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Integration of Modal Test Results of a Composite Wing into the Introductory Aerospace Vibrations Course

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ABSTRACT

Often students do not see the relevance of concepts and methods that are taught in introductory mechanics classes. For many students, their academics are a blur of equations and theory. To improve students' engagement and retention of concepts, a real-world problem was introduced in the undergraduate vibrations course, which is a required course for all aerospace engineering students at Mississippi State University. The course centered on an overarching, interesting, and realistic problem that was motivated by the author's research in determining the modal characteristics of a full-scale composite aircraft wing; thus, measured vibration data and characteristics were available for comparison with results from students' analyses. During the semester, as concepts relevant to the problem were covered, students determined the first natural frequency of the complex wing using different models. In this way, concepts were reinforced and students became familiar with mathematical models that were representative of the physical models of the actual structure. This paper reports on the design and development of this activity by presenting details of the spring-mass system and a beam model with different loadings that were used to determine the first natural frequency of the wing structure. By integrating research results into the classroom, many engineering mechanics and mechanical vibration concepts can be reinforced by (a) analyzing a "real-world" problem through simple mechanical models to simulate a complex structure and (b) by highlighting the relationships between physical and mathematical models of an actual aerospace structure.

I. Introduction

Several papers have reported on the importance and benefit of including research into the undergraduate curriculum. Phillips and Schroeder [1] note that undergraduate research that complements course topics is innately interactive and can enable student interest in engineering. Prince et al [2] highlighted the potential of research integration in undergraduate curriculum to positively impact the quality of education. The first recommendation of the Boyer Report [3] was to "make research-based learning the standard". Integrating research into the classroom can also address several ABET [4] outcomes. For example, engineering graduates should be able to design and conduct experiments, analyze and interpret data, and have an ability to identify, formulate and solve engineering problems. However, in many engineering undergraduate curriculums, research is typically integrated into education mainly through university seminars and capstone projects. This paper presents an example of integrating research results in the classroom in a no-cost manner. By so doing, all students in the course have some exposure to actual research practices, subsequently satisfying several of the above ABET criteria to enable the professional formation of engineers.

As a way to increase student interest and motivation, research was integrated into the required aerospace undergraduate vibrations course, which is typically taken by juniors in aerospace engineering. To establish a theme and create a thread that would bring cohesiveness to the course, the following course problem was posed at the beginning of the semester: *Determine the modal characteristics of a full scale wing of an all-composite ultralight aircraft.* This activity was inspired by the author's previous study involving the determination of the vibration characteristics of an ultralight optionally piloted aircraft (OPA) shown in Fig. 1[5]. Many Mississippi State University students are familiar with this aircraft as it is housed at the Raspet Flight Research Laboratory. Elements of this research study were integrated over the semester at appropriate times.

In the following sections, the overall program development, objectives of the study and a discussion linking ABET student outcomes to student learning are presented. A brief description of the wing vibration research is given, along with student activities that use the results of this research. A complete summary showing the integration of this research in class topics is shown in Appendix A.

II. Program Development, Learning Objectives, and Relationship to ABET Outcomes

Vibrations (EM 3413) is a required course that is taken by undergraduate aerospace students (primarily juniors). Prerequisites for the course include introductory courses in dynamics, differential equations, and linear algebra. In fall 2014, 40 students were enrolled in this course.

Figure 1. All-Composite Optionally Piloted Aircraft

Program Development

The main focus of this activity is to integrate aspects of the wing vibration research into the traditional lecture-based vibrations course to reinforce fundamental mechanics and vibration concepts. This is a work in progress as Phase I, which is being reported here, involves the design and development of the active-learning activities. This includes the formulation of the in-class exercises as well as their scheduling in the semester-long course. Phase II will include the full implementation of all activities and the development of assessment tools to obtain student feedback and impact on student learning with regards to this research integration activity.

Learning Objectives

The primary learning objectives of this activity are to enable students to:

1. Increase their understanding of the mathematical models that are used to obtain the vibratory response of simple beam structures.
2. Increase their comprehension of fundamental mechanics and vibration concepts.
3. Enhance their analysis skills.
4. Gain an understanding of the experimental process in vibration testing.

