

## **2006-1198: INTEGRATING APPLICATIONS IN THE TECHNION CALCULUS COURSE: A SUPPLEMENTARY INSTRUCTION EXPERIMENT**

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# **Integrating Applications in the Technion Calculus Course: A Supplementary Instruction Experiment**

## **Introduction**

Science and Engineering students in the course of their academic studies and further carriers continue applying calculus as their professional tool. In higher education, the ability to apply mathematics has been recognized as one of the main learning outcomes required from graduates of the engineering programs<sup>1</sup>. The mission of mathematics education is more than to impart the knowledge of mathematical rules, theorems and procedures, but to develop the ability to put mathematical knowledge and skills to functional use in a multitude of contexts<sup>2</sup>. Applications and modeling are a central theme in mathematics education and mathematics education research. "Very many questions and problems, concerning human learning and the teaching of mathematics affect, and are affected by relations between mathematics and the real world"<sup>3</sup>.

There is an intensive debate on applications and modeling in mathematics curricula for engineering students. Many educators believe that applied mathematics skills can be developed in undergraduate mathematics courses, particularly in calculus<sup>4-6</sup>. Kumar and Jalkio<sup>7</sup> proposed a conceptual framework for teaching mathematics from the application point of view. It concerns the mathematical skills required by the engineering disciplines, mathematics courses for developing these skills, and relevant applied problems.

A number of mathematics with applications textbooks has been recently published<sup>8-10</sup>, which implemented the following principles:

- Topics are presented geometrically, numerically, analytically, and verbally.
- Formal definitions and methods evolve from the investigation of practical problems.
- The real world problems are open-ended and may have more than one solution.

This paper reports a study of applications-integrated Multivariable Calculus course at the Technion. In the study we developed and tested different methods of integrating applications in the calculus course without affecting its mathematical level and scope. The study examined the effect of learning applications on students' understanding calculus concepts and attitudes towards the course.

## **Educational concepts**

Cognitive psychologists noted that instruction should refer to individual characteristics of learners. The educational approach which coordinates student's abilities and teaching methods is Attitude Treatment Interaction<sup>11-13</sup>. ATI points that students can be convergent or divergent thinkers, short-term or long-term memorizers, extraverts or introverts, more or less confident, etc. ATI offers a variety of instructional methods and gives students opportunities to choose those which fit their learning styles. The educational approach emphasizes team-based inquires and project assignments in which the students can select their preferred learning strategies. The ATI indicated that integrating different instructional methods provided more students with opportunities of successful and motivated learning. It gave rise to substantial examination of different learning styles and approaches to address them in curriculum and instruction.

The two central aspects of the multiplicity of intelligence are cognitive performances and learning styles. Sternberg<sup>14</sup> defined three levels of cognitive performances: human adaptation to environment, learning and experimentation, and information processing. Sternberg moves the focus from formal learning processes to those emerged when studying and solving real practical problems. His theory supports the integration of applications in science education.

Gardner<sup>15</sup> pointed out a number of different intelligences which characterize different learning styles: linguistic, musical, logical-mathematical, spatial, body-kinesthetic, intrapersonal, and interpersonal. He aspired to build learning processes which implement multiple intelligences in order to fit learning methods to different students and provide harmonic development of their abilities. The Gardner theory directs us to integrate various teaching methods and emphasizes the value of experiments and applications.

Wilson<sup>16</sup> pointed that learning and knowing are integrally and inherently situated in the every day world of human activity and that the context in which something is learned is very important part of learning. The theory of Situated Cognition supports meaningful learning and enriches the learning process by providing practical experiences of real life situations<sup>17</sup>. Educational studies indicated that situated cognition is a way to "address difficulties students have in retention and generalization"<sup>18</sup>.

### **Education Design Experiment Methodology**

The term Education Design Experiment (EDE) denotes a certain type of educational research methodology, which utilizes the similarity of educational and engineering process<sup>19-20</sup>. The EDE objective is to engineer (create, test, and revise) a new learning environment and examine (implement, evaluate, and conceptualize) educational processes rising in the environment. In this section we consider the features of design experiments and their relevance for our study.

The education design experiment removes certain biases of the traditional educational study:

- The primary motivation for the experiment is to improve the educational process and examine efficiency of the proposed approach.
- The environment, instruction, and learning are studied as the whole.
- The investigated factors and research methods are goal-oriented and can be changed during the experiment.
- The EDE explicitly delimits the range in which its results are significant.

The EDE is an adequate framework for research in which the learning environment is developed along with the educational process and its characteristics largely determine the efficiency of learning. The design experiment can be conducted at individual, class, and educational program levels. Following Cobb et. al.<sup>20</sup>, the EDE consists of three stages: preparation, experiment, and retrospective analysis.

