AC 2012-3731: INTERACTIVE MATH LEARNING FOR STEM STUDENTS

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Interactive Math Learning for STEM Students

Abstract:

The rapid development of computer and information technology impacts almost every aspect of our daily lives. It has been witnessed that the microprocessor changes from 8-bit to 64-bit and the display from monochrome monitors to millions of colors LED screens. Educators have also enjoyed the benefits on improved pedagogy in teaching STEM courses by integrating both the computer technology and up-to-date research results into the classrooms. However, the teaching of college level STEM obligatory math courses does not keep pace with the rapid development of the computer and information technology. To better prepare STEM students' math training still remains a challenge. Another simple reality is that today's college students are grown up in a multimedia world and they are enthusiasts of internet, video games, iPhones and iPads, MySpace, facebook, etc., even way before they enter colleges. This makes the math learning less attractive which further affects the enrollment and retention rate of every STEM program.

To address this issue, a group of faculty members with multidisciplinary background have investigated the teaching of the traditional math courses unconventionally. In addition to the traditional blackboard-textbook teaching, we integrate new teaching approaches that allow students to learn math concepts by interactively working with specially designed math teaching and learning modules. Our attention is focused on building learning modules for entry level college math courses, such as College Algebra and Calculus I. For each course a certain amount of time will be set aside for students to reinforce the concepts they just learned during the normal lecture time. The learning modules are designed to be user friendly in order to attract students' attention to math learning instead of texting in classrooms. The benefits of this pedagogy include: 1) interactive modules make students actively involved in the math learning process; 2) the unlimited randomly generated questions and examples give students more opportunities on practicing and reinforcing the concepts they just learned; 3) the quick answer checking function helps students build confidence by immediately identifying their learning progress; and 4) the mobility of the modules ensures that students can learn and practice their math concepts any time anywhere with their laptop PC. Vizard—a popular virtual reality programming environment—is chosen to design the course teaching modules to make the math learning full of fun.

In this project, faculty members developed new interactive teaching and learning modules and introduced them into the corresponding college math classes. Preliminary results on STEM students' opinion were obtained and obstacles were discussed. Based on feedbacks of our preliminary exploit, we will further improve our current teaching and learning modules and develop more modules to enhance STEM students' math learning.

1. Introduction

Nowadays, with the advancement of computer technology, the number of jobs requiring math and science is soaring. The rapid development of cutting-edge technology also demands well-prepared and highly-qualified workforce, with recent undergraduates in particular, in the fields of science, technology, engineering, and mathematics (STEM). Identifying this trend, most of countries in the world are investing heavily in math and science education at all levels. If the mathematics performance of 8th-grade students in the U.S. is compared to those of their counterparts in other industrialized countries in the year of 2007, US students scored only 508, which is ranked number 9 and 90 points less than the top-performed students from Chinese Taipei [1].

Another important indicator is the average mathematics scale scores of all nations in rank-order of fifteen-year-old students from across the world who took part in the Program for International Student Assessment (PISA), which gauges their math literacy [2]. Among the 34 nations under OECD jurisdictions, the US ranked 25th. By the time US students graduate from high school, their average mathematics scale score is near the bottom of all industrialized nations.

What causes all these problems? In fact, American students simply lost their interest in learning mathematics. According to a survey commissioned by Raytheon in 2009, more than 1,000 students between the ages of 10 and 15 participated to uncover the attitudes and behaviors of today's US middle school students towards math. The survey found that “72% of US middle school students spend more than three hours each day outside of school in front of a TV, mobile phone or computer screen rather than doing homework or other academic-related activities. By contrast, just 10% of students spend the same amount of time on their homework each day with 67% spending less than one hour on their math homework.” It continued revealing that “while most middle school students believe that math is important to their futures, they fail to understand the connection between the subject and potential careers.” [3]

To address this issue, a group of faculty members with multidisciplinary background have investigated the teaching of the traditional math courses unconventionally. As educators, it is our responsibilities to find a workable solution. It is widely accepted that increasing students' interest in mathematics is the foundation for tackling the above problem. In this paper, we concentrate on how to use modern technology to improve mathematics teaching and learning in the classroom. The general strategy we present here is to use 3D computer graphics, simulation and visualization technology to add a fun-filled dimension to mathematics learning and to make mathematics problems easy to understand and solve.

2. Existing Systems

There is currently no existing 3D computer graphics based college math teaching systems, to the knowledge of the authors. Similar systems include math software tools, such as Matlab, Maple, and 2D Java-based simulation software which are available throughout the Internet. All of these systems implement a 2D system, which lack the capability of presenting objects in a 3D coordinate system. Based on these observations, the authors developed an interactive 3D graphics college math system to tackle the abovementioned problems.

3. Illustration of a Sample Module Design

In this section, we illustrate by a typical basic topic in college algebra, namely, rational functions and inequalities, about the design of such a module corresponding to rational functions and rational inequalities. Before we introduce our teaching and learning module, it is worth taking a look at the math background first.

3.1 Rational Functions

A rational function is a function that can be written as a quotient of two polynomials. To be more precise, a function of the form $f(x) = \frac{P(x)}{Q(x)}$
is called a rational function, where both $P(x)$ and $Q(x)$ are polynomials, with $Q(x) \neq 0$.

An important concept for a rational function is asymptotic behavior. An **asymptote** of a curve is a line such that the distance from a point on the curve to the line approaches zero when x approaches to a number or $x \to \infty$. In the case of rational functions, there are three types of asymptotes: vertical, horizontal, and oblique. Below, the definitions for these three asymptotes are given:

Let $f(x) = P(x)/Q(x)$ be a rational function in its most reduced form.
If $|f(x)| \to \infty$ as $x \to a$, then the vertical line $x = a$ is a **vertical asymptote**.
The line $y = a$ is a **horizontal asymptote** if $f(x) \to a$ as $x \to \infty$ or $f(x) \to a$ as $x \to -\infty$.
An asymptote that is neither horizontal nor vertical is called an **oblique asymptote** or slant asymptote.

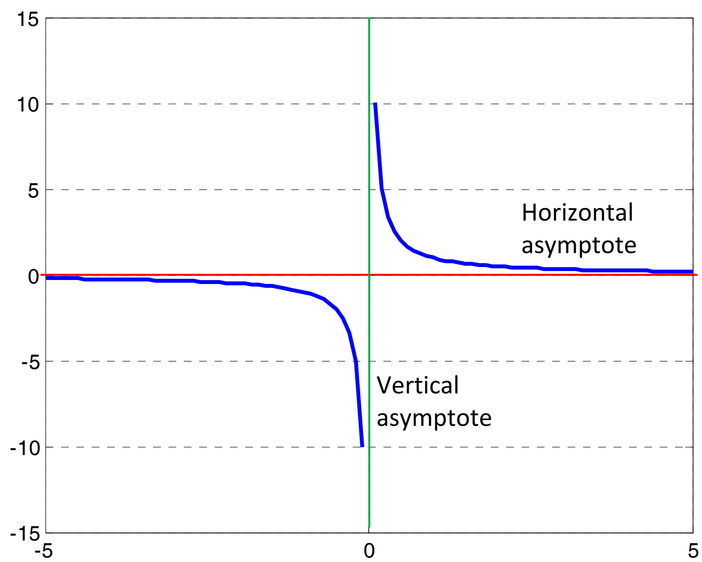


Figure 1: Example of asymptotes

An example of asymptotes is shown in Fig. 1. In this example, for the rational function $f(x) = 1/x$:

- $x = 0$ is a vertical asymptote, because when $x \to 0$, $|f(x)| \to \infty$.
- $f(x) \to 0$ when $x \to \infty$ or $x \to -\infty$, so, the $y = 0$ is a horizontal asymptote.

To find asymptotes for a rational function, the following procedure can be used.

Let $f(x) = P(x)/Q(x)$ be a rational function in its most reduced form with the degree of $Q(x)$ at least 1.

1. The graph of f has a vertical asymptote corresponding to each root of $Q(x) = 0$.
2. If the degree of $P(x)$ is less than the degree of $Q(x)$, then the x-axis is a horizontal asymptote.
3. If the degree of $P(x)$ equals the degree of $Q(x)$, then the horizontal asymptote is determined by the ratio of the leading coefficients.
4. If the degree of $P(x)$ is exactly 1 greater than the degree of $Q(x)$, then use division to rewrite the function as $mx + b + \frac{R(x)}{Q(x)}$. The graph of the equation formed by setting y equal to the quotient, i.e., $y = mx + b$, is an asymptote. This asymptote is an oblique or slant.

To graph a rational function in lowest terms:

1. Determine the asymptotes and draw them as dashed lines.
2. Check for symmetry.
3. Find all intercepts, if any.
4. Plot several selected points to determine how the graph approaches the asymptotes.
5. Draw curves through the selected points, approaching the asymptotes.