Relationship to ABET Outcomes

Currently, this course is listed as an engineering mechanics course for which no ABET outcomes are specifically measured. However, the integration of the wing vibration research addresses several ABET outcomes (a, b, e, i, and k) [4] . Elements of this research integration and their relationship to the outcomes are discussed in Table 1. In future offerings of the course, measurement tools to evaluate outcomes (e) and (k) will be developed. Outcome (e) refers to most of the activities that pertain to the modeling and assessment of simple vibration models. Outcome (k) can be addressed by assessing activities in which programming / computational skills are required.

Table 1. ABET Outcomes Addressed from Integration of Wing Vibration Study.

ABET Outcome	Description	Discussion
(a)	An ability to apply knowledge of mathematics, science and engineering	Analysis of vibratory motion involves math, particularly linear algebra and differential equations. Additionally, a number of engineering practices can be discussed due to the incorporation of an actual aerospace problem.
(b)	An ability to design and conduct experiments, as well as to analyze and interpret data	Incorporating the wing vibration research introduces various elements of experimental design. This includes discussion of test apparatus to simulate service conditions, measurement sensors such as accelerometers, load cells, deflection gages, force transducers, and data acquisition equipment.
(e)	An ability to identify, formulate, and solve aerospace or related engineering problems, and to assess the solutions obtained critically and objectively	By modeling the aircraft wing with various loading configurations, and assessing the solutions by comparison with the experimental data, students can gain insight into the basic engineering process.
(i)	A recognition of the need for, and an ability to engage in life-long learning, including the ability to assess critically and objectively information so obtained;	Integrating a study involving an actual full scale aircraft structure brings relevance to the course, which fosters interest and promotes life-long learning.
(k)	An ability to use the techniques, skills, and modern engineering tools necessary for aerospace or related engineering practice.	Research integration allows for the introduction of a number of experimental tools and practices. Additionally, further offerings of the course will include a computational segment in which the measured vibration data will be utilized.

III. Integration of Vibration Research of an Aircraft Wing into the Aerospace Vibrations Course

To integrate the wing vibration research into the vibrations course, a series of in-class exercises were developed and implemented. Overall, the geometry of the wing was modeled as a cantilevered beam and the loading of the beam was changed to observe the change in the first natural frequency. This considers the vibration of the beam in the direction perpendicular to its length, called transverse or flexural vibration. All cases considered undamped motion.

This section describes the active learning exercises as Activities 1-5. Activities 1-3 were implemented as in-class activities. Due to time limitations, Activities 4 and 5 were integrated into the lectures. In future course offerings, all activities will be in-class, team (2 or 3 students) exercises.

An overview of the research project was given to the class at the beginning of the semester. As the course progressed, more in-depth presentations and discussions regarding the relevant topics followed. For example, during the portion of the course in which vibration measurement was covered, a presentation detailing the experimental method was given. This included discussion of the components of the test fixture, data acquisition system, and measurement sensors such as accelerometers, load cell, deflection gages, and force transducers. In Appendix A, this is listed as a Lecture (presentation).

The integration of this research activity was inspired by the vibration testing of a large scale (length = 204 in), ultralight (35 lb) carbon composite OPA wing, which was tested using a shaker table approach, as shown in Fig. 2 [6] . Wing response was obtained from the surface mounted accelerometers, identified by A1 through A17. Wing data and the first natural frequency, obtained from the modal testing shown in Fig. 2, is listed in Table 2; this was the primary data given to the students. Measurements were given in US Customary units to convert dimensions and understand the difference between units of pound-force and pound-mass.

Table 2. Wing data used in wing vibration analysis in course.

