AC 2009-51: INTEGRATION OF NUMERICAL ANALYSIS AND EXPERIMENTAL TESTING INVOLVING HEAT TRANSFER FOR A SMALL HEATED CYLINDER DURING COOLING

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Integration of Numerical Analysis and Experimental Testing Involving Heat Transfer for a Small Heated Cylinder During Cooling

Abstract

Heat transfer projects can be focused on either experimental measurements or numerical analysis. Due to time constraints in a class it can be difficult to solve complicated problems using both methods. The project described in this paper involves both experimental work and numerical simulations used to determine the temperature of a small aluminum cylinder while it is cooling from a temperature of approximately 80ºC to ambient room temperature of approximately 23ºC. The project spans two courses in the undergraduate mechanical engineering curriculum: Heat Transfer where the numerical analysis was performed, and Systems and Measurement where students address the problem with experimentation. This implementation is intended to reinforce fundamental heat transfer concepts by working on a project through two different approaches. In the junior year the students first complete a numerical analysis of the cooling cylinder by solving the convection and conduction equations in radial coordinates using a finite difference approach and determining the temperature of the cylinder as it cools. In the senior year the same students look at the physical system in a laboratory exercise, utilizing thermocouples, computerized data acquisition, and processing to experimentally measure the temperature of the cooling cylinder. A description of both the theoretical problem and the experimental problem are given in this paper as well as results from a survey conducted after the students completed the project to help determine the effectiveness of the approach and the possibility of using a similar approach for other topics outside of heat transfer.

Introduction

This paper documents a heat transfer project that incorporates both numerical analysis (finite difference) as well experimental testing of the cooling of a small aluminum cylinder. This was done over the span of two undergraduate courses, the required introductory heat transfer lecture course (ME336 Heat Transfer) and a senior level technical elective on instrumentation (ME 491 Systems and Measurement). The goal is to reinforce concepts of conduction and convection heat transfer. The problem was divided into two courses for the following reasons. First, in a single course it is often not possible to have enough time to conduct detailed numerical analysis and time intensive experiments. Second, by using two separate courses the students can see the relationship between different engineering courses and strengthen their appreciation of their curriculum.

In recent years, many studies have been presented on the effectiveness of using computational methods to enhance the teaching of heat transfer [1,2,3,4]. Though numerical analysis is an integral part of engineering education, it is largely agreed that simulation can not replace hands-on learning[5]. As a result, there is an effort to establish laboratory work that supplements numerical investigations in the field [6].

In both the numerical analysis and the experimental testing, students work in groups of two to four students. This was done to promote teamwork and also give the students the chance to learn from each other. Furthermore it has been shown that group sizes play a factor in group performance and learning, and with one study indicating that groups close to four were preferential[7].

Although experimental testing is an important part of undergraduate mechanical engineering studies, often there is may not be a strong link between tests conducted in lab and numerical analysis. Also, since studies have shown that a strong majority of engineering students are either logical thinkers (type 2) or hands-on thinkers (type 3), it could be helpful to look at the same problem from both the experimental side and the analytical side[7].

This paper is divided into five sections. Following this introduction, the numerical analysis is given along with the results for that part. Next, the experimental testing is described in detail along with the associated results. . After that, the results of a student survey designed to quantify the effectiveness of the two-part project are presented with a discussion of the results. This is followed by the conclusion.

