AC 2009-287: INTRODUCING BIOMEDICAL ENGINEERING TO MECHANICAL ENGINEERS THROUGH THERMAL DESIGN PROJECTS

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Introducing Biomedical Engineering to Mechanical Engineers through Thermal Design Projects

Introduction

There is no question that an important part of the future of engineering will be in biomedical applications. Due to resources and/or politics many engineering schools cannot introduce an undergraduate program in Biomedical Engineering. More and more of the graduates from the mechanical engineering program at Michigan State University go on to careers in the biomedical industry with companies including: Stryker, Eli Lilly, General Electric Medical, and Abbott. Though the mechanical engineering program has, for more than twenty years, required a life science course, it still faces the challenge as to how to provide its graduates with some biomedical background without displacing topics needed for more conventional mechanical engineering careers in energy, aerospace, or manufacturing. The Department has taken two approaches to address this issue. In the first approach the Department has created a Biomechanical Engineering concentration, which appears on the student's transcript, and utilizes the four technical electives required in the mechanical engineering program. Over time, the faculty members of the department conducting biomedical research have developed courses in this discipline. These courses include Biofluid Mechanics and Heat Transfer, Tissue Mechanics, and Biomechanical Design. Additionally, students involved in undergraduate research in Biomedical Engineering may take the department's independent study course. Also, there are also several biomedical engineering courses offered by other departments that may be used. The requirements for the Biomechanical Engineering concentration for the BSME are shown in Figure 1.

The second approach, the focus of this paper, which has been implemented, involves the utilization of biomedical engineering projects in a traditional thermal design course. This paper continues with a brief description of the ME 416 course. This is followed by detailed project descriptions. Results of these projects, including student performance results will then be presented. Next, student reaction to the biomedical engineering theme is discussed through a survey that was administered. The paper concludes with lesson learned observations and recommendations for implementation.

About the Course

ME 416, Computer Assisted Design of Thermal Systems, is a 3 credit, senior level course that serves as a design elective in the mechanical engineering curriculum. It has a typical enrollment between 35 and 55 students. The course meets three lecture hours a week and two computer lab hours. The lectures are taught by a faculty member, while the computer lab is supervised by a graduate teaching assistant. There are three course goals for the course:

1. Development and practice of design skills as they apply to thermal systems.
2. Development of modeling skills.
3. Development and refining of computer skills

Figure 1 Biomechanical Concentration

Department of Mechanical Engineering

BIOMECHANICAL CONCENTRATION

(16-17 Credits)

A mechanical engineering degree with the biomechanical concentration prepares students for both traditional mechanical engineering as well as bioengineering. Engineers trained in biomechanical engineering find work designing, for example, prosthetics, artificial joints, automotive safety equipment, robotics for telemedicine, heart valves, left ventricle assist devices, and the whole range of medical devices. Research by biomechanical engineers includes studying the strength of bones and soft tissues, the motion of cells, the kinematics of human motion, and the flow of blood.

To complete a Bachelor of Science degree in mechanical engineering with a biomechanical concentration, students must complete the requirements for the B.S. degree, including the following:

•PSL 250 Introductory Physiology–4 credits (Fall and Spring)
•BS 111 Cells and Molecules–3 credits (Fall, Spring and Summer)

Plus _9 credits_ from the following list:

•ME	490 Independent Study[1]	1-4 credits (Fall, Spring, Summer)
•ME	491 Selected Topics[1]	1-4 credits (Fall, Spring, Summer)
•ME	494 Biofluid Mechanics & Heat Transfer	3 credits (Fall Only)
•ME	495 Tissue Mechanics	3 credits (Spring Only)
•ME	497 Biomechanical Design	3 credits (Spring Only)
•BE	445 Biosensors for Medical Diagnostics[2]	3 credits (Spring Only)
•ECE	445 Biomedical Instrumentation	3 credits (Fall of Even Years Only)
•MSE	425 Biomaterials & Biocompatibility[3]	3 credits (Spring Only)

CREDIT DISTRIBUTION: PSL 250 will be applied to the Bioscience requirement, and BS 111 will be applied to Other Electives. The nine engineering credits will be applied to the Senior Elective requirement (_not_ the "design intensive" course component, however). Completion of the concentration will be noted on the final transcript.