3.2 Rational Inequalities

An inequality that involves a rational expression, such as $\frac{6-x}{x-3} \leq 0$, is called a rational inequality. To solve the inequality, two different methods can be used: sign graphs or test-points. To illustrate graphically, this simple problem will be solved by using the sign graph method.

Problem: Solve $\frac{6-x}{x-3} \leq 0$. State the solution set using the interval notation.

Solution. Before such an inequality is solved, there must have 0 on one side. In this example, there is already a 0 on the right hand side. However, if there is not, subtraction and simplification can be used to make sure that a 0 is on one side.

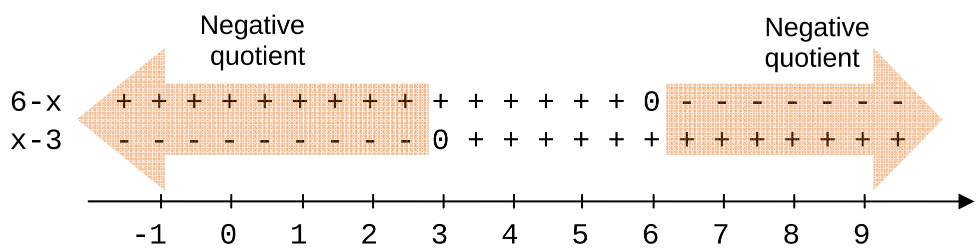


Figure 2: Sign graph

As illustrated by using the + and – signs in Fig. 2, the sign graph shows that $6 - x > 0$ if $x < 6$ and $6 - x < 0$ if $x > 6$. If $x > 3$, then $x - 3 > 0$ and if $x < 3$ then $x - 3 < 0$.

The inequality $(6 - x)/(x - 3) \leq 0$ is satisfied if the quotient of $6 - x$ and $x - 3$ is nonpositive, or $6 - x$ and $x - 3$ have opposite signs. The graph indicates that $6 - x$ and $x - 3$ have opposite signs and therefore a negative quotient when $x > 6$ or $x < 3$. If $x = 6$ then $6 - x = 0$ and the quotient is 0. If $x = 3$ then $x - 3 = 0$ and the quotient is undefined. So, 6 is in the solution set but 3 is not. The solution set is $(-\infty, 3) \cup [6, \infty)$.

This example shows how the signs of the linear factors determine the solution to a rational inequality. The sign graph method works only if the numerator and denominator can be factored. The test-point method works on any rational inequality for which all zeros of both the numerator and the denominator have to be found.

4. Module Design

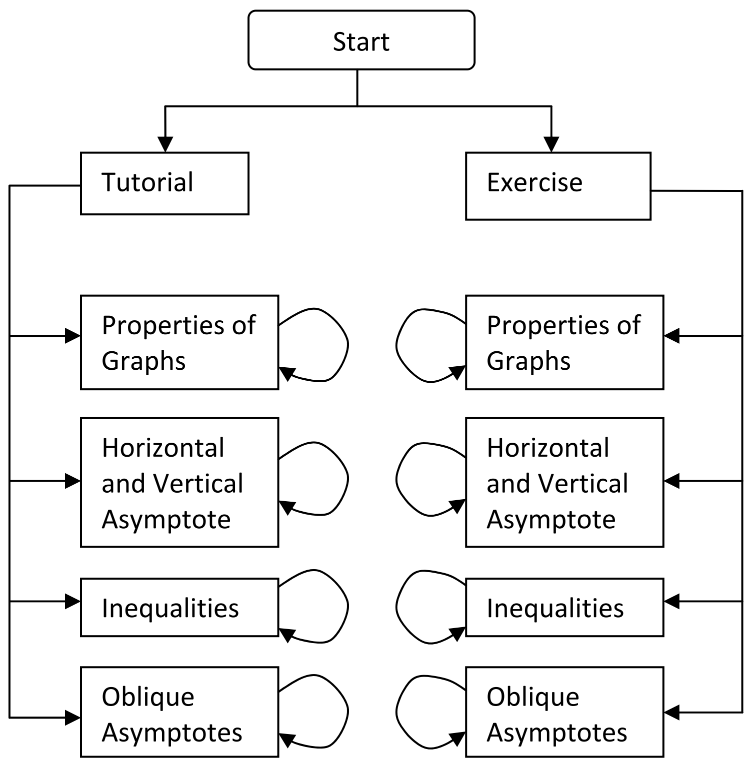
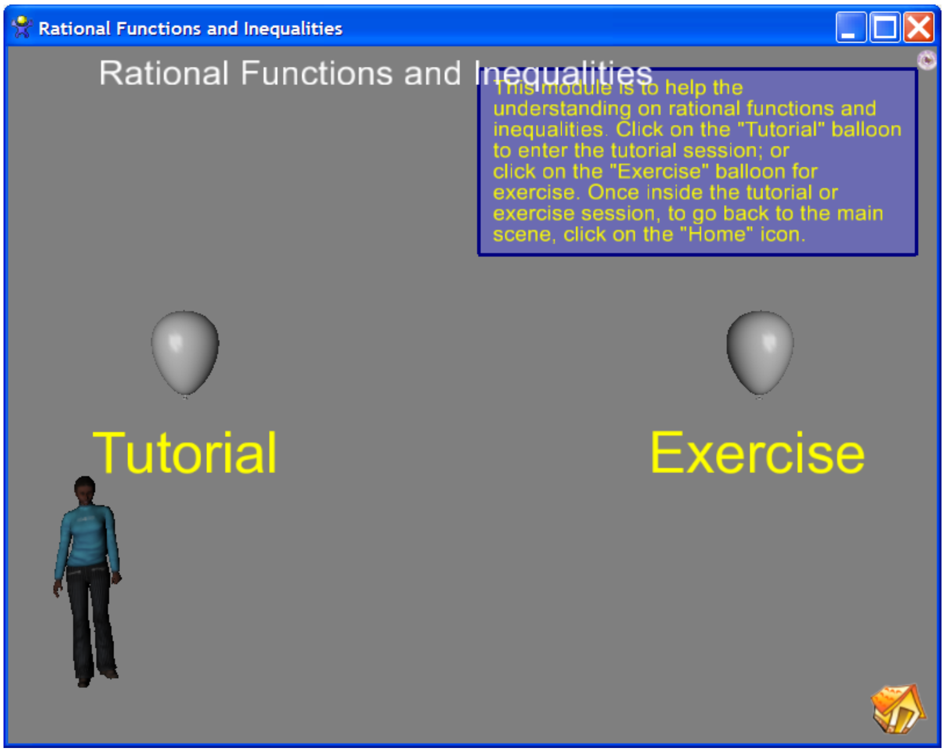


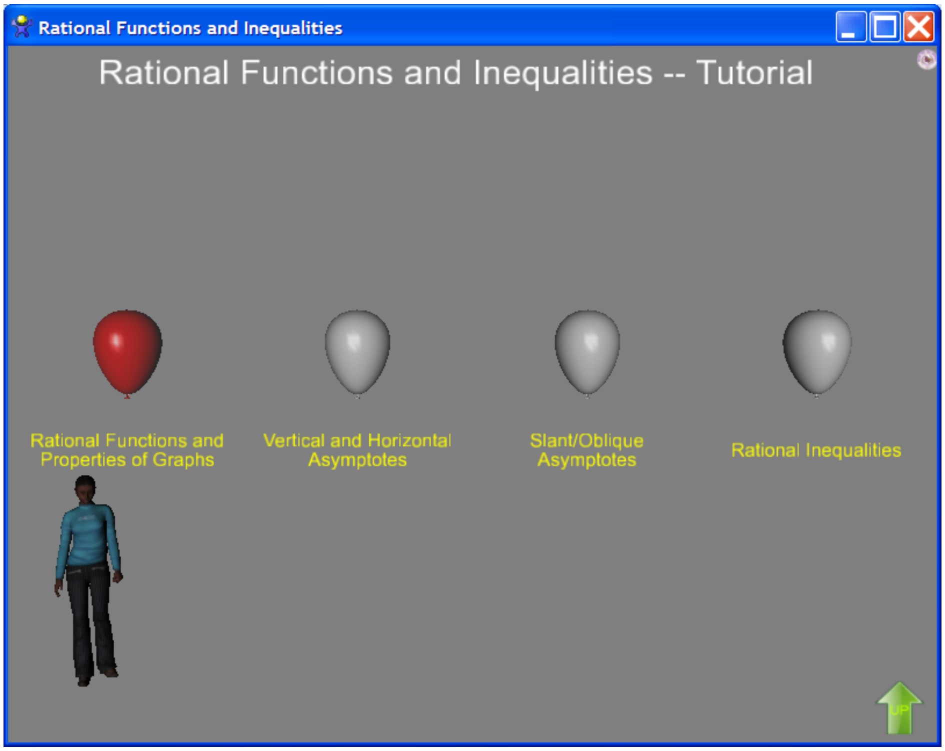
Figure 3: Block diagram of the teaching and learning module

To help students understand the math problem described in Section 3, we designed the "Rational Functions and Inequalities" math teaching and learning module. With the help of computer graphics technology, the teaching and learning module can display rational function graphs in a real-time manner. The module was designed by using Wordviz Vizard. To help understanding of the concepts better, two scenes—tutorial and exercise—are provided. In the tutorial scene, users can learn or review the math concepts by looking at randomly generated examples. In the exercise scene, users have the opportunities to interactively practice the math concepts. To make sure that every individual concept is included in the module, four topics—Property of Graphs, Horizontal and Vertical Asymptotes, Oblique Asymptotes, and Inequalities—are designed in each scene, as shown in the block diagram in Fig. 3. From Fig. 3, it can be seen that, after the module is started, the users can choose one scene to stay, tutorial or exercise. Once the users select the scene, they can choose one topic from the four to learn or practice, depending on which scene they are in.