Theoretical Analysis and Finite Difference Simulation

A semester project involving writing a finite difference code to determine the temperature (centerline and surface) of a small aluminum cylinder was assigned to introductory heat transfer students. The project demonstrates heat transfer concepts of conduction and convection as well as numerical solutions to time-dependant partial differential equations. The geometry of the cylinder is shown in Figure 1 below.
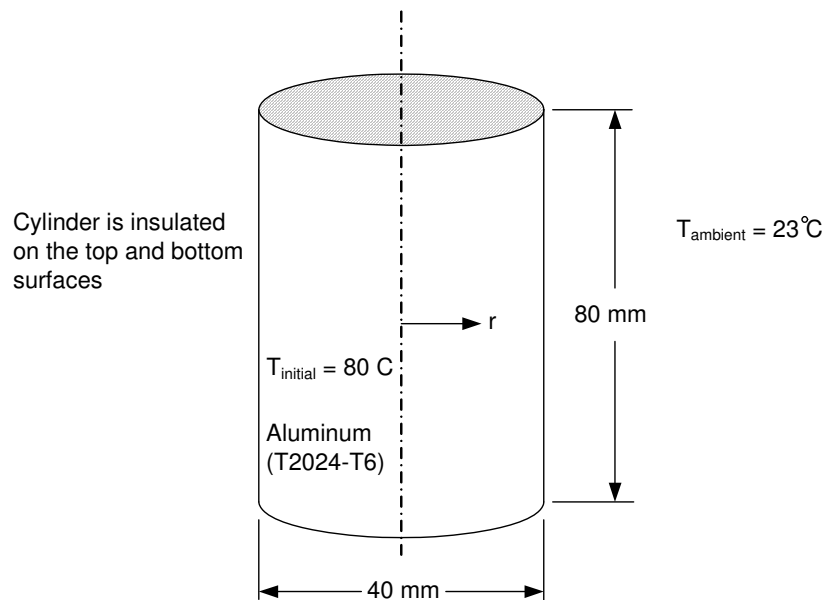

Cylinder is insulated on the top and bottom surfaces

$T_{ambient} = 23°C$

80 mm

$T_{initial} = 80 C$

Aluminum (T2024-T6)

r

40 mm

Figure 1: Geometry of cylinder.

The diameter of the cylinder is 40 mm, the height is 80 mm and the material is T2024-T6 aluminum. The initial temperature of the cylinder is 80°C. The cylinder is assumed to be

insulated on the top and bottom, so that heat transfer occurs only through the sides of the cylinder. The cylinder is allowed to cool by free convection to the room which is at a temperature of 23ºC.

Inside of the cylinder, the governing equation for the temperature is given below assuming only radial conduction[8].

$$\rho c_p \frac{\partial T}{\partial t} = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right]$$

(1)

At the surface of the cylinder, conduction within the cylinder equals convection as given below.

$$q_{conduction} = q_{convection}$$

$$-k \frac{\partial T}{\partial t} \bigg|_{r=R} = -h \left(T_{surface} - T_{surroundings} \right)$$

(2)

The temperature distribution in the cylinder is symmetric, therefore at the centerline the following equation can be used:

$$\frac{\partial T}{\partial t} \bigg|_{r=0} = 0$$

(3)

Equations (1), (2), and (3) are the governing equations for the problem. To solve the set of equations, a finite difference approach is used for the time and spatial derivatives. A first order forward difference is used for time along with second order centered differences for the spatial terms as listed in equations (4), (5), and (6) below[9].

$$\frac{\partial T}{\partial t} = \frac{1}{\Delta t} \left(T^{t_{new}} - T^t \right)$$

(4)

$$\frac{\partial^2 T}{\partial t^2} = \frac{1}{\Delta r^2} \left(T_{r+1} - 2T_r + T_{r-1} \right)$$

(5)

$$\frac{\partial T}{\partial r} = \frac{1}{2\Delta r} \left(T_{r+1} - T_{r-1} \right)$$

(6)

The subscripts in the equations above represent node numbers for the finite difference solution while superscripts represent the time for each nodal temperature. Also, Δr is the spacing between each radial element and is specified to be 1.0 mm. The time step, Δt, is not specified in the problem, but students experiment to determine a good value. To provide numerical stability of the solution a Δt of approximately 0.0002 seconds or less is needed.

The overall results of the project were successful. Each group of students submitted a final report and they also demonstrated their finite difference program to the instructor. Most students utilized C/C++ or MATLAB to create a program to iteratively solve for the temperature distribution. A few students used Microsoft Excel to solve for the temperature distribution. The finite difference equations can be entered into Excel relatively easily. However, the version of used (Excel 2003) has a limitation of approximately 65,000 rows per worksheet. This project typically requires over 500,000 time steps, therefore, multiple worksheets had to be used. Calculation time can be somewhat lengthy using Excel as well, depending on the computer used it could require several minutes to calculate or save the document.

The students were not given the heat transfer coefficient (h) for the problem, but they were able to use their program along with a set of experimental data provided to estimate the heat transfer coefficient. A trial and error approach was used. Modeling the system as free convection with air, students were able to predict the heat transfer coefficient will likely be in the range of 2-25 W/m^2K. The students found the convection coefficient to be approximately 13 W/m^2K.

