Academic achievement in Mathematics

The Mathematics Department in our University does not have a departmental final test. The comparison presented in Table 3 is among freshman students enrolled in the first calculus course in other semesters. For two groups (A and B) the final exam was exactly the same as the one administered in the Fall 2012 to the Fis-Mat students. For the other two groups (C and D), some of the problems were different, therefore, here we only present the result of the problems that were the same.

Table 3.
Percentages of correct response on the final test problems for five different math groups

|  | Fis-Mat <br> 20 students | 40 students | 40 students | 40 students | 40 students |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Prob 1 | $89 \%$ | $74 \%$ | $62 \%$ | $80 \%$ | $76 \%$ |
| Prob2 | $96 \%$ | $82 \%$ | $81 \%$ | $80 \%$ | $81 \%$ |
| Prob3 | $84 \%$ | $85 \%$ | $85 \%$ | $86 \%$ | $76 \%$ |
| Prob4 | $89 \%$ | $83 \%$ | $73 \%$ | $84 \%$ | $75 \%$ |
| Prob5 | $98 \%$ | $88 \%$ | $90 \%$ |  |  |
| Prob6 | $63 \%$ | $53 \%$ | $59 \%$ |  |  |
| Prob7 | $88 \%$ | $76 \%$ | $65 \%$ | $74 \%$ | $69 \%$ |
| Prob8 | $93 \%$ | $63 \%$ | $59 \%$ | $66 \%$ | $58 \%$ |

The students in the Fis-Mat course were able to respond to the same exam and in many problems do as well or better than those students in a regular calculus course. This is encouraging because during this initial experience, students were able to learn and apply the main calculus concepts, despite the fact that the instruction placed less emphasis on typical math problems and more on the applications.

## Collaborative attitude

During every session of the Fis-Mat course, students were asked to work in groups. Accordingly, two of the four partial exams included a collaborative portion and the final project was also collaborative. Students were asked to respond to a 45-question survey on how they perceived themselves in collaborative work. We are in the process of analyzing the responses, but in general the results show that most of the Fis-Mat participants consider themselves to be part of a group, value their peers' comments, and care (academically) for each other.

## Discussion and Conclusions

In general, the participant faculty members were pleased with the experience. We have taken note of aspects that need improvement and of others that will be retained for future implementations. Moreover, we are revising and adjusting the activities so that the course content is presented smoothly, with fewer abrupt changes between topics.

The general comments of the students are positive and reassuring that this was a good learning experience. Students commented that the reduced boundaries between physics and math helped them to better understand the application and need for the math content.

We believe that in future implementations we will do better and that students' learning of physics and mathematics will significantly improve. We are confident that with the data collected, we will be able to identify some other desirable skills, such as problem solving strategies. Moreover, we are planning on following the students to observe whether some of the strategies learned in the Fis-Mat course are still being used and we hope to reunite students for another interdisciplinary experience in some other of their courses.

## Acknowledgements

The authors acknowledge the support received from our Institution through a research chair grant CAT140 and the School of Engineering. We also recognize and express our appreciation to Eric Brewe and his Physics Education Research Group for sharing material they have developed as support for teaching a Physics course using Modeling Instruction.

## Bibliography

1. M. W. Ohland, R. M. Felder, M. I. Hoit, G. Zhang, T. J. Anderson, "Integrated Curricula in the SUCCEED Coalition", Proceedings 2003 Annual Meeting of the American Society for Engineering Education, ASEE (2003).
2. J. Carpenter, "Integrating Calculus and Introductory Science concepts", Proceedings 2007 Annual Meeting of the American Society for Engineering Education, ASEE (2007).
3. L. Gentile, L. Caudill, M. Fetea, A. Hill, K. Hoke, B. Lawson, O. Lipan, M. Kerckhove, C. Parish, K. Stenger, and D. Szajda, "Challenging Disciplinary Boundaries in the First Year: A New Introductory Integrated Science Course for STEM Majors", J. College Science Teaching, 41(5), 2012, pp. 44-50.
4. M. L. Temares, R. Narasimhan and S. S. Lee, "IMPaCT - A Pilot Program", 1996 ASEE Annual Conference Proceedings, ASEE (1996).
5. D. Pines, M. Nowak, H. Alnajjar, L. I. Gould, D. Bernardete, "Integrating Science and Math into the Freshman Engineering Design Course", Proceedings 2002 Annual Meeting of the American Society for Engineering Education, ASEE (2002).
6. P. Salinas, et al, "Elements of Calculus; Conceptual Reconstruction for Learning and Teaching", Mexico: Trillas (2005).
7. T. Dray, B. Edwards and C. A. Manogue, "Bridging the gap between mathematics and physics", (2008).
8. M. S. Sabella and E. F. Redish, "Literature Review: Student Understanding in Calculus", 2002. Retrieved on august 24, 2007 from http://www.physics.umd.edu/perg/math/calc.htm.
9. J. Martínez-Torregos, R. López-Gay and A. Grass-Marti, "Mathematics in Physics Education: Scanning Historical Evolution of the Differential to Find a More Appropriate Model for Teaching Differential Calculus in Physics", Science \& Education, 15, 447-462 (2006).
10. R. R. Hake, "Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses", Am J of Phys, 66(1), 64-74 (1998).
11. C. Meyers, and T. Jones, Promoting active learning: Strategies for the college classroom. San Francisco, CA: Jossey-Bass Publishers, (1993).
12. D. Hestenes, "Toward a modeling theory of physics instruction", Am J of Phys, 55(5), 440-454, (1987).
13. M. Wells, D. Hestenes, \& G. Swackhamer, "A modeling method for highschool physics instruction", Am J of Phys, 63(7), 606-619, (1995).
14. D. Hestenes, "Modeling methodology for physics teachers", Proceedings of the International Conference on Undergraduate Physics Education, College Park, MD: University of Maryland, (1996).
15. E. Brewe, "Modeling theory applied: Modeling instruction in introductory physics", Am J of Phys, 76(12), 1155-1160, (2008).
16. M. Niss, W. Blum, P. and Galbraith, " Introduction". En W. Blum (Ed.), ICMI Study 14: Applications and Modelling in Mathematics Education, New York: Springer, pp. 3-32, (2007).
17. L. C. McDermott, P. Shaffer, y Physics Education Group. Tutorials in Introductory Physics. Upper Saddle River, NJ: Prentice Hall, (2002).
18. R. K. Thornton, and D. R. Sokoloff, "Assessing student learning of Newton's laws: The Force and Motion Conceptual Evaluation and the Evaluation of Active Learning Laboratory and Lecture Curricula", Am J of Phys, 66(4), 338-352, (1998).
19. R. Beichner, J. Saul, D. Abbott, J. Morse, D. Deardorff, R. Allain, , et al, "Student-centered activities for large enrollment undergraduate programs (SCALE-UP) project". In E. F. Redish and P. J. Cooney (Eds.), PER-Based Reform in University Physics. College Park, MD: AAPT, (2007).
20. R. Beichner, "North Carolina State University: SCALE-UPW". In D. G. Oblinger (Ed.), Learning Spaces. Boulder, CO: EDUCAUSE, (2006).
21. Y. J. Dori, and J. Belcher, "How does technology-enabled active learning undergraduate students understanding of electromagnetism concepts?", J of the Learning Sciences, 14(2), 243-279, (2005).