After the program is started, the users choose which scene they want to go, as shown in Fig. 4a. By choosing tutorial, a window like Fig. 4b will be seen. If exercise is selected, an almost identical window will be displayed, with the same 4 topics for users to choose.



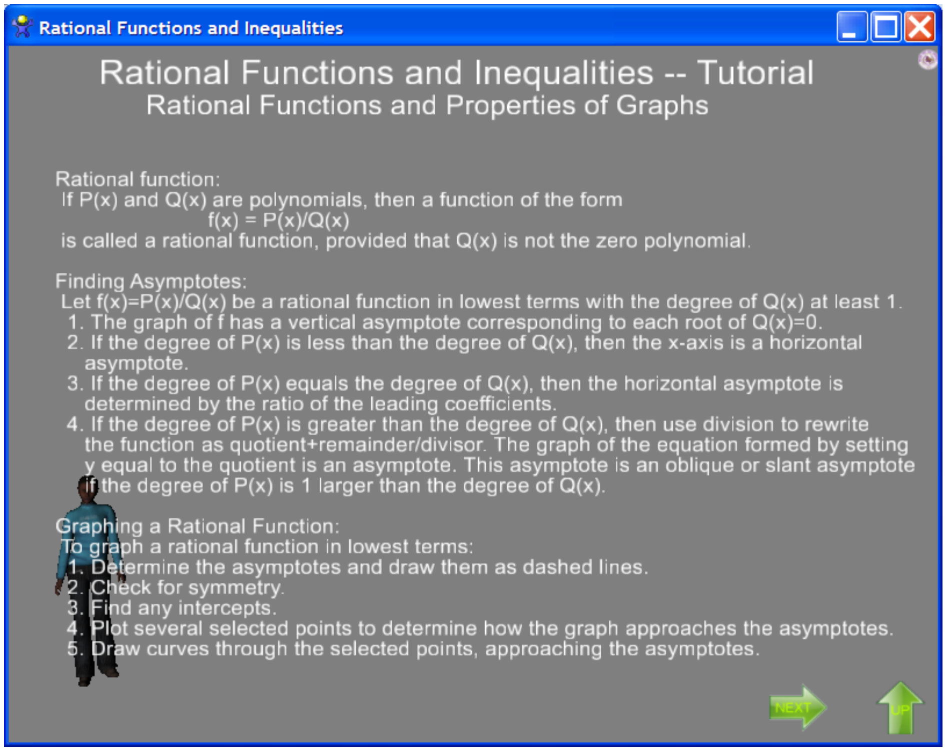
(a)



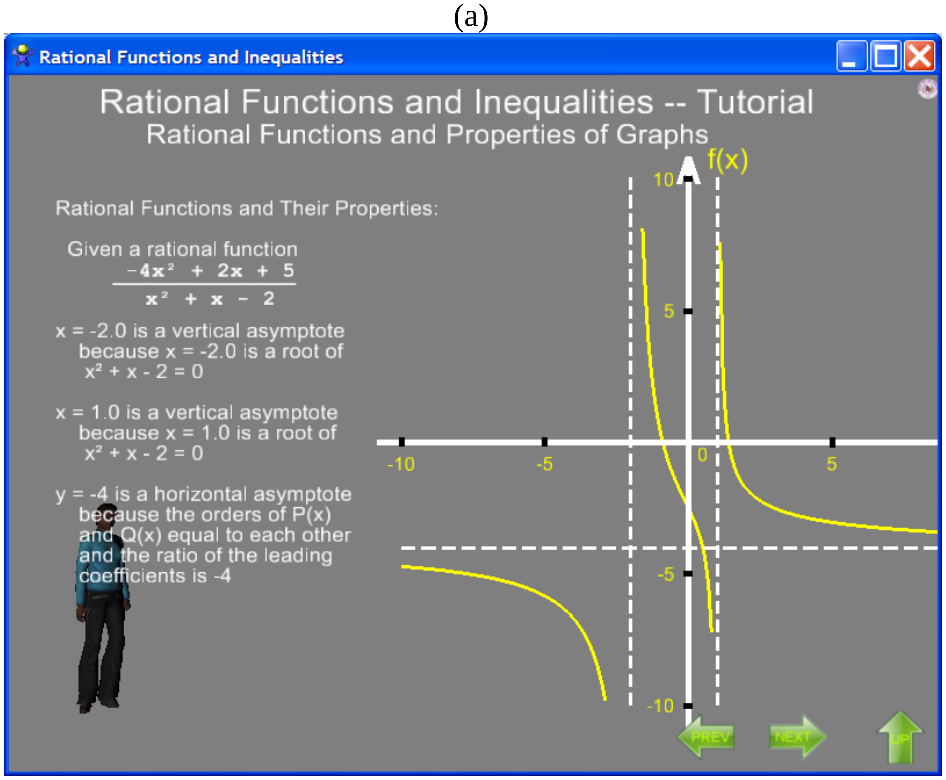
(b)

Figure 4: Starting of the program

In the tutorial scene, if topic “Properties of Graphs” is chosen, first a window as shown in Fig. 5a will be seen. Here the users can learn how to find asymptotes and how to graph rational functions. By clicking the “next” button, the module will randomly generate a rational function, graph it, and display the asymptotes, as shown in Fig. 5b. The same procedure applies to the other two topics: horizontal and vertical asymptotes (Fig. 6) and oblique asymptotes (Fig. 7).

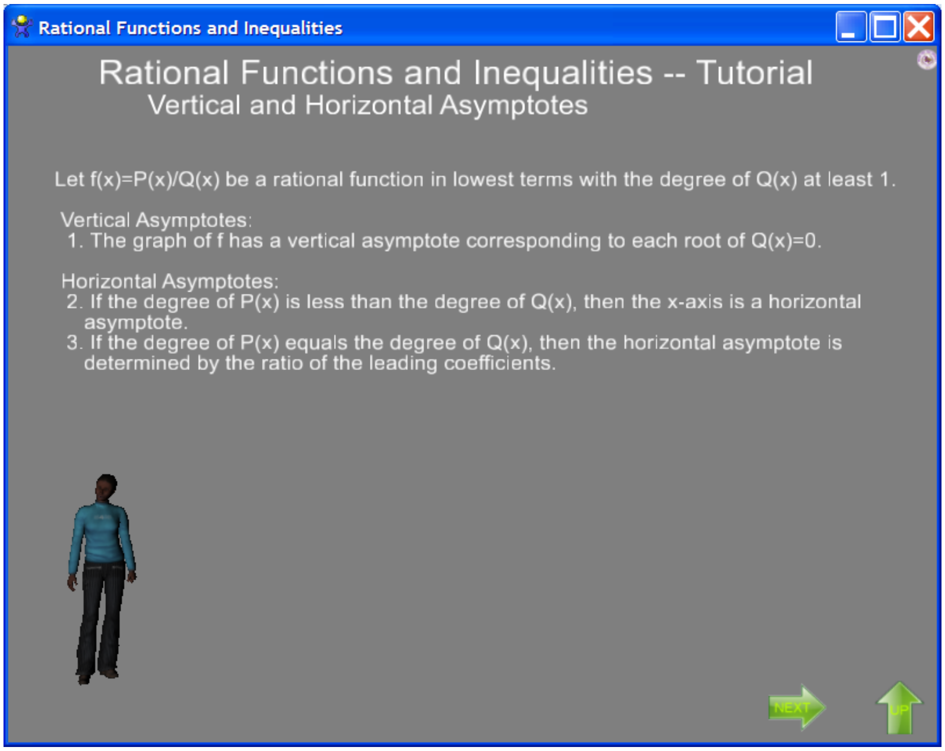


(a)