Representative final results for the project are shown in Figure 2. The calculated centerline and surface temperature are plotted as a function of time along with the experimental data. It can be seen that the centerline and surface temperature of the aluminum cylinder are almost identical. The students were asked to comment on this observation. Due to the high thermal conductivity of the aluminum and the dimensions and heat transfer coefficient involved, there is a very small temperature gradient inside of the cylinder. This can be predicted ahead of time by calculating the Biot number (Bi) of the cylinder.

$$Bi = \frac{h \cdot \left(\frac{R}{2}\right)}{k} \tag{7}$$

Where, Bi: Biot number
        h: heat transfer coefficient (13 W/m^2K)
        r: radius of cylinder (0.02 m)
        k: thermal conductivity of cylinder (177 W/m·K)

This results in Bi = 0.000734 and for Bi < 0.1, it can be assumed that an object will have a uniform temperature distribution which was observed in this case.
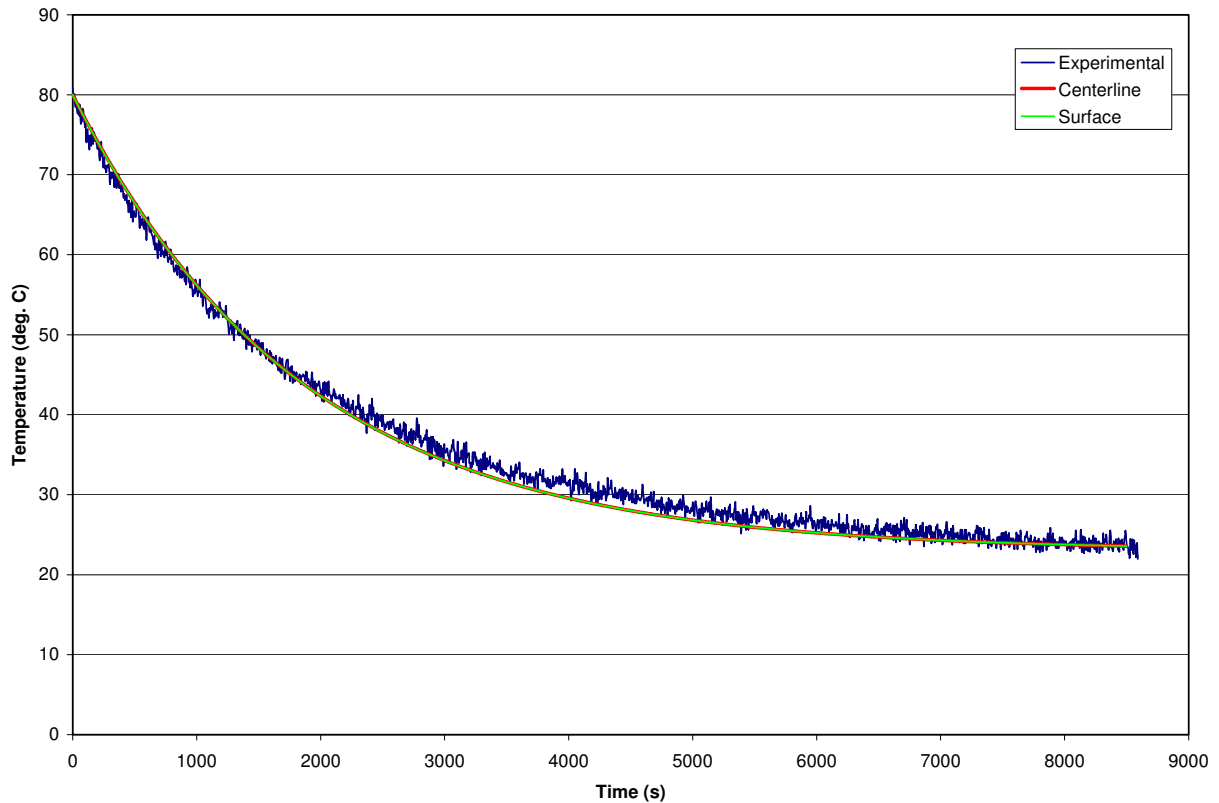
Figure 2: Temperature distribution of cylinder with time.

Experiment

In the Systems and Measurement course, the same students conducted an experiment, cooling the same cylinder that was analyzed previously using finite difference techniques. The primary goal of the laboratory was to determine an estimate of the heat transfer coefficient of the cooling cylinder under free convection cooling. This was to be done by fitting a trend line to collected cooling data, where the coefficients of the trend line are directly related to the desired coefficient. The exercise was also used to give the students an early-semester exposure to the many facets of experimental measurement.