Length (in)	**Weight (lb)**	**Flexural Rigidity *EI* (lb-in^2)**	**Stiffness *k* (lb/in)**	**1st Natural Frequency (Hz)**
204	35	3.353E+07	11.85	4.77

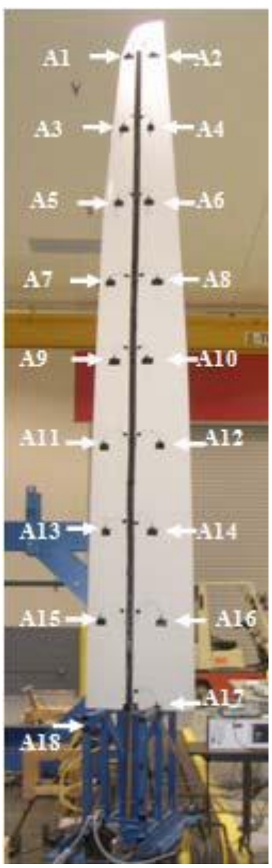


Figure 2. Wing Test

Prior to each student activity, a brief lecture on the topic was given. After introduction of the activity, students (working in groups of two or three) were given 5-15 minutes in class to address the activities described below. All activities were initiated in class; some were completed in class while other activities, which required more computation time, were completed as parts of homework assignments.

In this section, student activities with brief overviews of lecture topics are presented. In summary form, the integration of the vibration research over the entire course is shown in Table A in Appendix A.

A. Modeling of the UAV wing as a spring-mass system (Case A)

The introduction to vibrations includes the topic of structural models of realistic and complex systems. Simple models of a number of structures such as motorcycles, cars, buildings, bridges are discussed. This topic led to the first student activity.

Student Activity 1: Model the composite wing using simple structural elements.

Students were encouraged to consider wing geometry and boundary conditions that simulate the attachment of the wing to the fuselage. The cantilevered beam was the natural choice. Students were reminded that the beam is an elastic element and it has stiffness, which is a property of the material from which it is fabricated. Therefore, the beam can also be represented by an elastic spring with a stiffness *k* and a mass *m*, as shown in Fig. 3.

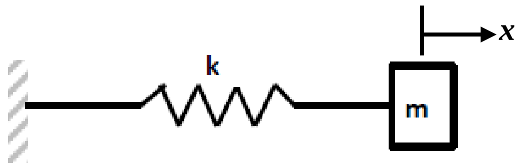


Figure 3. Case A: Spring-mass system.

Once students understood the genesis of the model for the aircraft wing, the mathematical model for the spring-mass system was developed. For a linear spring, the stiffness k is related to the force applied by the spring F_s and the deflection x as

$$F_s = k\,x \tag{1}$$

Thus, the class was introduced to the fundamental spring-mass system and its relationship to a complex structure such as an aircraft wing. The development of the free vibration equation of motion (EOM) for the spring-mass system exists in numerous texts [7-9] on mechanical vibrations and can be expressed as a second order homogenous differential equation as

$$\ddot{x}(t) + \frac{k}{m}\,x(t) = 0 \tag{2}$$

where $\ddot{x}(t)$ is the acceleration (second time derivative of the displacement $x(t)$), k is the stiffness and m is the mass of the system. To predict the response of a periodic vibratory motion of amplitude A, a harmonic solution of Eq. (2) can be assumed as

$$x(t) = A\ \sin(\omega_n t + \varphi) \tag{3}$$

where ω_n is the angular natural frequency and φ is the phase. By successive differentiation of Eq. (3), the velocity and acceleration of the motion are determined. For Eq. (3) to be a solution of the EOM (Eq. (2)), the frequency ω_n in rad/sec is determined to be the square of the coefficient of the second term in Eq. (2) as

$$\omega_n = \sqrt{\frac{k}{m}} = 2\pi\, f_n \tag{4}$$

The frequency in cycles/sec (Hz) can be expressed as

$$f_n = \frac{\omega_n}{2\pi} \tag{5}$$

The natural frequency of the spring-mass system can now be computed.

Following the development of the mathematical model for the spring-mass system, students were asked to compute the natural frequency of the wing structure and compare their result with the measured data, which leads to the next student activity.