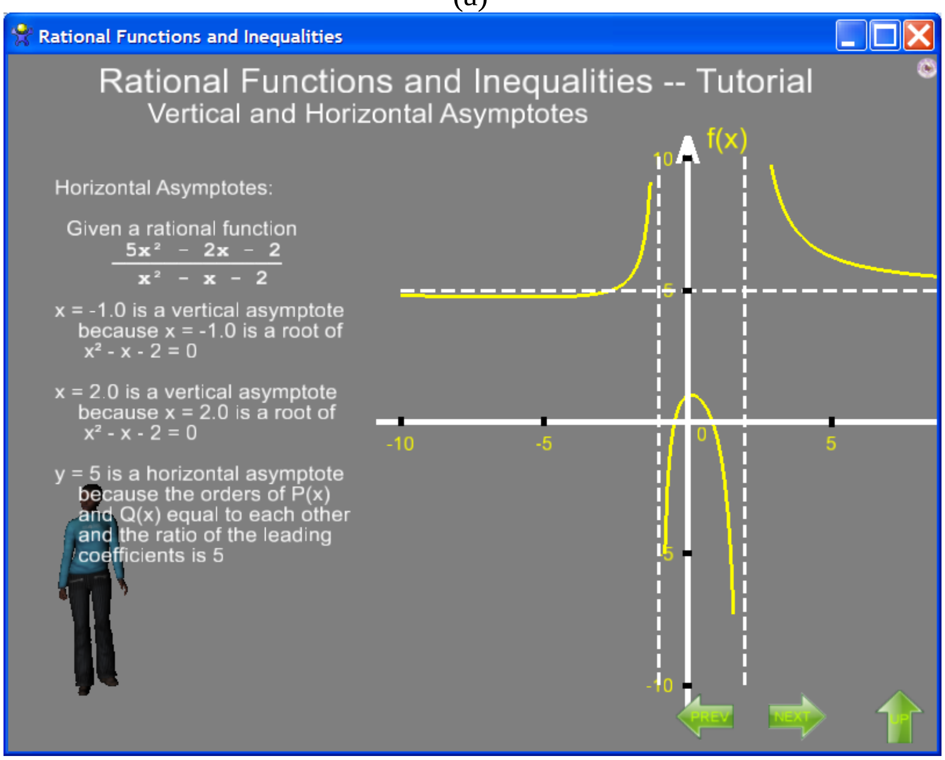


(b)

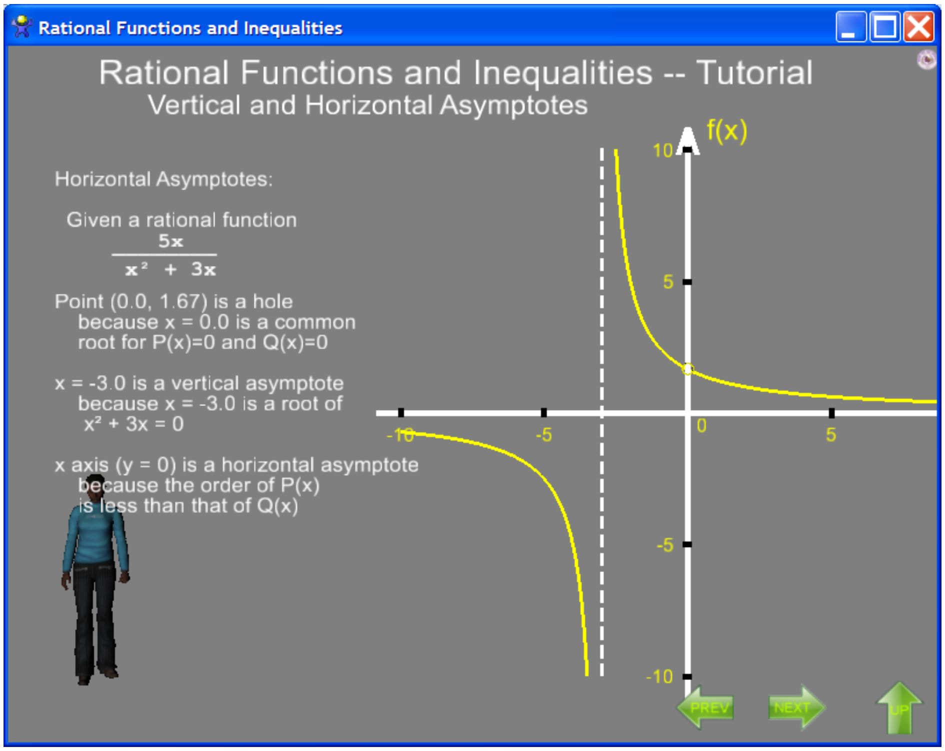
Figure 5: Tutorial—Properties of Graphs


Rational Functions and Inequalities
Rational Functions and Inequalities -- Tutorial
Vertical and Horizontal Asymptotes
Let f(x)=P(x)/Q(x) be a rational function in lowest terms with the degree of Q(x) at least 1.
Vertical Asymptotes:
1. The graph of f has a vertical asymptote corresponding to each root of Q(x)=0.
Horizontal Asymptotes:
2. If the degree of P(x) is less than the degree of Q(x), then the x-axis is a horizontal asymptote.
3. If the degree of P(x) equals the degree of Q(x), then the horizontal asymptote is determined by the ratio of the leading coefficients.
NEXT
UP

(a)

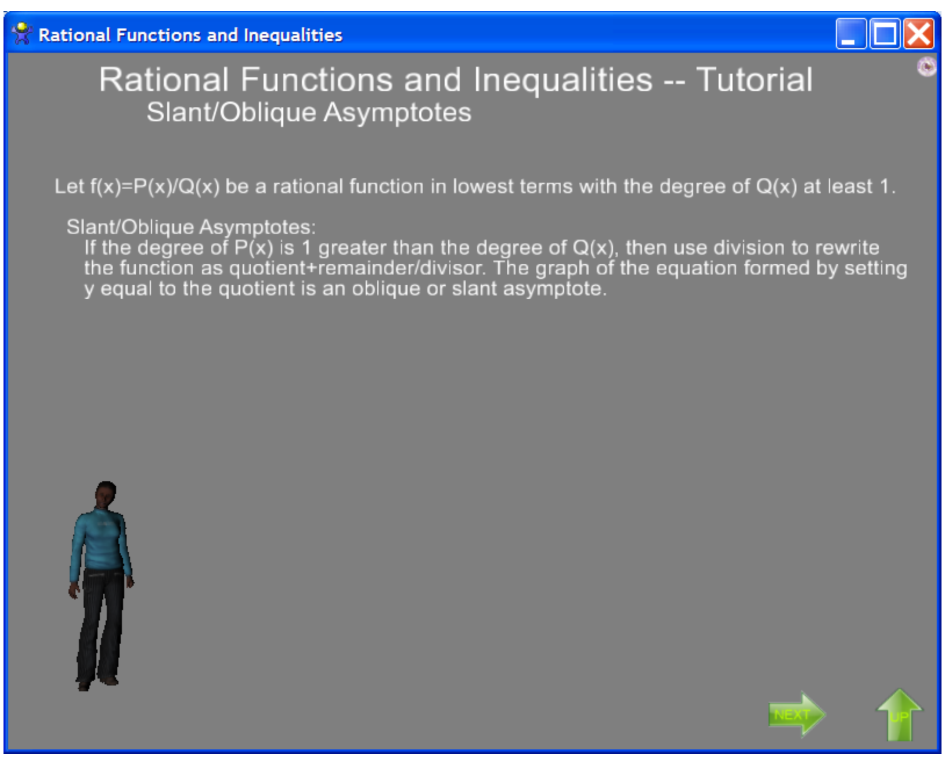

Rational Functions and Inequalities
Rational Functions and Inequalities -- Tutorial
Vertical and Horizontal Asymptotes
Horizontal Asymptotes:
Given a rational function
5x² - 2x - 2
x² - x - 2
x = -1.0 is a vertical asymptote because x = -1.0 is a root of x² - x - 2 = 0
x = 2.0 is a vertical asymptote because x = 2.0 is a root of x² - x - 2 = 0
y = 5 is a horizontal asymptote because the orders of P(x) and Q(x) equal to each other and the ratio of the leading coefficients is 5
f(x)
10
5
-10
-5
0
5
-5
-10
PREV
NEXT
UP

(b)