[1]Requires department approval.
[2]One of the prerequisites is (BS 111).
[3]One of the prerequisites is (PSL 250 or concurrently).

Curriculum\BiomechCon.pmd
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ME 416 is a project intensive course with the students completing five projects during the semester. The projects take on the following forms:

- Project 1: Development of a spreadsheet data base for the properties of a specified substance.
- Project 2: Development of a computer model for a turbomachine, design analysis for a turbomachine, selection of a turbomachine for a specified flow system.
- Project 3: Climate control design for a space.
- Project 4: Design of a thermal system.
- Project 5: Design of a heat exchanger within the thermal system of Project 4.

Project 1 is done on an individual basis, so as to raise all students' computer skills to an acceptable level, while projects 2, 3, 4, and 5 are done in teams of two. Projects 4 and 5 are done together, so as to emphasize the feedback between system design and equipment selection. Each project requires a technical memo documenting the students' work. Projects 1 and 2 also require the submission of the software developed by the student teams, which are tested by the instructional staff.

Each semester the projects are connected through a theme. Previous themes have included automotive systems and air processing systems. During the fall semester of 2005 and the fall semester of 2008, a biomedical engineering theme was used for the course. The projects are also framed in the context of a contract for an engineering firm, which provides the students with some insight into that career path.

About 50% of the lecture time is spent introducing and discussing the projects. The remaining 50% addresses the technical aspects of property analysis, turbomachinery, engineering economics, optimization, thermal system modeling, and heat exchanger design. Four quizzes are given during the semester, which are intended to draw the some of the students' attention to the lecture material.

Biomedical Engineering Projects

Five biomedical engineering projects were developed and utilized for the course. A brief description of each project is given below with some detailed attention given to the thermal system design project (Project 4). Details for all of the projects and software provided may be accessed through the course web site at

http://www.egr.msu.edu/classes/me416/

Though the students come into the class having completed a required life science course, it is important to provide them with additional background on biomedical issues tied to the projects. This is accomplished through lectures, the project descriptions, and additional handouts. One or two lecture periods (50 minutes) are devoted to the introduction of a project to the students. During these presentations the pertinent biomedical background is discussed. This may include the physical nature of blood, tissue damage due to burns, or kidney function. As in seen in the description for Project 4, provided in Appendix I, biomedical background is provided in this medium. Finally, some focused handouts are posted separately on the course web site.

<u>Property Project: Computerized Data Base for Human Blood Properties</u>

The student is asked to develop an Excel spreadsheet that will allow the calculation of a number of human blood properties at specified hematocrit (over a range from 0 to 0.60), temperature (over a range from 280 K to 330 K), and pressure (over a range from 80 kPa to 150 kPa). The set of thermodynamic properties to be calculated include yield stress, specific volume, density, specific heat, internal energy, enthalpy, entropy, and the thermal expansion coefficient. For the thermodynamic properties constitutive equations are provided to the students. The students are directed to use the symbolic manipulator associated with MATLAB to evaluate thermodynamic properties that require integration or differentiation. The following transport properties are to be included in the spreadsheet: thermal conductivity, thermal diffusivity, dynamic viscosity; kinematic viscosity, and Prandtl number. Tabular data are provided for the transport properties. The students are directed to utilize a curve fit for one transport property and a table look-up for a second transport property. The other three transport properties can then be evaluated through the three relationships among the transport properties.

To assist the students and ease the burden of grading, a template is provided, which is shown in Figure 2.