Student Activity 2. Given the wing data in Table 2, compute the natural frequency of the spring-mass system which is a model of the aircraft wing. Compare the calculated value with the experimental frequency and show the percent difference. Discuss the factors that contribute to this difference.

This activity enabled a discussion of the assumptions made in the development of this spring-mass model and the characteristics of the complex aircraft wing that are not being captured by the simple model, i.e., the characteristics of a non-homogenous, anisotropic, geometrically complex structure are being represented by a single spring stiffness and mass.

B. Modeling of the wing as a cantilevered beam (Case B)

The next opportunity to integrate the wing vibration study was on the topic of effective stiffness. The experimental determination of the flexural rigidity of the actual wing was presented during this topic, listed in Appendix A as a Lecture (presentation).

Using a mechanics of materials approach, students determined the elastic curve for a cantilever beam with a tip load. The elastic curve represents the deflection of the beam from its original unloaded position, as shown in Fig.4.

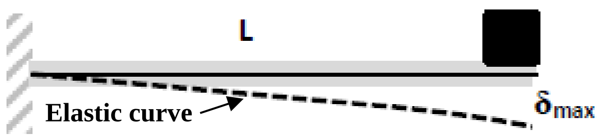


Figure 4. Cantilevered beam subjected to a concentrated tip load.

Once the elastic curve was determined, the maximum deflection was obtained and used to present the topic of effective stiffness. As a way to refine the wing model, wing loading was discussed, resulting in the consideration of three different loadings, as shown in Fig. 5. Modeling the wing as a prismatic, homogeneous cantilevered beam, students determined the maximum deflection for each case using a mechanics of materials approach.

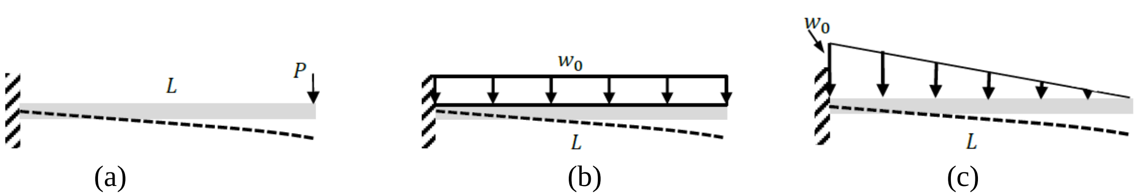


Figure 5. Wing modeled as a cantilevered beam with (a) Case B1. Concentrated tip load (b) Case B2. Uniform load and (c) Case B3. Triangular load.

The tip or maximum deflections of the wing modeled as a cantilevered beam of length L are δ_{B1} for the concentrated load, δ_{B2} for the uniform load, and δ_{B3} for the triangular load, and can be expressed as [10,11].

$$\delta_{B1} = \frac{PL^3}{3EI}, \qquad \delta_{B2} = \frac{w_0 L^4}{8EI}, \qquad \delta_{B3} = \frac{w_0 L^4}{30EI} \tag{6}$$

where E is the modulus of elasticity, I is the moment of inertia, the product EI is the flexural rigidity, P is the concentrated load, and w_0 is the load per unit length. The effective stiffness for these load cases can be determined by putting Eqs. (6) in the form of Eq. (1) to obtain

$$k_{B1} = \frac{3EI}{L^3}, \qquad k_{B2} = \frac{8EI}{L^3}, \qquad k_{B3} = \frac{30EI}{L^3} \tag{7}$$

The natural frequency for each case could now be determined, leading to the next in-class activity:

<u>Student Activity 3:</u> Given the experimentally determined flexural rigidity EI and the length and weight of the wing from Table 2, compute the natural frequency using Eq. (5) for the three load cases shown in Fig. 5. Obtain the percent difference between the experimental and analytical values and discuss (a) the factors that contribute to the difference in the analytical and experimental values and (b) the factors that contribute to the difference in the frequencies from the different loadings in Cases B1, B2,and B3.

Discussion on this activity included boundary conditions to simulate the wing-fuselage attachment correctly, wind loading distributions and the geometric and material property differences between the actual composite wing and the prismatic homogeneous beam models.