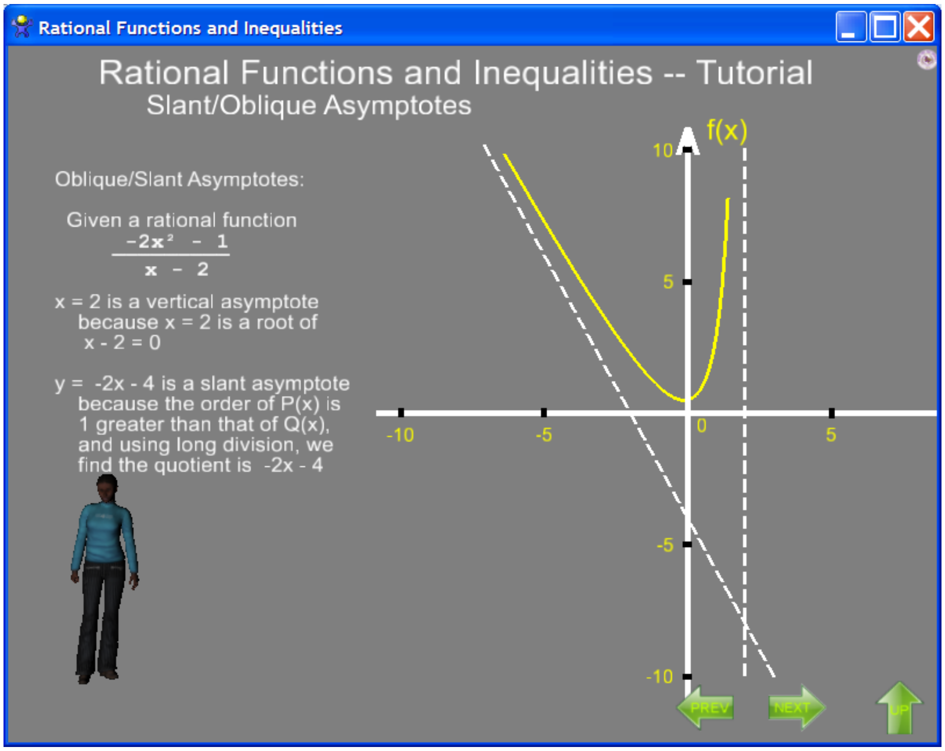


(c)

Figure 6: Tutorial—Vertical and Horizontal Asymptotes



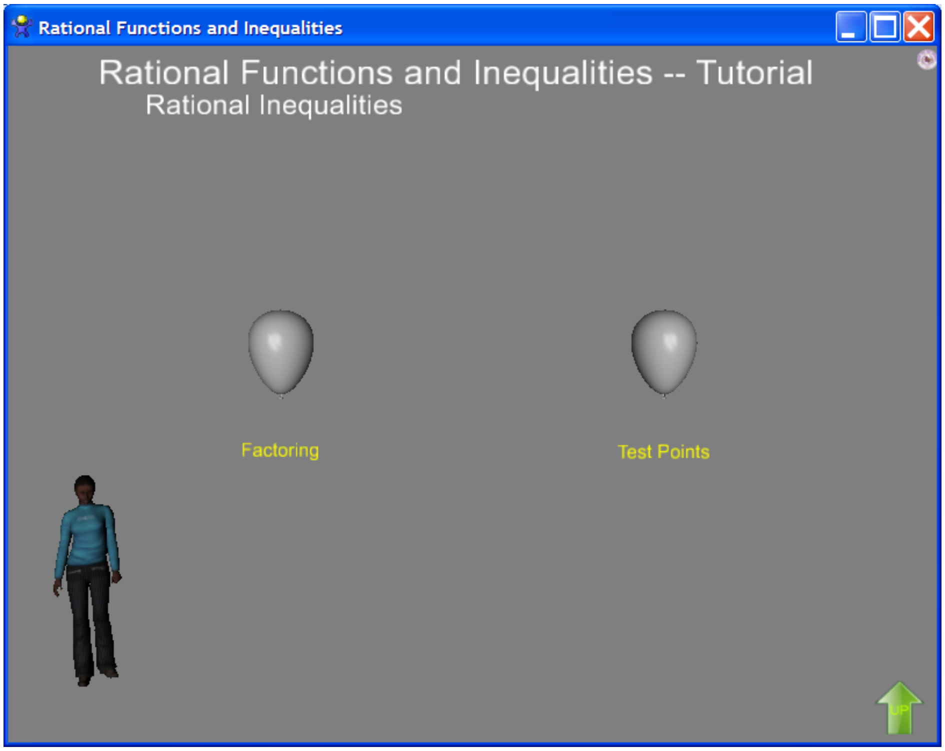
(a)

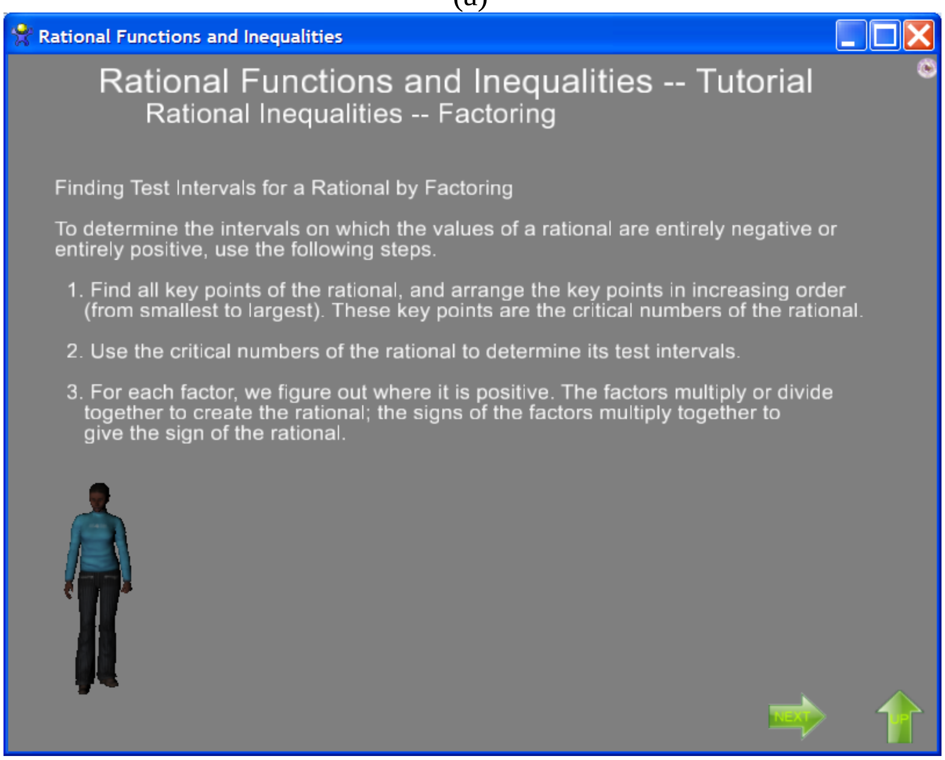


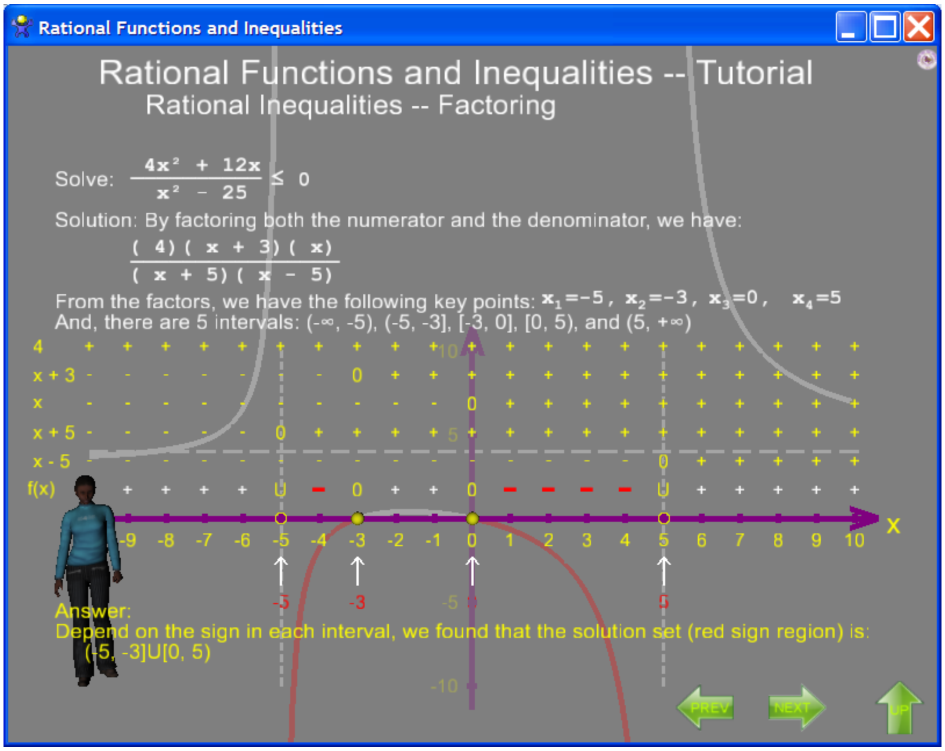
(b)