Figure 2. Excel Template for Blood Property Project

ME 416 CAD of Thermal Systems Fall 2008
Student: TECAD Rhino Thermophysical Properties of Blood

		Case #1		
		Hematocrit	Temperature (K)	Pressure (kPa)
Property	Units	Student Values	Course Values	% Difference
Yield Stress	N/m^2			#DIV/0!
Density	kg/m^3			#DIV/0!
Specific Volume	m^3/kg			#DIV/0!
Specific Heat	kJ/(kg·K)			#DIV/0!
Internal Energy	kJ/kg			#DIV/0!
Enthalpy	kJ/kg			#DIV/0!
Entropy	kJ/(kg·K)			#DIV/0!
Thermal Expansion	1/K			#DIV/0!
Thermal Conductivity	W/(m·K)			#DIV/0!
Thermal Diffusivity	m^2/s			#DIV/0!
Dynamic Viscosity	$N{\cdot}s/m^2$			#DIV/0!
Kinematic Viscosity	m^2/s			#DIV/0!
Prandtl Number	NONE			#DIV/0!

Dark shaded cells will be user input. Values for light shaded cells will be inputted during testing by the course instructor.

Once the spreadsheet is completed the student is required to assess its correctness through three comparison studies.

1. Quantitative comparison with transport property data provided in project statement.
2. Quantitative comparison with blood properties found in the literature.
3. Quantitative comparison of thermodynamic properties with those of water.

<u>Turbomachinery Project: Computer Simulation of Heart-Lung Bypass Pump Operation</u>
The student team is required to develop a MATLAB program that can be utilized to perform the pump calculations for a cardiac bypass machine. The program should be able to predict the power required, isentropic efficiency, and exit conditions for either

axial flow pumps or centrifugal pumps. The software should model any one of three pump cases: an ideal pump, an adiabatic pump with an isentropic efficiency specified by the user, or an actual axial flow or centrifugal flow pump represented by equations provided by the instructor.

Additionally, the student team is to use a MATLAB program and an Excel spreadsheet program provided by the instructor to perform pump design studies. Using the MATLAB program, the team will develop a graph of efficiency versus specific speed for the two pumps of interest, axial flow and centrifugal flow. Using the spreadsheet program, the team will design a pump for each of the two flow systems provided: an average size adult male patient and a neo-natal patient.

Climate Control Project: Climate Control Design of a Burn Patient Hospital Room
The student team is asked to design an optimum system for the heating system of a burn ward with respect to the insulation thickness of the walls, ceiling, and floor of the building, insulation type, and radiant heater height above the patient. Cost is used as the objective function for the optimization. An Excel spreadsheet program is provided that performs all of the needed calculations for a design case, which includes built-in cost information. Once the optimal design is identified, the team is asked to consider the robustness of the optimal design for the base conditions provided by varying the ambient temperature, building life, and different interest rate. Additionally, the team will determine the electricity cost that makes one of the non-preferred insulation types an equal choice to the preferred insulation type for the base case design and the insulation costs that makes the two non-preferred insulation types equal choices to the preferred insulation type.

Thermal System Design Project: Computer Assisted Design of a Kidney Dialysis System
The complete project statement for this project is provided in Appendix I. It begins by placing the project in the context of an engineering firm with the statements:

> The Rhino Thermal Engineering Company has recently received a contract with a major biomedical company to design a kidney dialysis system. Your project team has been assigned the task of developing a preliminary design for a hemodialysis system.

A background is provided on the hemodialysis process. Two design outcomes are identified: minimizing the patient time and minimizing the energy consumed by the machine. An objective function is identified to handle these two outcomes based upon a 75% weighting for the time and a 25% weighting for the energy consumed. It is defined by

Objective Function = (0.75)(normalized cycle time)
+ (0.25)(normalized energy consumption)

The two terms in the objective function are normalized with respect to the highest values for cycle time and energy consumption found from calculations. Calculations are performed over a range of dialysate volume flow rates and ultrafiltration coefficients for two blood flow rates. The preferred system configuration will be the case with the smallest objective function.