C. Modeling the aircraft as a three DOF system (Case C)

The topic of two and multiple degrees of freedom (DOF) vibration systems provided an opportunity to model the complete aircraft as a three DOF system. Due to time limitations, this exercise became part of the in-class lecture. Attention was directed to considering the stiffness and mass of the components. In the next offering of the course, this will be an in-class activity, with student teams formulating the model and determining the equations of motion. The response of the system and the determination of the modal characteristics will be assigned as part of a computational homework assignment. Student Activity 4 was initiated in class as:

<u>Student Activity 4.</u> Model the aircraft as a three DOF system. As given in Table 2, the aircraft has a total weight of 155 lb. and each wing weighs 35 lb. Consider proper boundary conditions and model the masses of the wings and fuselage appropriately using simple structural elements.

Class discussions were directed to result in modeling the aircraft using cantilevered beams with a lumped mass at the tip, resulting in the use of the stiffness k_{B1} (Eq. 7). The cantilevered beams are then depicted as spring-mass systems to continue the link from the beginning of the course when cantilevered beams were represented as spring-mass systems. Including the wing mass *m* and scaling the mass of the fuselage (*2.4 m*) appropriately, the resulting model is shown in Fig. 6.

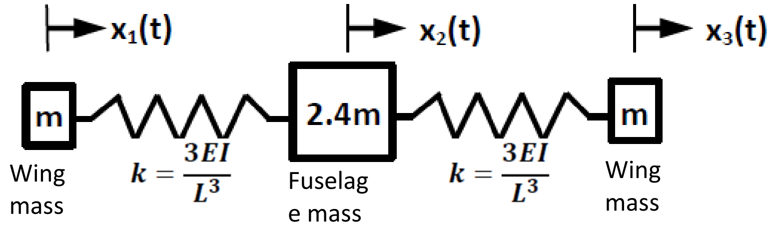


Figure 6. Three DOF spring-mass model of the complete

The first nonzero frequency for this system is determined to be

$$\omega_n = 1.732\sqrt{\frac{EI}{mL^3}} \tag{8}$$

The results from all activities are listed in Table 3; the results show that the natural frequency for Cases A, B1 and C is identical. This allowed a discussion of the similarities between the models and the fact that modeling the complete aircraft with three DOFs does not improve the result when compared to that from the 1 DOF spring-mass model. The system is characterized by its natural frequency, which depends on the stiffness and mass of the system and in each case, elements of the same stiffness and mass are used as in the previous activities.

D. Modeling of the wing as a cantilevered beam under its own weight (Case D)

Similar to Activity 4, the following activity was also presented in lecture form in class due to time limitations as this topic is presented toward the end of the semester. The wing was modeled as a cantilevered beam under its own weight. The natural frequencies of vibration for a slender beam in transverse vibration can be determined from [7-9]

$$\omega_n = (\beta_n L)^2\sqrt{\frac{EI}{\rho L^4}} \tag{9}$$

where ρ is the mass per unit length and β_n depends on the boundary conditions. A table [7,8], listing the numerical values of the quantity $(\beta_n L)^2$ for common end configurations, was given to the class for computation of the analytical result. This led to the last research integration activity (Activity 5).

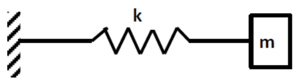
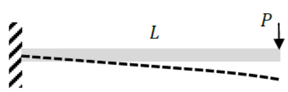
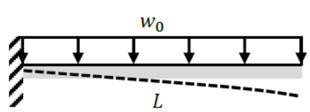
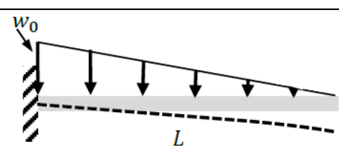
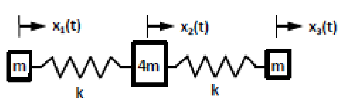
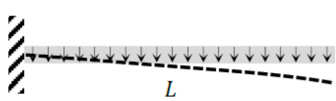
<u>Student Activity 5.</u> *Determine the natural frequency for the aircraft wing under its own weight. Obtain the percent difference between the experimental and analytical values and discuss (a) the factors that contribute to the difference in the analytical and experimental values and (b) the factors that contribute to the difference in the frequencies from all cases considered. List the assumptions of the Euler-Bernoulli beam theory.*