Figure 7: Tutorial—Oblique Asymptotes

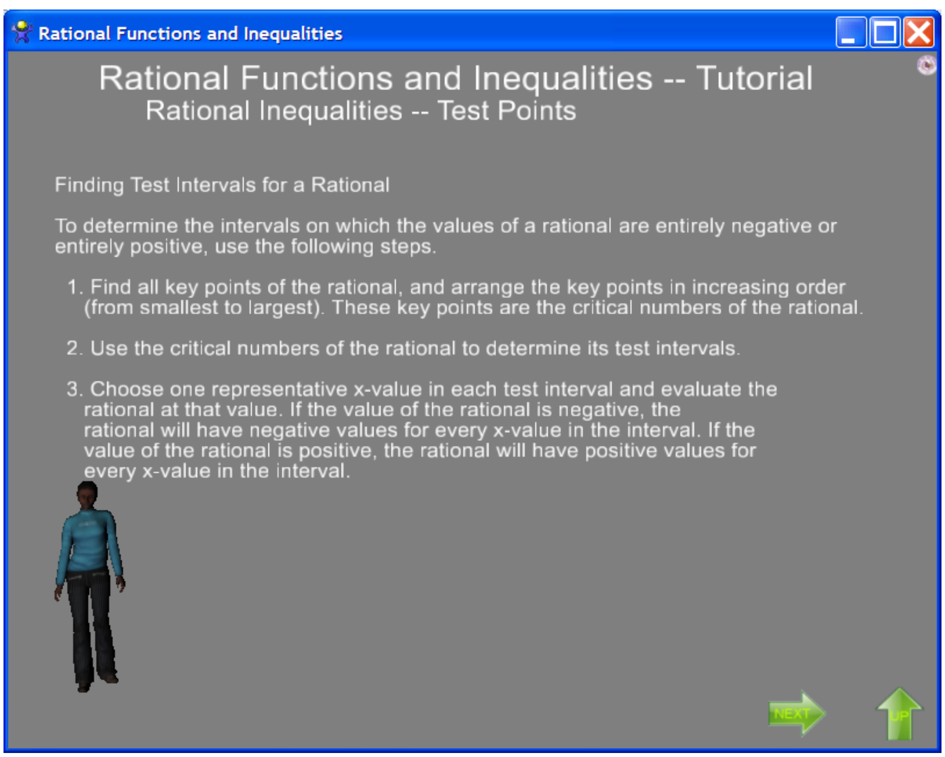
The inequality topic is different because there are two methods for solving rational inequalities: sign graphs (factoring) and test-points. If this topic is selected, the users will see a window with these two methods options, as shown in Fig. 8a. When sign graphs method is selected, the users will see windows as shown in Figs. 8b and 8c; while for test-points method, windows like Figs. 8d and 8e will be displayed.


Rational Functions and Inequalities
Rational Functions and Inequalities -- Tutorial
Rational Inequalities
Factoring
Test Points
(a)

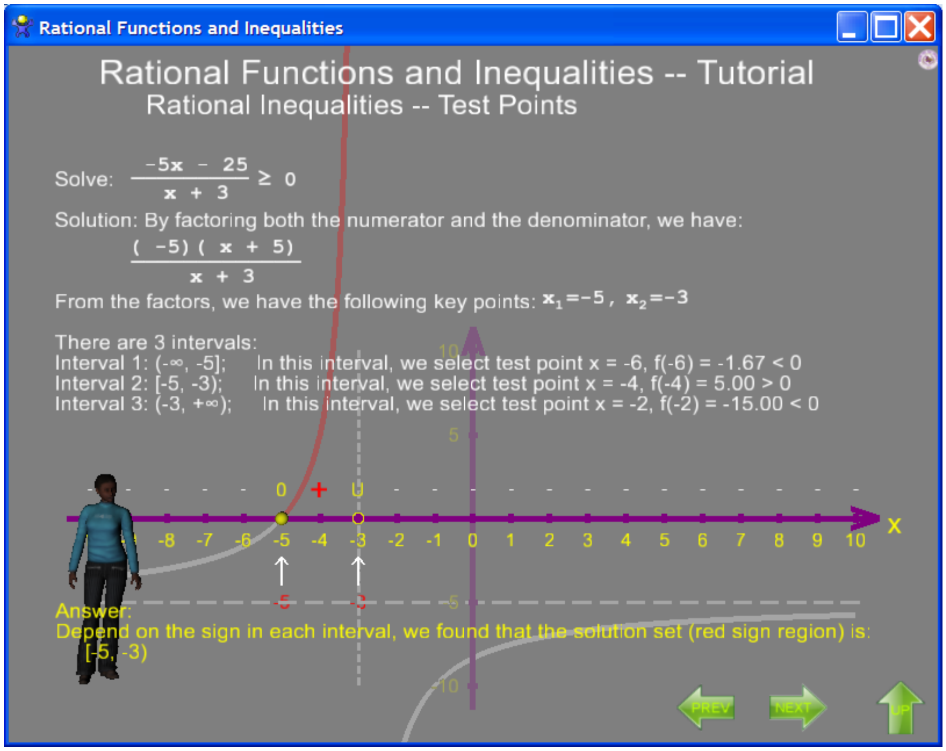

Rational Functions and Inequalities
Rational Functions and Inequalities -- Tutorial
Rational Inequalities -- Factoring
Finding Test Intervals for a Rational by Factoring
To determine the intervals on which the values of a rational are entirely negative or entirely positive, use the following steps.
1. Find all key points of the rational, and arrange the key points in increasing order (from smallest to largest). These key points are the critical numbers of the rational.
2. Use the critical numbers of the rational to determine its test intervals.
3. For each factor, we figure out where it is positive. The factors multiply or divide together to create the rational; the signs of the factors multiply together to give the sign of the rational.
(b)


Rational Functions and Inequalities
Rational Functions and Inequalities -- Tutorial
Rational Inequalities -- Factoring
Solve: $\frac{4x^2 + 12x}{x^2 - 25} \leq 0$
Solution: By factoring both the numerator and the denominator, we have:
$\frac{(4)(x+3)(x)}{(x+5)(x-5)}$
From the factors, we have the following key points: $x_1=-5$, $x_2=-3$, $x_3=0$, $x_4=5$
And, there are 5 intervals: (-∞, -5), (-5, -3], [-3, 0], [0, 5), and (5, +∞)
4
x + 3
x
x + 5
x - 5
f(x)
X
Answer:
Depend on the sign in each interval, we found that the solution set (red sign region) is:
(-5, -3]U[0, 5)
PREV
NEXT
UP

(c)


Rational Functions and Inequalities
Rational Functions and Inequalities -- Tutorial
Rational Inequalities -- Test Points
Finding Test Intervals for a Rational
To determine the intervals on which the values of a rational are entirely negative or entirely positive, use the following steps.
1. Find all key points of the rational, and arrange the key points in increasing order (from smallest to largest). These key points are the critical numbers of the rational.
2. Use the critical numbers of the rational to determine its test intervals.
3. Choose one representative x-value in each test interval and evaluate the rational at that value. If the value of the rational is negative, the rational will have negative values for every x-value in the interval. If the value of the rational is positive, the rational will have positive values for every x-value in the interval.
NEXT
UP

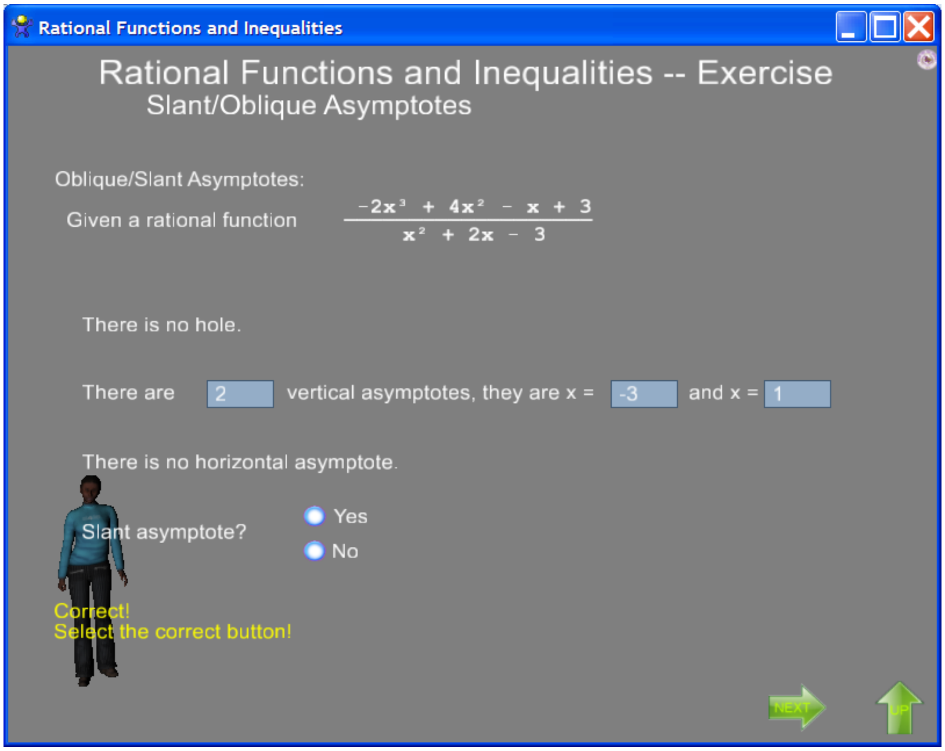
(d)