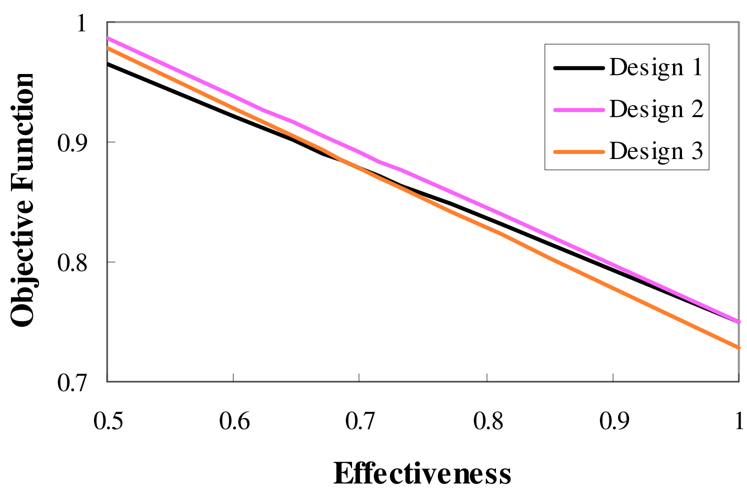
An in-house MATLAB program is provided to the students to perform the calculations. An example mapping of the objective function is shown in Table 1.

Table 1 Objective Function Values for a Blood Flow Rate of 400 cm^3/min

Dialysate Volume Flow Rate (cm^3/min)	Ultrafiltration Coefficient (mL/hr·kPa)				
	300	350	400	450	500
450	0.89075	0.79149	0.69222	0.64308	0.59262
500	0.90924	0.80793	0.70661	0.65643	**0.55511**
550	0.92774	0.7735	0.72099	0.61728	0.56642
600	0.94623	0.78891	0.73538	0.62961	0.57772
650	0.91016	0.80432	0.74976	0.64228	0.58902
700	0.92762	0.81974	0.7102	0.65461	0.60032

In addition to the optimization, the team is asked to perform two robustness studies on the preferred system: one for the blood pump pressure boost and the other for the heat exchanger effectiveness. Example results for the heat exchanger effectiveness robustness study are shown in Figure 3.

Figure 3 Heat Exchanger Effectiveness Robustness Results



It is of interest for the students to note the crossover point between Design 1 and Design 3 at an effectiveness of about 0.7.

The project is graded with the grading sheet shown in Figure 4. This also includes the grading of Project 5, since a single technical memo is required for the two projects

Figure 5 Grading Sheet for Projects 4 and 5

Project Grade Evaluation
Projects #4 & #5 Dialysis Machine and Heat Exchanger Optimization

