

Integrating fracture mechanics into undergraduate design

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Abstract

This paper describes the work that is under progress to develop instructional material and a computer program that will automate linear stress analysis in two-dimension. The computer program can be used in a variety of ways to educate students about the stress behavior near a variety of stress raisers, though the initial effort will be near cracks and other stress raisers in homogenous materials. By automation of stress analysis it is implied that the user will not need to know the methodology used in the computer program and will not need to know how to create the mesh needed to solve the problem. The user would only describe the boundary value problem, which will be facilitated by a user friendly interface and all analysis decision will be made inside the program. Development of e-handbooks on stress concentration factors and stress intensity factors will further reduce the demands on the user in describing the boundary value problems thus shifting the focus from analysis to use of analysis results in design.

1. Introduction

The importance of fracture mechanics, interface mechanics, and stress raisers in homogenous and composite materials is highlighted by the extensive research that has been and is being conducted in each of these areas. Books have been written and there are journals devoted exclusively for publishing research on each of these topics. Yet this impressive pool of knowledge has had little impact on the undergraduate engineering education and hence had little impact on industrial practices. One reason for the small impact of mechanics research on undergraduate education is the required mathematics to teach the mechanics concepts related to fracture, interface stresses, and stress gradients in composites is well beyond the exposure that undergraduate get in a typical engineering curriculum. The mathematics that is so essential in explaining stress behavior near stress raisers is of lesser importance than the intuitive appreciation and understanding of stress behavior in creative *preliminary* structure and machine design as has been demonstrated by the use of stress concentration factor over the past hundred years. If the students could be taught to appreciate the stress behavior near cracks, material interfaces, holes, corners and load discontinuities in traditional materials (isotropic) and composites (anisotropic) and taught how to incorporate this understanding of stress behavior in design, then the door is opened to utilize and apply the research in mechanics to engineering applications.

The pioneering work of Inglis, Griffith, Westergaard, Williams, Irwin, Rice [1-6] established many of the fundamentals of fracture mechanics by 1970. Similarly the pioneering work of Goodier, Eshelby, Dundurs [7-9], the research in adhesive bonding [10] have established many of the fundamentals related to stresses near material interface. Stress concentration due to holes, notches, grooves, corners, sudden changes in cross-section etc. are long recognized [11] as important in any strength based design whether the material is traditional (isotropic) or composites (anisotropic). Journal papers, handbooks [12-16] and monographs [17-23] are testimony to the vast amount of information that is available on fracture mechanics, interface mechanics, and stress raisers in composite materials. But this information is usually beyond most undergraduate engineering students who in their undergraduate curriculum have not been exposed to the required mathematics and the theory of elasticity.

The challenge of transforming mechanics research into information that undergraduate and practicing engineers can use in design was successfully met by educators in the last century. The key to the educational success in the past century was the recognition that the mathematics that is so essential in explaining stress behavior near stress raisers is of lesser importance than the intuitive appreciation and understanding of stress behavior in *preliminary* creative design of structures and machines. Though the state of stress near stress raiser is complex, one can convey a sense of its behavior in a phenomenological manner using the concept of lines of force. The second element in the educational success of the past century was the use of empiricism in the form of charts and tables of stress concentration factor. These charts and tables permitted extrapolation of nominal stress results from simple models taught in a ‘mechanics of materials’ course to obtain a maximum stress value for purposes of strength design. Once a new preliminary design configuration is created, then it can be analyzed, refined, optimized using experimental and numerical techniques. The third element in the educational success of the past century was the simplicity and unobstructive nature of the concept of stress concentration factor in design. Fundamental concepts in the introductory course of mechanics of materials could be taught without the distraction of the complexity of stress states in regions of stress concentration. Thus the three elements of past educational success of incorporation of mechanics research into undergraduate design were:

1. Phenomenological explanation of complex stress states.
2. Empirical modification of stress values from simple models to predict failure.
3. Simplicity and unobstructive accounting of complexity that does not distract the student from the important mechanics concepts.

The work under progress aims to replicate the above three elements in a more modern and significantly more effective manner in incorporating mechanics research in fracture and stress raisers in homogenous bodies into undergraduate design. The phenomenological explanation of complex stress states can be shown visually, for example, students on computer screens can see dominant regions of tensile stresses and shear stresses in which cracks may grow in mode I and mode II to produce failure. The empiricism of handbooks on stress concentration factor could be used only to predict maximum tensile stress, but now the software will give all the stress components making it possible to account for the differing tensile, compressive, and shear strength. The simplicity and unobstructiveness will lie in the fact that the stress analysis is automated and will require little knowledge to use it.