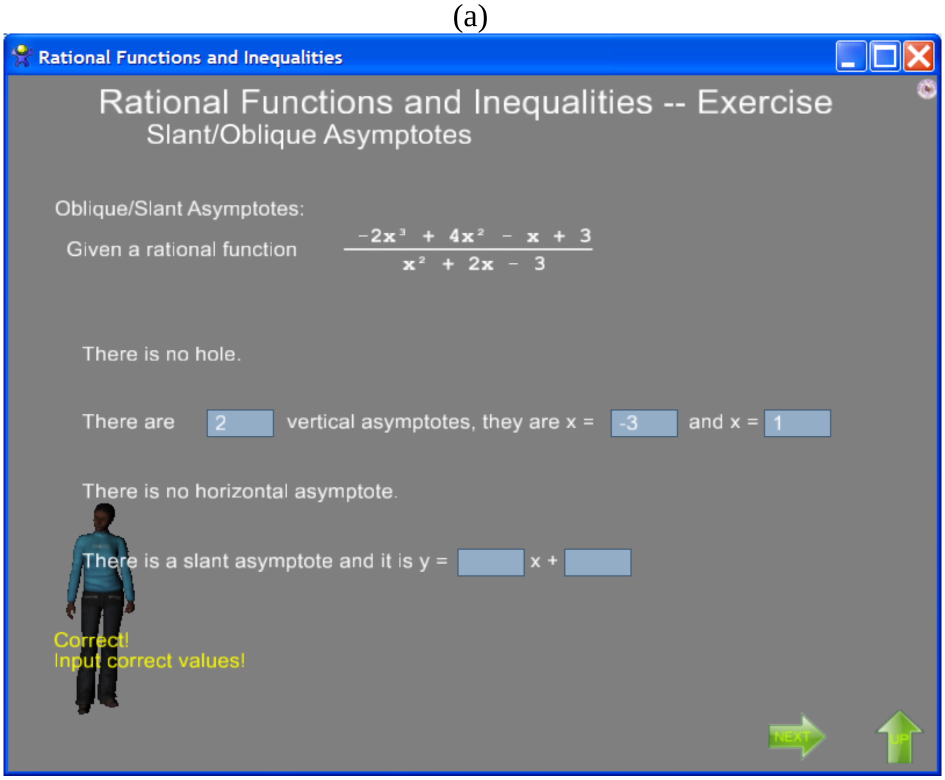
(e)

Figure 8: Tutorial—Inequalities

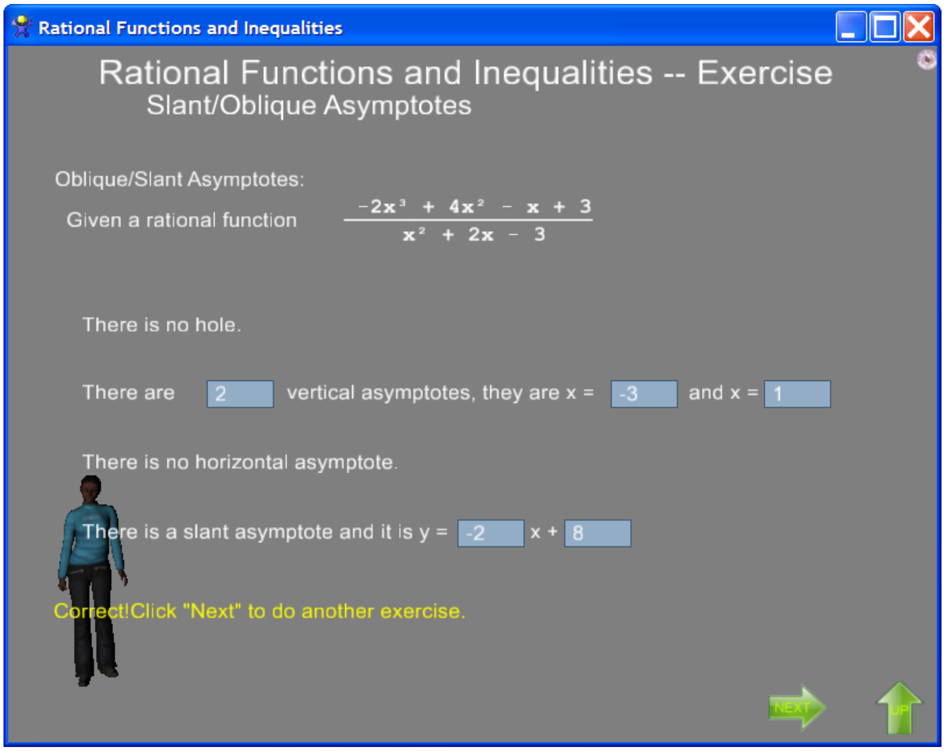
In the exercise scene, the same four topics are provided. For example, if “oblique asymptotes” is chosen, the users will see windows as shown in Fig. 9. The users have to answer the questions by filling out the blanks. To make the program interesting, if the answer is correct, the avatar will clap her hands; and if it is wrong, the avatar will dance. Fig. 10 shows the windows that will be displayed when the users choose to practice rational inequalities.


Rational Functions and Inequalities
Rational Functions and Inequalities -- Exercise
Slant/Oblique Asymptotes
Oblique/Slant Asymptotes:
Given a rational function
$\frac{-2x^3 + 4x^2 - x + 3}{x^2 + 2x - 3}$
There is no hole.
There are 2 vertical asymptotes, they are x = -3 and x = 1
There is no horizontal asymptote.
Slant asymptote?
Yes
No
Correct!
Select the correct button!
NEXT
UP

(a)

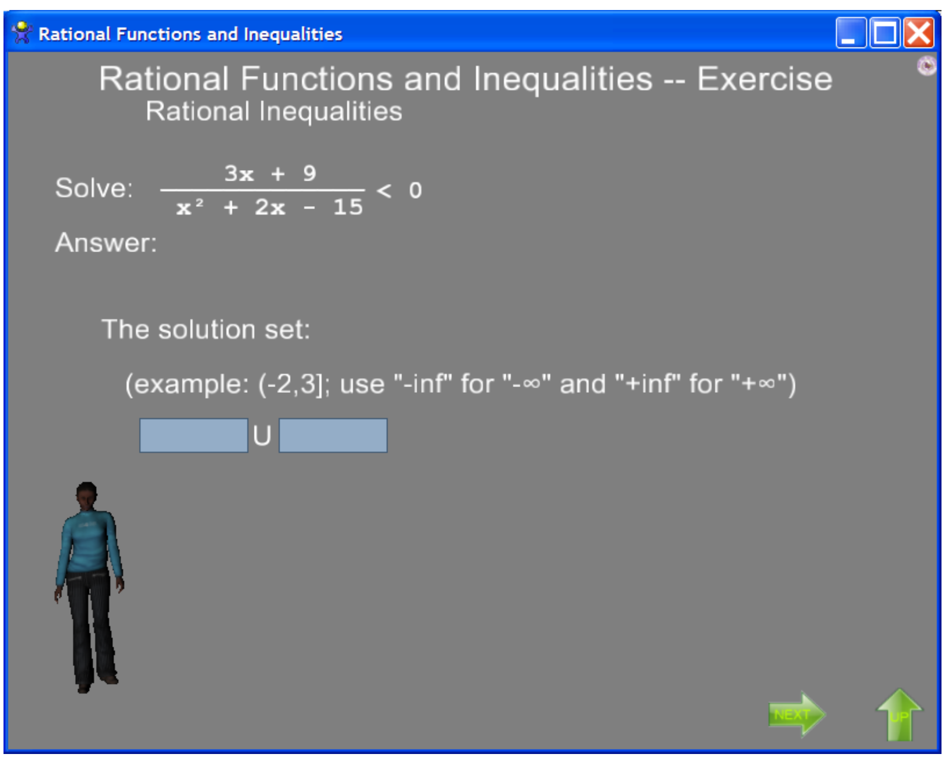

Rational Functions and Inequalities
Rational Functions and Inequalities -- Exercise
Slant/Oblique Asymptotes
Oblique/Slant Asymptotes:
Given a rational function
$\frac{-2x^3 + 4x^2 - x + 3}{x^2 + 2x - 3}$
There is no hole.
There are 2 vertical asymptotes, they are x = -3 and x = 1
There is no horizontal asymptote.
There is a slant asymptote and it is y = x +
Correct!
Input correct values!
NEXT
UP