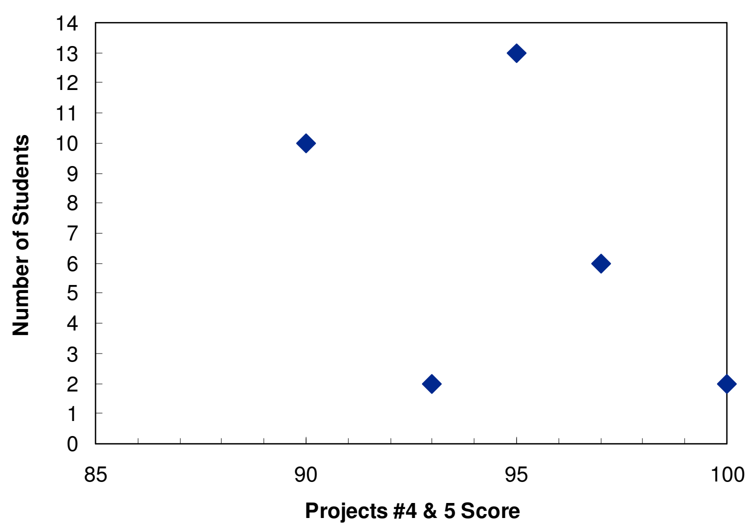
Student Name: ____

Student Name: ____

Topic	Assigned Score	Maximum Score
P4: Optimization for Blood Flow Rate, Dialysate Flow Rate, and Filter Type		20
P4: Analysis for Pressure Difference		10
P4: Analysis for Heat Exchanger Effectiveness		10
P5: Initial Estimation of Heat Exchanger Effectiveness		5
P5: Cost Optimization for Plate Width, Flow Gap, and Cold Fluid Exit Temperature		25
P5: Analysis for Dialysate Flow Rate		10
Quality, including connection between the two projects		20
Total		100

Student performance on the project is shown in Figure 6. The average score of 94 and the median score of 95 show outstanding student performance on the project

Figure 6 Student Scores for Project 4



<u>Heat Exchanger Project: Hemodialysis Heat Exchanger Design</u>
The project team is to select an appropriate heat exchanger for the heat exchanger of the dialysis machine (Project #4 above) by using a spreadsheet program provided by the instructor that will allow the user to evaluate different plate frame heat exchangers. In

designing the heat exchanger, the project team needs determine the plate width, flow gap between plates, and the fresh dialysate exit temperature that will minimize the cost of the heat exchanger. The cost will include material and fabrication costs, along with operating cost due to the required pumping power and the energy that must be used for the heater of the dialysis machine to raise the fresh dialysate temperature to 310 K.

Student Feedback

Student reaction to the use of biomedical projects in the ME 416 was solicited through the survey shown in Figure 7 that was administered the last day of class. Table 2 shows the results of question 1. Clearly, students have a strong interest in Biomedical Engineering, second only to Aerospace and just ahead of Automotive. The results to questions 2 and 3, shown in Figure 9, give an indication that the students felt positive about their learning from these projects.

Figure 7 Student Survey

Survey on Biomedical Theme

1. Several different themes could be used in ME 416 to tie the projects together. Rank the themes below from 1 (has the most interest to you) to 7 (has the least interest to you)

____ Steam Power Systems

____ Air Processing (Heating/Air Conditioning Systems)

____ Biomedical Systems

____ Automotive Systems

____ Aerospace Systems

____ Gas Turbine Power Systems

____ Computer Systems

Please provide a response to the following questions.

2. The biomedical theme enhanced my learning of thermal design.

4	3	2	1	0
Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree

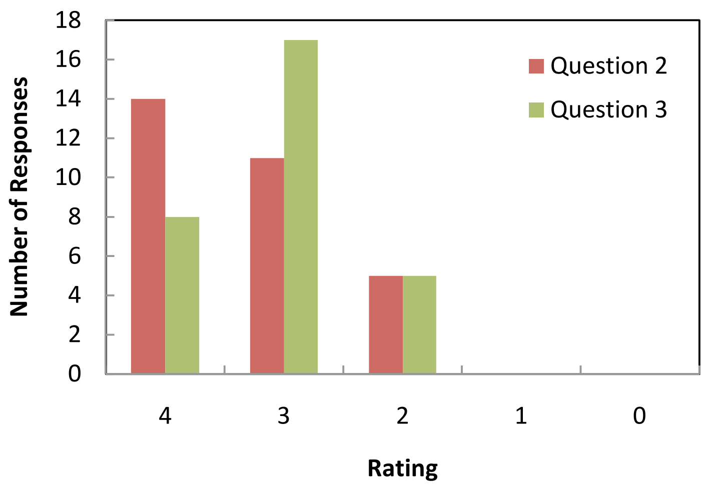
3. I learned a lot about biomedical engineering in this course.