For the experiment, students were instructed to assemble the hardware and software necessary to collect useful temperature data. The experimental set up is shown in Figure 3. Using a Fluke Type-K thermocouple probe module (Fluke 80TK) with a Type K immersion probe (Fluke 80PK-22) in connection with a National Instruments CompactDAQ (NI cDAQ-9172) containing the Analog In module (NI 9215), students were able to sample analog voltages proportional to the core temperature of the cylinder, which for the experiment is assumed to be the same as the surface temperature. The data was recorded using a Lenovo T61 laptop using National Instruments LabVIEW 8.5.
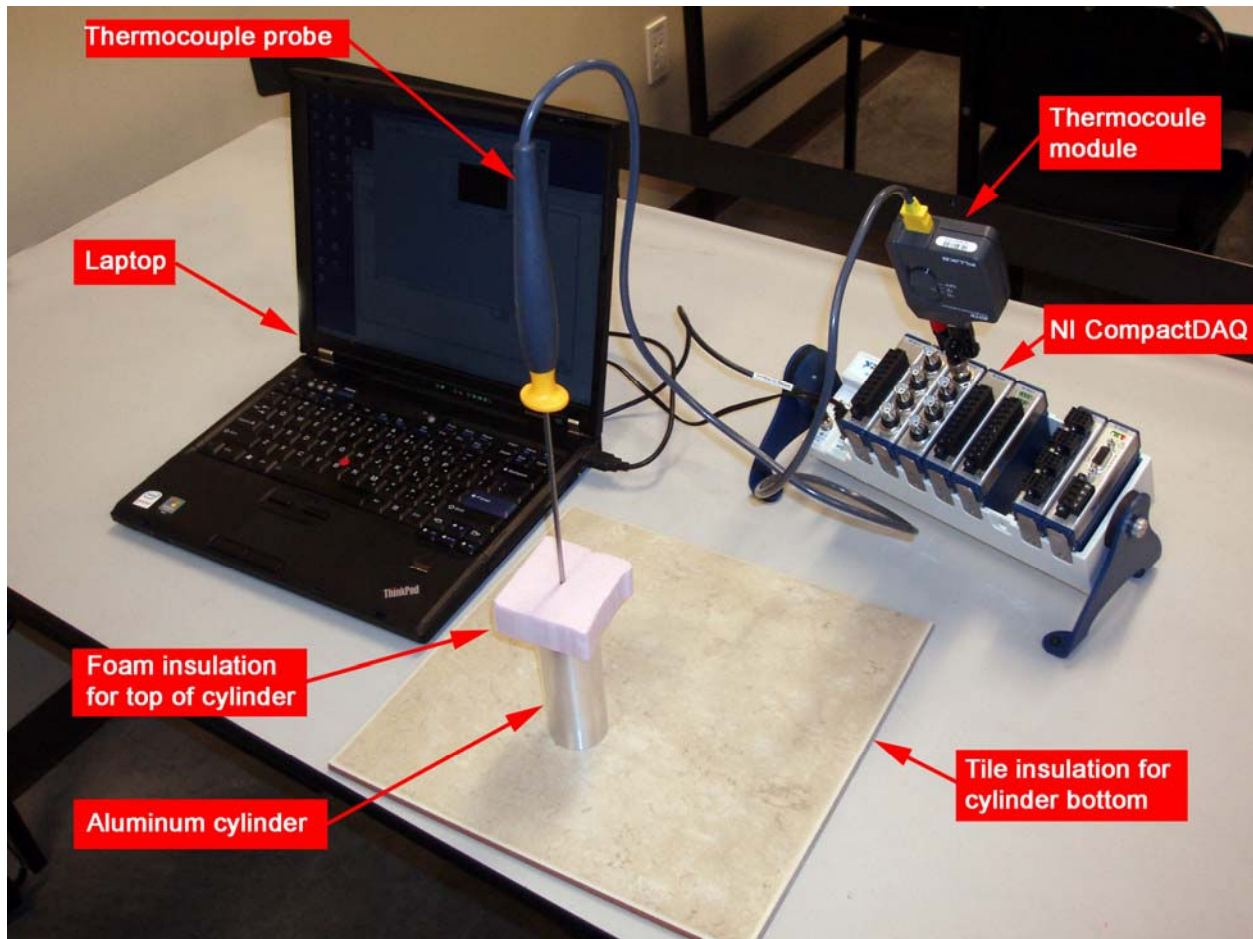
Figure 3: Photo of experimental set-up.