The key to replicating past century's success in incorporating mechanics research into undergraduate education as elaborated in the above paragraph is the automation of stress analysis. The process of automating stress analysis by Boundary Element Method (BEM) was described by the author in reference [33]. A brief description of the BEM software is given in Section 3. In the next section the instructional material that will be developed to use the BEM software is described.

2. Instructional material

There are two types of instructional material that is being developed: the class notes and the manual with exercises and problems for computer laboratory.

Class notes: Class notes will cover the basic concepts of fracture mechanics including but not limited to: fracture modes, stress intensity factor, critical stress intensity factor, fracture toughness, energy release rate, J-integral, displacement discontinuity, branch cracks, and stress singularity. Appropriate formulas will be presented but the emphasis will be on use of these formulas rather than the theoretical derivations. Underpinning assumptions of the formulas will be discussed in phenomenological sense to ensure that the limitations of these formulas are clearly understood.

A student survey¹ of reading habits showed that only 76% read text but 100% read numerical examples. For this reason, great attention will be paid in selecting the example problems and in presenting the solution. Each example problem will start with a section called *Plan* and end with a section called *Comments*. Developing a plan before solving a problem is essential for the development of analysis skills. Comments are observations deduced from the example highlighting concepts discussed in the text before the example. This format of 'example' presentation in effect utilizes numerics to teach concepts and exploits student predisposition to read examples.

Another predisposition of engineering students is toward applied work. Thus, it is critical for student motivation that the practical relevance of fracture mechanics concepts is shown in the text. Today's technology permits easy incorporation of photographs into text and will be used extensively in examples and post-text problem. The following example from textbook [24] on mechanics of materials highlights some of the points in this section.

Example 1: The propeller shaft of a submarine is subjected to a tensile axial stress and a torsional shear stress when the submarine reverses its direction. The propeller shaft of a submarine on display shown in Fig.1 showed a crack at an angle of 27° to the axis of the shaft. At the point where crack was seen, the stresses are estimated as shown. The shaft material has a critical stress intensity factor of $140ksi\sqrt{in}$. If the submarine was still in operation, then at what crack length would you recommend that the submarine be pulled out of water for repairs, assuming: (a) the detected

1. The author conducted an informal survey of student reading and learning habits and incorporated the survey result to address student pedagogical needs into design of his book [24] on 'Mechanics of Materials'. Details of the pedagogical features can be found on the web page <http://www.me.mtu.edu/%7Emavable/book.html>.

crack could grow. (b) there is no pre-existing crack.

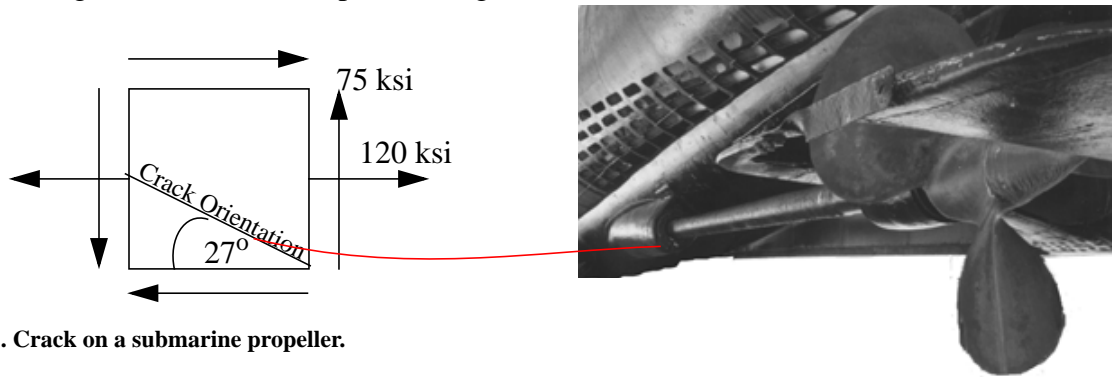


Fig. 1. Crack on a submarine propeller.

Students understand the state of stress shown. They can see that it will arise from combined axial and torsional load on a shaft. They can be exposed to the ideas of various modes of fracture, mixed mode, and the role of critical stress intensity factor. They can calculate normal and shear stress on the plane containing the crack and using mixed mode to answer part (a) of the question. They can also calculate principal stress one and answer part (b) of the problem. Problems of these type help integrate fracture mechanics with the material they already know. The photographs show the students the practical relevance of the subject material.