4	3	2	1	0
Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree

Table 2 Student Interest in Project Themes

Theme	Average
Steam Power Systems	4.67
Air Processing	4.37
Biomedical Systems	3.55
Automotive Systems	3.73
Aerospace Systems	2.97
Gas Turbine Power Systems	4.00
Computer Systems	4.52

Figure 9 Survey Results



Each semester 18 questions are asked of students in an attempt to evaluate the course learning objectives. The following four questions seem especially pertinent to the course goals stated in the "About the Course" section of this paper.

1. Ability to evaluate design alternatives
2. Ability to perform a simple optimization
3. Ability to calculate the performance of turbomachinery
4. Ability to calculate the performance of heat exchangers
5. Ability to design the operating conditions for a thermal system
6. Ability to write programs in MATLAB
7. Ability to program and graph in Excel

Questions 1, 2, and 5 would address course goal #1. Questions 3 and 4 would address course goal #2. Questions 6 and 7 would address course goal # 3. The students are asked to evaluate his/her level of confidence on the topics listed using a 1-4 scale, with 1 indicating complete confidence and 4 indicating no confidence. Results from these questions are shown in Table 3.

Table 3 Student Evaluation of Selected Course Learning Objectives

Question	Average Score
1	1.31
2	1.32
3	1.97
4.	1.86
5.	1.90
6	1.76
7	1.21

These numbers are solid and indicate in the students' opinion the goals of the course are pretty well achieved. It is clear that the students are very confident in their design and computer skills, but somewhat less so with their modeling skills. Anecdotally, the faculty of the program have, for several years, expressed concerns with the students modeling skills.

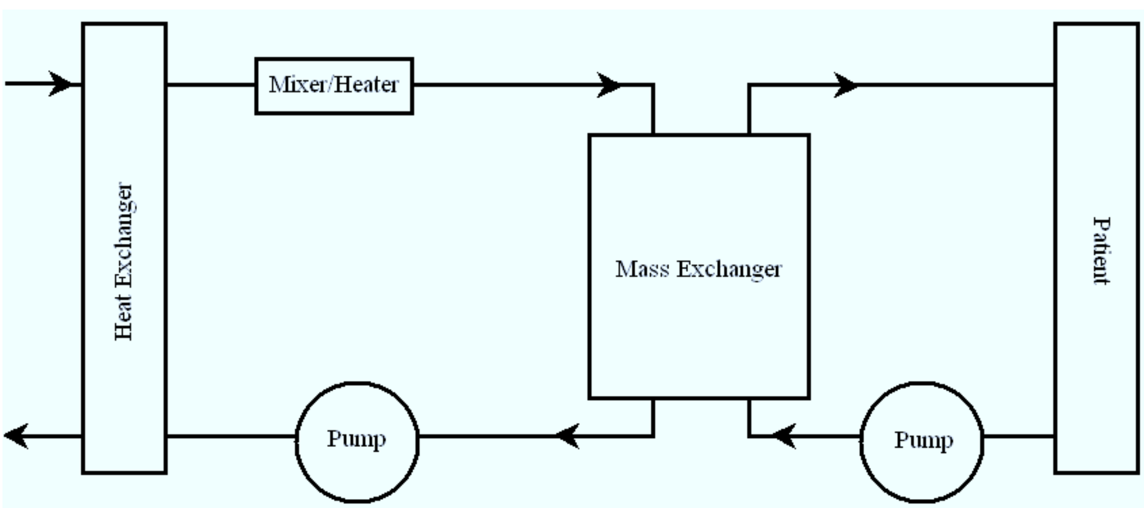
Lessons Learned and Recommendations

- It is important to include adequate biology background in using the biomedical projects, so that students can understand the processes they are modeling and the devices they are designing. This can be done by augmenting the course lectures and with handouts.
- Mechanical engineering students react in a positive fashion to the incorporation of biomedical engineering into a traditional course.
- The biomedical projects provide a good vehicle to teach the basic fundamentals of thermal design.
- A second set of projects with a biomedical orientation could be developed, which could then lead to the teaching of a bio-thermal design class. This class could be used explicitly as part of the Biomechanical Concentration or even as a course in a biomedical degree program.