Based on an in-class development, LabVIEW code was created and implemented for data collection. An understanding of the static and dynamic characteristics of the sensor and the temperatures to which it would be exposed allowed students to make intelligent selections for input range and sample rate. Calibration information for the sensor was used to convert the input voltages to a temperature in degrees Celsius. Before conducting the test, the students first measured the ambient room temperature with the thermocouple modules. It was noted that significant noise was in the analog signal and this gave students insight into the level of filtering and averaging that would allow them to collect meaningful time response data. The LabVIEW program displayed the data on the Front Panel for real-time monitoring while saving the recorded data for post processing.

Students were then instructed to heat the small aluminum cylinder to a temperature of 80°C in the oven provided (Cole Parmer StableTemp Gravity Convection Oven). Then removing the cylinder from the oven, students placed the thermocouple in the tap drilled to the cylinder's center along the centerline of the cylinder. With this, they recorded the temperature as the cylinder cooled under ambient conditions. An example of student collected data is given in Figure 4.
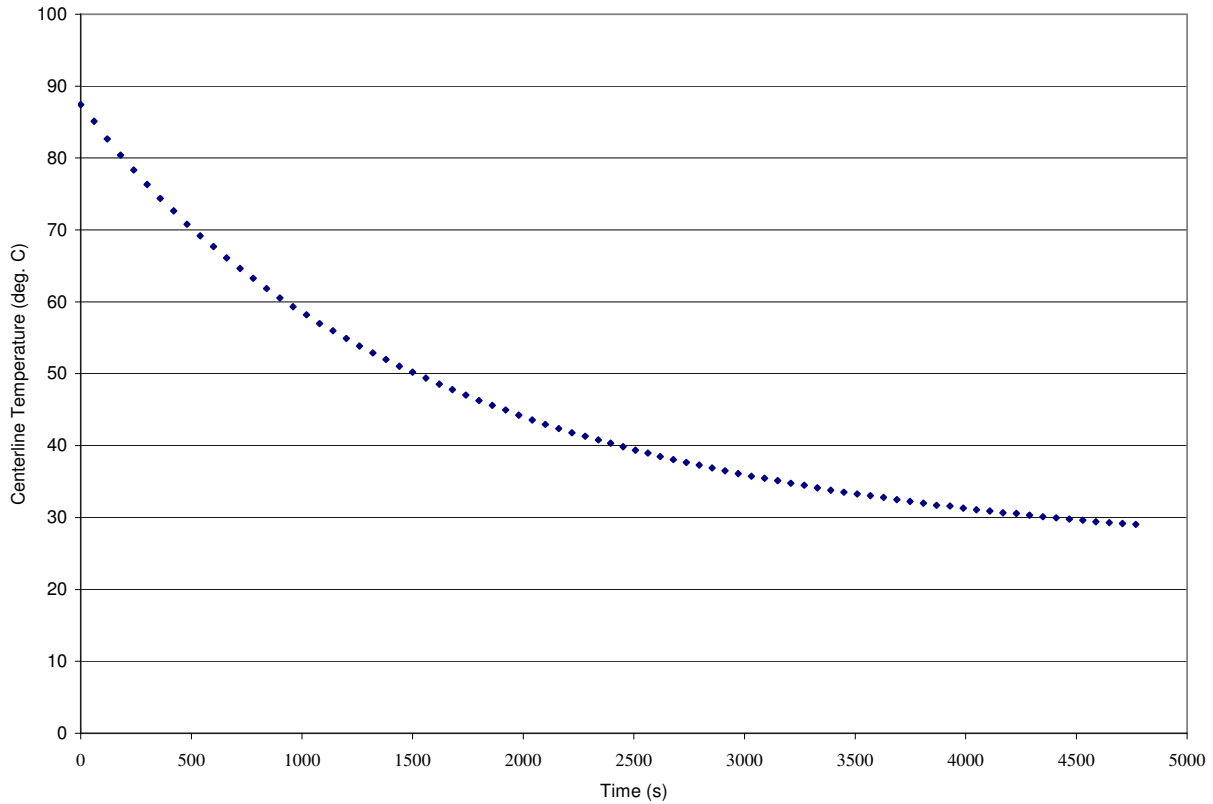
Figure 4: Experimental data.