Preparations for a design experiment include the following activities:

1. Identifying learning behaviors, which can indicate the contribution of the proposed learning method and research instruments to be used for measuring the outcomes.
2. Determining the concepts and subjects, in which the proposed approach can lead to efficient learning. This can possibly affect developing new frameworks that match the approach.

3. Ascertaining, students' prior knowledge and attitudes and specifying their prospective progress in the learning subject.
4. Formulating the experiment's outline.

At the experiment stage, the primary objective is "to improve the initial design by testing and revising conjectures informed by ongoing analysis of both the student's reasoning and the learning environment" <sup>20</sup>. At the retrospective analysis stage, the central challenge is to generate a coherent conceptual framework that accounts the effects observed in the course of the experiment and gives directions for dissemination and future development of the proposed educational approach.

The reasons for grounding our study on the EDE methodology are as follows:

- Our primary research motivation was to test the effectiveness of teaching mathematics with applications as a way to improve learning in the Multivariable Calculus course.
- Dependency on the existing course constraints in relation to time, curriculum, assessment, involvement, and student access limited our opportunities of using conventional research frameworks.
- We developed, tested and revised the learning environment and methods of teaching applications in parallel with teaching the course.
- The study strived for drawing conclusions based on real experience.

These limitations and factors influenced the action research experiment framework of our study.

### Study Framework

The evolution of the Multivariable Calculus course outline at the Technion is presented in Figure 1. Conventionally, the course weekly schedule included four lecture hours and two hours of classes (Figure 1A). With development of the Technion Mathematics Web tutoring system the classes were reduced to one hour a week (Figure 1B).

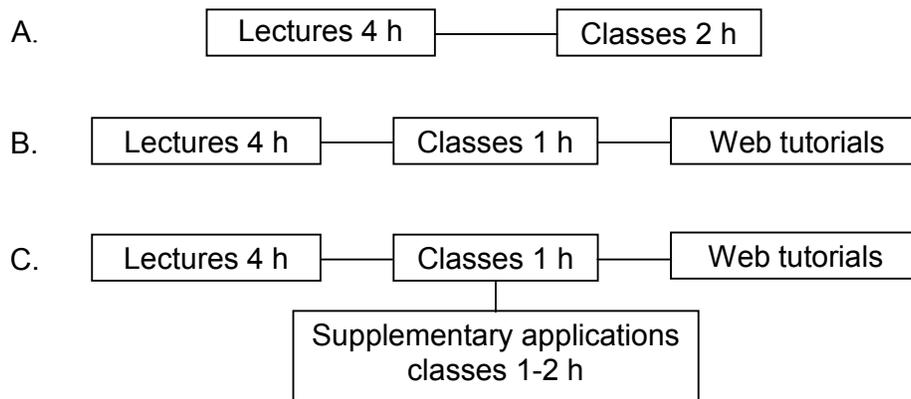


Figure 1. Multivariable Calculus outlines: A. Conventional; B. Computerized; C. Applications integrated

In our study the course schedule was extended by supplementary applications classes (Figure 1C). The applications classes were given voluntarily in parallel with teaching a conventional Multivariable Calculus class. The study included three teaching experiments which examined different forms of supplementary applications classes.

In the first (pilot) experiment Supplementary Applications Classes (SACs) were given by one of the authors (Aroshas) in the fall semester 2002-2003. The classes were designed for 30 students, but in practice about 75 students participated regularly. The students came from different science and engineering faculties. The SACs were coordinated with the main calculus class so that theoretical concepts and their use in real problems were studied at the same week. Condensed subject matter descriptions were given when needed.

The classes were followed up by a pilot study. In parallel we developed and tested in class applied problems and teaching methods. To characterize the applied problem solving skills we asked a number of experts from science and engineering faculties at the Technion for their opinion regarding the need for learning applications in the mathematics courses, and how important it is for their students. We also got their advice on how to present and analyze applied problems in the course.

The second (central) teaching experiment was conducted in the spring semester of 2003-2004 and designed in the following way. Two groups (experimental and control) of Multivariable Calculus students were created with the same number of students ( $N=33$ ). Both groups attended the same Calculus lectures with no emphasis on applications, but their classes were different. The control group had two hours per week conventional classes (without applications), while the experimental group had one hour conventional class and one hour SACs throughout the course. There were no significant differences between the experimental and control groups with regard to average grades in Calculus 1 and in the pre-course test, interest to study calculus with applications. Both groups represent the same engineering faculties. In this experiment the study focused on testing the effect of teaching calculus with applications through its comparison with the conventional teaching approach. The comparison related to learning achievements, understanding calculus concepts, and students motivation.