Computer laboratory manual: The primary focus of the manual will be on set of computer exercises to bring out important conclusions in the areas of fracture mechanics, for cultivation of intuitive appreciation of stress behavior in the vicinity of cracks and regions of stress concentration, and to provide empirical information for use of design of simple machines and structures. Consider a simplified linear analysis of mechanically fastened plate in which the mechanical fasteners is approximated as pin connection shown in Fig.2a. The three dominant modes of failure for metals are the bearing failure, tension failure, and shear out failure as shown in of Figs. 2b, 2c, and 2d. A particular failure mode depends upon the ratio of W/d and h/d .

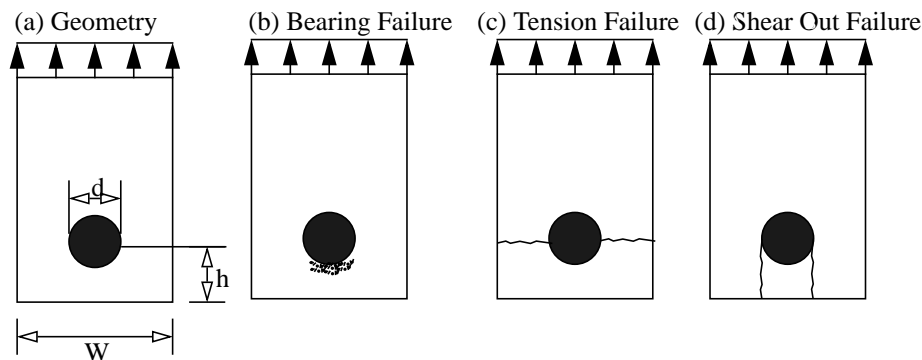


Fig. 2. Pin connection and failure modes.

For purpose of design of mechanically fastened members, plots of maximum load vs. h/d with W/d as a parameter are to be constructed. Plots can be created by conducting stress analysis starting with a defined geometry i.e., defined values of h/d and W/d . In a traditional analysis, equivalent von-Mises stress would be found and compared to failure stress from tension test to determine the failure load. To incorporate fracture mechanics¹ the students would be asked to use critical stress

intensity factor for mode I for tension failure and mode II for shear failure, and mixed mode if necessary. These plots can be used for designing simple structures as described in the Table 2.

Example 2: The landing wheel of the plane is modeled as shown to the right in Fig.3. Pin at C is in double shear and has an allowable shear stress of 12 ksi The allowable axial stress for link BC is 30 ksi. Using plots developed above, determine the diameter of pin C and the effective area of cross-section of link BC.

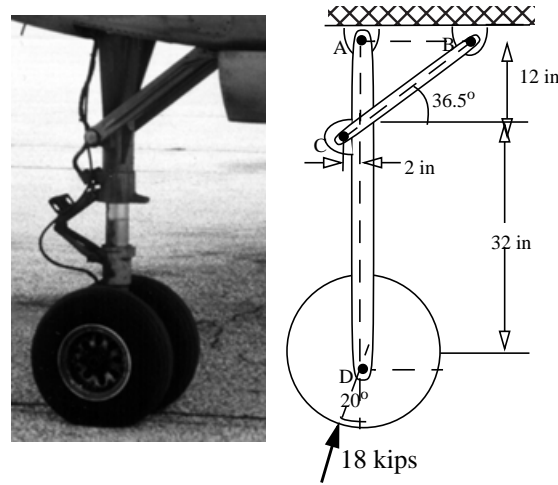


Fig. 3. Model of landing wheel.

In a manner similar to the above example, fracture mechanics with other types of stress raisers can be incorporated in design problems from mechanics of material textbook [24].

3. Boundary element method software

Boundary element method (BEM) does not have the versatility of finite element method but has extremely good resolution of stress gradients that arise near stress raisers such as cracks, inclusions, interfaces, sudden changes in loads and/or geometries. The author with his students has developed BEM algorithms for fracture mechanics problems [25], for stress analysis in composite materials [26-28], and for interface problems [29,30]. There are several versions of BEM. Each version having certain advantages for a class of problems [31]. An efficient mesh refinement technique that can be used with any version of BEM has been developed [32] that for most problems converges within two iterations to produce accuracies better than 0.1%. However, this mesh refinement technique is applicable to only homogenous materials. These algorithms have been incorporated into program BEAMUP¹ and form an integral part of automating the stress analysis process as described in reference [33]. The objective of the automation is to relieve the user of all decisions related to the analysis process. The user need only describe the boundary value problem in two-dimension which will consist of x and y coordinates of boundary geometry, the boundary conditions, and material properties. Inside the program, decisions will be made regarding the appropriate version of boundary element method to use for a piece of boundary, the appropriate

1. The pin could be modeled as an elastic body for modules on interface stresses. Failure criterion accounting for differing tensile, compressive, and shear strength could be considered in modules for stress raisers in composites.
 1. Additional details of BEAMUP can be found at <http://www.me.mtu.edu/%7Emavable/BEAMUP/index.html>.

mesh and approximation of the unknown. Some research and development work is needed to automate the process as described in reference [33]. A numerical example is presented below to demonstrate the effectiveness of these algorithms in context of fracture mechanics.