An analysis of the data was done to determine the heat transfer coefficient for the cylinder. Assuming a constant temperature throughout the cylinder, as was confirmed in the theoretical analysis, the temperature of the cylinder can be related to the heat transfer coefficient with the following equation[8].

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{t}{\tau}\right)\right] \tag{8}$$

Where T is the temperature of the cylinder, T_∞ is the air temperature, T_i is the initial temperature, t is time, and τ is the time constant of the system, $\tau = \rho V c/(hA_s)$. In the equation for the time constant, h is the heat transfer coefficient, A_s is the surface area, ρ is the density, V is the volume, and c is the specific heat of the cylinder. In this way, the heat transfer coefficient was related to the time constant of the experimental system. An exponential curve fit resulted in an estimate of the time constant τ. With this estimation, the cylinder's known geometry, and tabulated values for the density and specific heat of aluminum, the heat transfer coefficient was calculated.

Results and Discussion
Upon completion of the experiment the eleven participating students were given a survey to rate the effectiveness of the two part exercise. The questions and results are shown in Table 1. For

questions one through five students were asked to respond based on the following scale: 5 – strongly agree, 4 – agree, 3 – no opinion, 2 – disagree, and 1 – strongly disagree. For questions six and seven, the scale was given with: 5 – full, 4 – high, 3 – medium, 2 – low, and 1 – none.

Table 1: Student Survey and Response.

Survey Statement	Response Average
1. The experiment helped to increase my understanding of heat transfer.	3.00
2. Similar experiments should be developed related to other topics discussed in lecture courses.	4.36
3. I clearly see the link between the experimental measurements and the numerical simulation completed in heat transfer	4.09
4. The experiment gave me a better understanding of the heat transfer coefficient and time constant.	3.18
5. Conducting the experiment before conducting the theoretical analysis would be more beneficial than the current order of events.	2.55
6. Level of understanding after completing the theoretical analysis in heat transfer (ME336)	4.27
7. Level of understanding after completing both the theoretical analysis (ME336) and the experimental analysis (ME491)	4.36

As shown with the scores in question one, the students had no net feeling that the experiment increased their understanding of heat transfer. This, however, is not an indication that the experiment was not of benefit. The response to question six indicates that the students felt they had a strong understanding before attempting the experiment, leaving little room for improvement. Still, responses to question seven do show a slight improvement in understanding was gained with the experiment.

Students were in agreement that the link between the simulation done in Heat Transfer and the experiment performed in Systems and Measurement was clear, indicating that the proper connection was made. Second, the students agreed that similar experiments would be of benefit as students make connections with other courses. This is supported by responses to questions three and two, respectively. Students did not perceive a benefit from changing the order of the experiment and theory, as indicated with the response to question five.

In contrasting the results from questions one and four, it is seen that students replied that they had no net increase in the understanding of heat transfer as a topic but did show a slight increase in understanding of the heat transfer coefficient and time constant. Similarly, a comparison of the results to questions six and seven indicates a slight improvement in the student's level of understanding. These seemingly contradictory results are an indication that the survey results would benefit from both questioning more students and perhaps a finer resolution in student response options (e.g. a ten point scale), thereby improving the confidence interval in the reported data. Another item to note is that there was a semester gap between the theoretical analysis and the experimental testing. This was due to the scheduling of courses, however it would likely be more beneficial to have the courses closer together in time.

Additional comments from the students indicated that the Systems and Measurement topics in the experiment were appreciated. Some stated they may have benefited from a more clear separation between the measurement component and the heat transfer component, suggesting breaking the lab into two parts with one focused on the data acquisition and processing and the other on the heat transfer phenomenon being revisited.

Conclusion

In this exercise students were given the theoretical development for a heat transfer phenomenon and asked to perform a simulation exploring methods for determining the heat transfer coefficient for a cooling cylinder. This topic was then revisited in a later course in hopes of making a strong connection with the previous learning. Results indicated that while the initial understanding was high, there was a slight overall increase in understanding. More significant was the indication that students did make a strong connection between the experiment and the previous work and that students felt strongly that they would benefit from more attempts to revisit learning from previous courses in an experimental setting. Future endeavors may be implemented in a way that helps students organize the material into aspects associated with measurement and those associated with the topic being revisited.

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