(b)

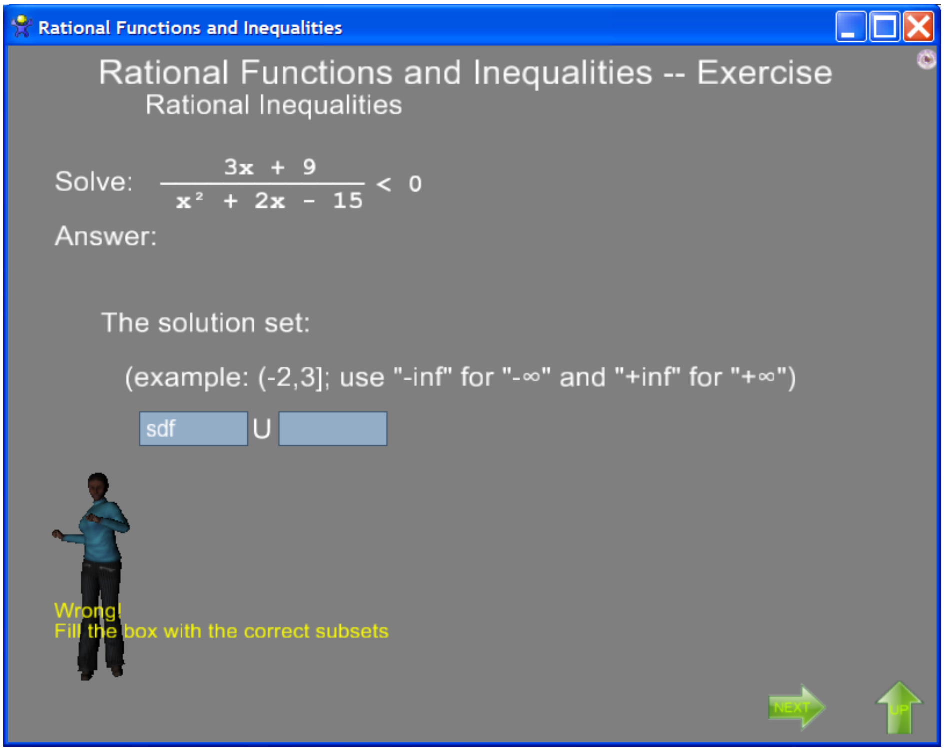


(c)

Figure 9: Exercise—Oblique Asymptotes



(a)



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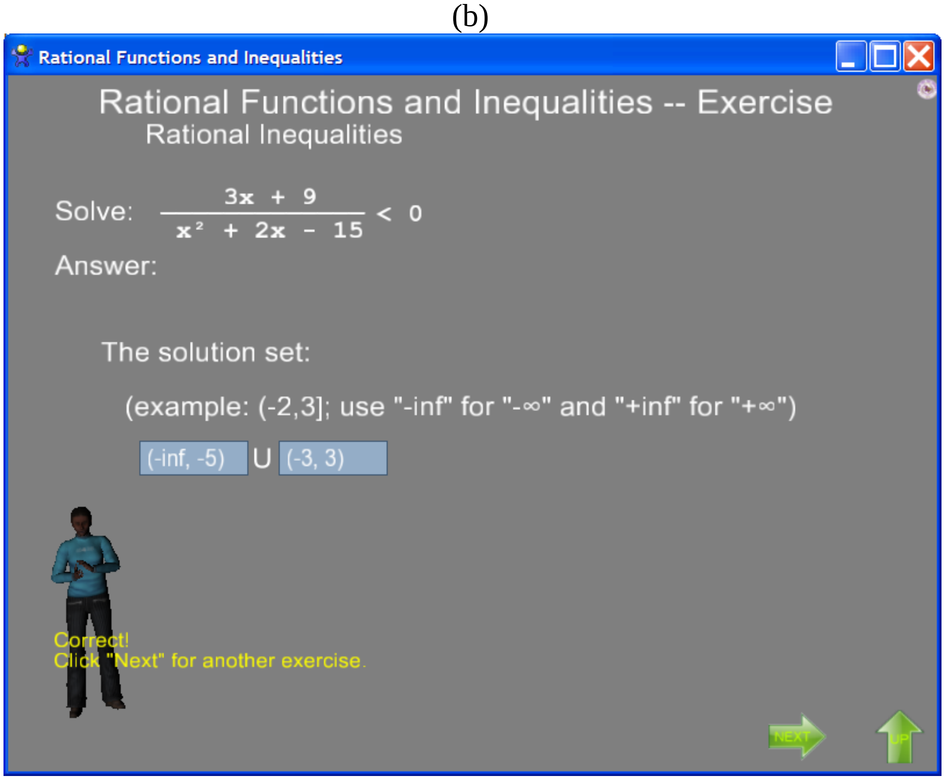

(c)

Figure 10: Exercise—Inequalities

5. Results

In the semester of fall 2011, 18 students in one College Algebra and Trigonometry section worked with the designed math modules. At the end of the semester the students were given a survey. The answers were collected and analyzed. Some of the survey questions and results are listed in Table 1 and the results are also shown in Fig. 11. Some students' feedbacks are interesting and worth to mention. For example, when answering the question: "In your opinion, what is the more effective way of learning math, namely, using the computer teaching modules or other technology or using the classical classroom (or whiteboard) instruction? Explain." one student indicated that "I like this method a lot, as it enables an individual to become sharper as a math student at PVAMU. However, I don't think anything beats a good old classroom math session."

Table 1: Student Survey Questions and Results

No.	**Question**	**Strongly Agree (%)**	**Agree (%)**
1	I wish I could use more of these modules	22	61
2	The modules have friendly platforms	33	67
3	These modules did help me understand and visualize the math concepts and notions easily	22	72
4	These modules enhanced my comprehensive math skills a lot	6	72
5	The use of the modules make math learning fun	11	72
6	I believe it is one of the best ways to learn math	22	33
7	I wish I would use the similar modules for my next upper level math course, if by anyway possible	28	56

For each question, students may select one answer within five options: (i) Strongly Agree; (ii) Agree; (iii) Disagree; (iv) Strongly Disagree; and (v) N/A. Students' responses are listed in the table as well as shown in Fig. 11. Obviously, the feedbacks are generally quite positive. For example, 100% students selected (i) Strongly Agree or (ii) Agree for question No. 2 "The modules have friendly platforms."

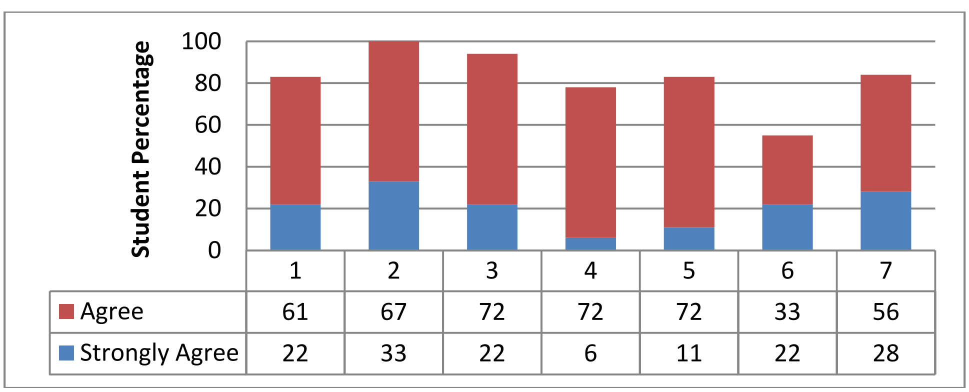

Figure 11: Students Survey Results

6. Conclusions

A computer graphics based interactive rational functions and inequalities teaching module has been introduced and designed. The learner-centered tutorial plus exercise model helps to motivate users and increase their learning confidence. The teaching and learning module is designed to be user friendly in order to attract students' attention to math learning instead of texting in classrooms. The benefits of this pedagogy include: 1) interactive modules make students actively involved in the math learning process; 2) the unlimited randomly generated questions and examples give students more opportunities on practicing and reinforcing the concepts they just learned; 3) the quick answer checking function helps students build confidence by immediately identifying their learning progress; and 4) the mobility of the modules ensures that students can learn and practice their math concepts any time anywhere with their laptop PC.

Based on the students' response and current success, we will further improve our current teaching and learning modules and develop more modules to enhance STEM students' math learning.

Acknowledgments. This research was partially supported by the U.S. Department of Education's Title III Part B Program under PR Grant # P031B070058.

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