This activity allows for an excellent review of important factors and concepts regarding the Euler-Bernoulli beam theory. Assumptions and limitations of the theory and the importance of using proper boundary conditions are emphasized. Particularly, that this theory is formulated for slender structures that are fabricated from isotropic materials and based on the cross-section being rigid in its own plane and remaining plane and normal to the deformed axis of the beam. Since the actual wing is complex both geometrically and materially, the experimental data showed mixed modes of vibration (not just transverse) and large differences between the predicted and measured data were expected.

IV. Discussion

A summary of the results from all activities is given in Table 3. As seen, the triangular loading (Case B3) gives the best result, followed closely by the Euler-Bernoulli beam (Case D). The difference between the computed and measured values in each case provides opportunities for class discussions and serves to highlight the characteristics and limitations of the models and the experimental process. Although the percent difference in the computed and measured values is large, the activities provide a demonstration of how the results improve by systematically refining the models and the model parameters that most impact the results. In summary, this includes modeling a complex composite (having a stiffness matrix) structure using a homogeneous, isotropic parametric beam or a spring, which has a single stiffness and mass. Also, these discussions add elements of a real-world problem to give insight into the experimental process and its limitations.

Table 3. Natural Frequency of OPA Wing models compared with the experimental frequency f_{exp}.

Case	**Model**	**Natural Frequency f_n**	**Computed Natural Frequency (Hz)**	**% Difference (f_{exp} = 4.77 Hz)**
A	**Spring-mass system** 	$\frac{1}{2\pi}\sqrt{\frac{k}{m}}$	1.82	61.8
B1	**SDOF massless beam with a tip load** 	$\frac{1}{2\pi}\sqrt{\frac{3EI}{m\,L^3}}$	1.82	61.8
B2	**SDOF massless beam with a uniform load** 	$\frac{1}{2\pi}\sqrt{\frac{8EI}{m\,L^3}}$	2.97	37.7
B3	**SDOF massless beam with a triangular load** 	$\frac{1}{2\pi}\sqrt{\frac{30\,EI}{m\,L^3}}$	5.76	20.7
C	**3 DOF Model** 	$\frac{1.7321}{2\,\pi}\sqrt{\frac{EI}{mL^3}}$	1.82	61.8
D	**Beam under its own weight (Euler-Bernoulli Beam)** 	$\frac{3.516}{2\,\pi}\sqrt{\frac{EI}{\rho L^4}}$	3.69	22.5

V. Conclusions

The primary goal of this effort was to integrate the experimental vibration study of an all-composite aircraft wing into the required undergraduate aerospace engineering vibrations course. A total of five active learning activities have been developed, with each activity followed by a class discussion that emphasizes the limitations of the models (simplified loadings, boundary conditions, geometry) and of the laboratory experiments in simulating actual service conditions.

Initially, there was concern that the course may become disorganized by integrating the research at intervals throughout the semester. Therefore, one geometric model was considered: a prismatic, homogeneous cantilevered beam and the loading configuration was varied. This also allowed for quick comparisons throughout the semester and the repetition emphasized many fundamental engineering mechanics concepts. Additionally, repeatedly linking the course topics to a common problem and structure brought cohesiveness to the course. Integration of this research study enabled insight into analytical, modeling and experimental methods. This activity provided opportunities to (a) introduce students to "real-world" issues and practices, (b) demonstrate the iterative process of modeling, (c) increase in-class student engagement, and (d) encourage students to work in groups of two to three students.

In the next offering of the course, all activities will be fully implemented as in-class exercises. An assessment protocol will be developed and implemented to obtain student response and impact of this research integration on student learning.