In the third (additional) teaching experiment two supplementary 2-hours SACs were given in the fall semester 2004-2005 and attended by more than 50 students from the multivariable calculus class. The goal of the classes was to introduce the calculus concepts of Lagrange multipliers and multiple integrals before their formal study in the lecture. In the sessions the concepts were recreated from the practical need and through the analysis of applied problems. In the follow-up we examined to what extent the SACs helped students to understand the concepts taught in the lectures.

### **Research Questions, Instruments, and Applied Problems**

The objective of our study was to develop methods and materials for teaching multivariable calculus with applications and examine the effect of applications on students' understanding calculus concepts, course achievements, and learning motivation. The research questions were as follows:

1. What is the effect of the applications-based mathematical instruction on learning achievements, understanding calculus concepts, and motivation in the course.

2. What are possible ways for integrating applications in the course while keeping constraints and limitations of the existing course.

### **Instruments**

In the first teaching experiment we used pre-course and post-course questionnaires. Four questions of the pre-course questionnaire were repeated in the post-course one. They tested student opinions related to the following aspects:

1. Anticipated effect of integrating engineering and science problems on understanding the calculus concepts.
2. Interest to solve calculus problems from the area of specialization.
3. Viewing the calculus capabilities as a condition to succeed in the area of specialization.
4. Interest to attend the applications motivated course in addition to the conventional calculus class.

The post-course questionnaire also inquired student opinions about the contribution of the three teaching methods used in the course: demonstrating mathematical problems of science and technology, constructing and solving mathematical problems in context, visualization through computer simulations.

The central teaching experiment applied pre-course and post-course questionnaires and tests. The pre-course questionnaire examined students' attitudes and learning styles, and collected personal information. This information and a One Variable Calculus applications test were used for creating experimental and control groups. The two post-course questionnaires were also conducted in both groups. The first one tested students' opinions about the value of tutoring sessions and their preferences in learning with applications. The second one was an understanding test which examined students' possible misconceptions in Multi Variable Calculus concepts. The middle term exam and the final course exam grades of students in the control and experimental groups were also collected.

In the third teaching experiment we used an attitude questionnaire which was conducted after each of the supplementary applications classes and the following lecture. In the questionnaire the students evaluated the contribution of the classes to understanding the calculus concepts given in the lecture.

### **Creating a Pool of Applied problems**

Through the literature search we found sources of applied calculus problems from various science and engineering domains which fit the scope and level of the course such as<sup>8</sup>. Other interdisciplinary texts<sup>21, 22</sup> and our professional experience served the sources of ideas for developing applied problems following the criteria proposed by Alsina<sup>23</sup>. The problems were discussed with highly experienced lecturers of Calculus, Science, and Engineering. Following are examples of problems given in the course:

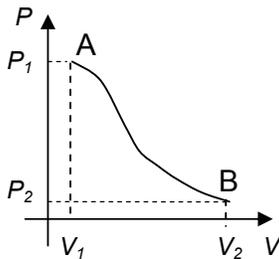
Problem 1 (Application of the double integral)

In a theater, the average waiting time in a ticket line is 10 minutes, and the average waiting time in a popcorn line is 5 minutes. Assuming that the waiting time in the two lines are

independent of each other, and their density functions are exponential  $f(t) = 0.1 \times e^{-t/10}$ ,  $g(t) = 0.2 \times e^{-t/5}$  for  $t \geq 0$  and  $f(t) = g(t) = 0$  for  $t < 0$ , then calculate the probability that a person coming to the theater will buy a ticket and popcorn in less than 20 minutes.

### Problem 2 (Application of the line integral)

The change of state of 1 mol mass of an ideal gas in a thermodynamic process from the initial state A ( $V_1, P_1$ ) to the final state B ( $V_2, P_2$ ) is described by the function  $T = \frac{P \cdot V}{R}$ . Here  $P$  is pressure,  $V$  is volume, and  $T$  is Kelvin temperature of the gas,  $R$  is the universal gas constant. The process is presented on the coordinate plain ( $V, P$ ) by a path  $AB$ . The increase in internal energy of the gas in the process is given by the line integral  $U = \int \frac{C_p}{R} \cdot P \cdot dV + \frac{C_v}{R} \cdot V \cdot dP$ , where heat capacities  $C_p, C_v$  are constants of the gas.



- Is the field  $\left( \frac{C_p}{R} \cdot P, \frac{C_v}{R} \cdot V \right)$  continuous on the path  $AB$ ?
- What condition provides that the field is conservative?
- Supposing that  $C_p \neq C_v$ , is the change of the gas state along the pass  $AB$  from point A to point B and back to A accompanied by heat transfer between the gas and the environment?

### Analysis of results

#### First teaching experiment

The pre-course and post-course questionnaires included four similar questions related to the following aspects:

- Anticipated effect of integrating engineering and science problems on understanding the calculus concepts.
- Interest to solve calculus problems from the area of specialization.
- Viewing the calculus capabilities as a condition to succeed in the area of specialization.
- Interest to attend the applications motivated course in addition to the conventional calculus class.