Example 3: Fig.4 shows a branch crack in an infinite plate subjected to uniaxial tension in the y-direction. The crack was modeled using displacement discontinuity density function and approximated by cubic hermite polynomials. Starting with a uniform mesh of 15 elements, convergence to the desired accuracy was obtained in one iteration for a total of 33 elements. The stress intensity factors at point A for mode I and at point B for mode I and II are shown in Table 1. The BEAMUP solution is within the range of accuracy of the solution reported in reference [13].

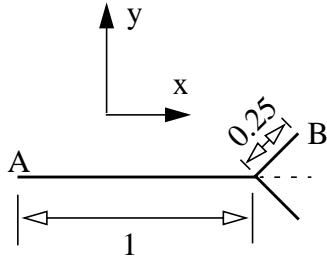


Fig. 4. Branch crack in an infinite plate in tension.

Table 1 Non-dimensional stress intensity factors

	BEAMUP	Ref. [13]
K_{IA}	1.384	1.383 ± 0.014
K_{IB}	0.703	0.705 ± 0.014
K_{IIB}	0.574	0.576 ± 0.014

4. e-handbook

Handbooks on stress concentration factors[12] and stress intensity factors[13,14] have proven to be invaluable resources in engineering design and analysis. In a similar manner e-handbook can be expected to be a valuable resource in today's computer dominated design environment. Fig.5 shows two typical geometries found in traditional handbooks. In the e-handbook the user would specify the values of geometric variables H, W, a, and θ and the load σ . From these values of the variables, the boundary value problem can be easily created and solved by the automated stress analysis software. Added features in the e-handbook could permit the user to change the uniform load values to varying values or include shear stress on the boundary. The e-handbook will significantly reduce the effort by the user in describing the boundary value problem.

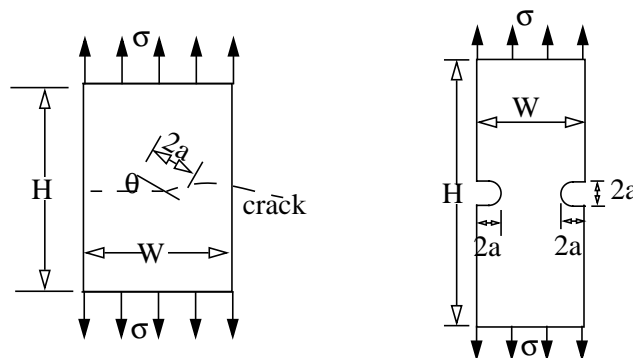


Fig. 5. Coupon geometries of handbooks.

5. Graphical user interface (GUI)

The author and his graduate students have developed two GUI for BEM in the past which are now outdated as the platform and the operating systems have changed. A new GUI is needed for automating the stress analysis software for the current work. A simple GUI is planned that will be easy to maintain in the fast changing computing environment.

The GUI will perform the following functions: (i) Provide easy access to the e-handbook library and solicit information on various parameters from the user. (ii) Obtain boundary value information from the user when the geometry of interest is not in the e-handbook library. The boundary value problem description will consist of x and y coordinates of boundary geometry, the boundary conditions, and material properties. This information will be imported by the GUI from a file, which either the user supplies or is created from commercially available geometric modeling software. (iii) Create output files in appropriate format for importing into commercial software for post processing.

6. Conclusions

This paper describes on going work to develop instructional material that can be used for incorporating fracture mechanics into design. It is anticipated that the instructional material modules that are developed could be incorporated in existing undergraduate and graduate mechanics and design courses. The key to the success of this enterprise is the development of an automated stress analysis software that is nearly complete. A faculty member in the education department has agreed to develop an assessment scheme and will implement it when the course material is introduced into the class room. Successful completion of this research and developmental effort will be followed by development of instructional material for other types of stress raisers such as for holes and cracks in composite materials and near interfaces of multiple materials.

7. References

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8. Biographical Information

Madhukar Vable, Associate Professor, has research interest in computational mechanics. He is a Fellow of Wessex Institute of Great Britain. He was named MTU Distinguished Teacher in 1998 and Distinguished Faculty Member from the Michigan State in 1999. He is author of 'Mechanics of Materials' textbook published by Oxford University Press. He is developing a stress analyzer called BEAMUP, details of which can be found at his webpage.