References

1. Phillips, M.; Schroeder, M. Integrating research and engineering education: A case study of undergraduate research in orthopaedics. Proceedings of the Spring 2007 American Society for Engineering Education Illinois-Indiana Section Conference, **2007**.
2. Prince, M.J.; Felder, R.M.; Brent, R. Does faculty research improve undergraduate teaching? An analysis of existing and potential synergies. *Journal of Engineering Education* **2007**, *96*, 283-294.
3. Boyer Commission on Educating Undergraduates in the Research University 9Carnegie Foundation for the Advancement of Teaching), Reinventing undergraduate education: A blueprint for america\'s research universities, 1998 (http://files.eric.ed.gov/fulltext/ED424840.pdf, accessed 20 March 2016).
4. ABET Engineering Accreditation Commission. Criteria for accrediting engineering programs, effective for reviews during the 2016-2017 accreditation cycle; ABET: Baltimore, MD, 2015.
(*http://www.abet.org/accreditation/accreditation-criteria/*, accessed 20 March 2016).
5. Sullivan, R.; Hwang, Y.; Rais-Rohani, M.; Lacy, T. Structural analysis and testing of an ultralight unmanned-aerial-vehicle carbon-composite wing. *Journal of Aircraft* **2009**, *46*, 814-820.
6. Simsiriwong, J.; Sullivan, R.W. Experimental vibration analysis of a composite uav wing. *Mechanics of Advanced Materials and Structures* **2012**, *19*, 196-206.
7. Inman, D.J. *Engineering vibration*. Pearson: 2014.
8. Thomson, W.T.; Dahleh, M.D. *Theory of vibration with applications*. Prentice Hall: 1998.
9. Rao, S.S. *Mechanical vibrations*. Pearson/Prentice Hall: 2004.
10. Pytel, A.; Kiusalaas, J. *Mechanics of materials*. Thomson: 2003.
11. Hibbeler, R.C. *Mechanics of materials*. Pearson Prentice Hall: 2005.

Appendix A

Table A. Integration of wing vibration research over the semester-long course in mechanical vibrations.*

Class Activity	Topic	Integration of Results from Vibration Testing of Composite Wing
	Introduction to Vibratory Motion	
1	**Free Vibration** Harmonic Motion Modeling Energy Methods ---►	• Introduction of course problem: *Determine the vibration characteristics of an ultralight all-composite UAV wing.* • Discuss models for the UAV wing
2	The spring-mass system ---► Equation of Motion (EOM) Solution of spring- mass system EOM: Deflection and Natural frequency Pendulum EOM of 1-DOF systems Energy Methods, Lagrange's Method	• Beam element can be modeled as an elastic spring with stiffness k • Give description (material and geometric) of wing and the experimentally determined natural frequency.
3	Springs and stiffness ---► Effective stiffness	• Model wing as cantilevered beam using different loadings (tip load, uniform load & triangular load). • Determine the effective stiffness and natural frequency f_n for each case.
	Viscous Damping EOM Solution of EOM (critically damped, overdamped, underdamped motion)	
Lecture (presentation)	Amplitude, RMS, Db, Log, Decrement Measurement ---► (moment of inertia, stiffness,)	• Describe the measurement of the flexural rigidity of the UAV wing
Lecture (presentation)	**Forced Vibration** Undamped and Damped System Base Excitation Rotating Unbalance Measurement Devices ---►	• Describe the experimental test apparatus (shaker-table, accelerometers, shaker, force transducer, etc.)
	General Forced Response Impulse Response Convolution Int., Laplace Transforms General Loading	
4	**Multiple DOF Systems** ---► 2 DOF Undamped, Damped Mode Shapes, Eigenvalues, Natural Frequencies	• Model wing as an undamped 3 DOF system and compare natural frequency to SDOF solution.
5	**Continuous systems** Vibration of string Euler-Bernoulli Beam ---►	• Model wing as a beam under its own weight and compare 1^{st} natural frequency with experimental result.

**Arrows depict the points of insertion of research into the course.*