An additional question of the post-course questionnaire related to the contribution of the three teaching methods used in the course to understanding calculus concepts. The methods were: demonstrating mathematical problems of science and technology, constructing and solving mathematical problems in context, visualization through computer simulations.

The pre-test findings were as follows:

- The absolute majority of students pointed their high level of expectations from integrating applied problems on understanding the calculus and interest to solve problems from the area of specialization.

- A majority of the students (70.7%) recognized the connection between successes in the first-year mathematics and the majoring subjects.

The post-test indicated:

- The students did not change their opinion about the effect of integrating applied problems, and continued to be interested in solving problems related to the area of their specialization (as given by the t-test). This shows that the course answered students' expectations.
- There was an insignificant increase in the average evaluation of the importance of the calculus capabilities for success in the area of specialization (as given by the t-test).
- All the students reported that they would recommend attending the applications course to their classmates.
- All three teaching methods significantly contributed to the understanding of calculus concepts (as revealed by F-test). The contribution of visualization through computer simulations was the highest.

### Second teaching experiment

The pre-course questionnaire was used to divide calculus students into two "equal" groups: the experimental group and the control group. As a result, there were no significant difference between the control and experimental groups in One Variable Calculus grades, interest to learn mathematics with applications, results of the one-variable calculus applications test, in representation of different faculties and in learning styles.

Results of the comparison between the experimental and control groups after the course are summarized in Table 1.

Table 1. Average grades of course exams and SACs contributions

Item	Control group	Experimental group
Midterm exam grade in Calculus 2	57.2	66.1
Final exam grade in Calculus 2	62.1	68.4
Contribution to understanding calculus concepts	3.96	4.66
Contribution to stimulating interest to calculus	3.52	4.03
Contribution to studying other disciplines	3.15	4.12
Contribution to presenting practical aspects	4.12	4.51
Contribution to broadening horizons	3.78	4.15
Contribution to exercising course material	3.66	4.60
Contribution to interest in application-driven learning	3.51	4.42

The groups were compared with regard to their average grades in the midterm and final course exams, and their answers to the post-course questionnaire. The average grades are presented in the first two rows of the table. The next rows show average grades given by the students in the post-course questionnaire for contributions of the course. These grades were given in the five-point Likert scale with 1 meaning no contribution and 5 indicating high contribution.

The features indicated by the table are as follows:

1. Average exam grades of the experimental group were 8.9 points higher than that of the control group in the midterm exam and 6.3 points higher in the final exam. The differences between the groups in both exams were found significant (t test).
2. The experimental group gave a significantly higher evaluation for the contribution of the course in relation to all the aspects mentioned in the table.
3. The two groups are of the same opinion about the need of addressing different learning styles and integrating applied problems in the course.

It worth mentioning, that in the teacher evaluation survey of the Technion Centre for Promotion of Teaching, average grades given by the control and experimental groups to the calculus teacher were similar (4.85 and 4.86 out of 5). This fact supports the view that the advantage of the experimental group was a result of studying calculus with applications.

The calculus understanding test consisted of 14 theoretical and applied questions related to the following concepts: equipotential lines, directional derivative, gradient, tangential plain, Lagrange multipliers, and extremum. The test was validated by two experienced practicing lecturers of the Technion Multivariable Calculus course.

Significant advantage of the experimental group over the control group in the percentage of correct answers to the test questions was indicated. For some of the questions correct answers were given by 70-80% of students in the experimental group vs. 25-35% in the control group. Typical student reflections to a question on the contribution of studying applications for understanding calculus concepts were as follows:

*"Through applications I grasped the complex calculus concepts".*

*"The impact of a one-hour application session is the same as of a regular (two-hour) tutorial".*

*"Sorry that applied problems were not given in the first Calculus course".*

The follow-up study using statistical and qualitative methods indicated that the groups which studied calculus with applications had significant advantage in the half-course and final exam grades. The students mentioned the high contribution of applications to understanding calculus concepts and their interest in the subject.

## **Conclusions**

In our ongoing study, an applications motivated course was given to freshmen engineering students as an optional extension of the Technion Multivariable Calculus course. Preliminary findings of the pilot study are as follows:

- The applications-motivated calculus course got highly positive evaluation by the students, as indicated by their diligent voluntary attendance, post-course questionnaires, and reflections.
- The majority of the students reported that the applications helped them in understanding theorems and formulas and that they were interested in the applied meaning of the mathematical concepts.
- The absolute majority of the students supported integrating applications in the Multivariable Calculus course, recommended to continue teaching the applications course in the future, and even extend it.

This positive feedback motivates the authors to continue the study by constructing families of calculus problems in science and engineering and measuring the effect of the applications motivated learning through comparison of experimental and control groups.

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