Appendix I

Project #4
Computer Assisted Design of a Kidney Dialysis System

The Rhino Thermal Engineering Company has recently received a contract with a major biomedical company to design a kidney dialysis system. Your project team has been assigned the task of developing a preliminary design for a hemodialysis system. Dialysis is an artificial way to perform the function of the kidneys by removing excess water and wastes in the blood through a semi-permeable membrane. The membrane in dialysis allows dissolved particles or solutes of certain sizes to pass through the membrane and while prohibiting others. The solutes that pass through the membrane are collected and disposed. The waste products of dialysis are transferred to a fluid called dialysate, which has many chemical similarities to blood. The dialysate carries the waste away and brings in other important dissolved particles to the blood. In hemodialysis, the semi permeable membrane is a filter made of extremely small porous straws encased in a plastic tube. The straws in hemodialysis have an inner or nominal diameter of approximately 200 micrometers. In this filter, blood passes through the straws and dialysate flows over them. A pressure differential in the blood and dialysate causes diffusion of small solute particles through the semi-permeable membrane of the straw wall. Typically, it can be though of as a mass exchanger. The solutes that pass through the mass exchanger include electrolytes, creatinine, vitamin B12 and urea. Particles that cannot pass through the mass exchanger include blood cells, bacteria, and proteins. The blood and dialysate in a mass exchanger flow in opposite directions. A simplified model of a hemodialysis system is shown below.



Two principals of the company, Mr. Aerts and Dr. Somerton, have developed a MATLAB program that models the operation of the simplified hemodialysis system and can be used to optimize its operation. To complete the contract a design team of two students have been assigned. Using the objective function below, the team should determine the mass flow rates of both blood and dialysate and the filter type that minimizes the objective function.

The objective function is based upon a 75% weighting for the time and a 25% weighting for the energy consumed. It could be defined as

Objective Function = (0.75)(normalized cycle time)
+ (0.25)(normalized energy consumption)

The two terms in the objective function are normalized with respect to the highest values for cycle time and energy consumption found from calculations. Calculations should be performed over a range of dialysate volume flow rates and ultrafiltration coefficients for two blood flow rates. The team may want to complete the matrix shown below for each blood flow rate. The preferred system configuration should then be the case with the smallest objective function.

Table I-1. Volume Flow Rate Matrix

Dialysate Volume	Ultrafiltration Coefficient (mL/hr·kPa)				
Flow Rate (cm^3/min)	300	350	400	450	500
450					
500					
550					
600					
650					
700					

A MATLAB program Dialysis.m has been made available to assist in the calculations. A brief user guide for the software is attached. The following base deign conditions should be used in the optimization.

Fresh Dialysate Inlet Temperature: 287 K
Heat Exchanger Effectiveness: To be estimated from Project 5
Pump Efficiency: 0.85
Dialysate Pump Pressure Boost: 10 kPa
Blood Hematocrit Level: 0.42
Blood Pump Pressure Boost: 30 kPa
Blood Flow Rates: 300 and 400 cm3/min
Mass Exchanger Surface Area: 2 m2
Urea Produced per Day: 30 grams
Number of Days Since Last Dialysis Visit: 3 days
Mass of Patient: 120-190 pounds
Maximum Mass of Urea Left in Blood: 2 grams

Once the optimal design (combination of volume flow rates and filter type) is determined, the design team should do a robustness study for the blood pump pressure boost. The range for the blood pump pressure boost is 15 to 50 kPa, and at least three different designs should be considered. A second robustness study should be done with the heat exchanger effectiveness (varying from 0.5 to 1.0). This second robustness study will be used by the team with the results of Project #5 to demonstrate the connection between equipment design (or selection) and system design.

A technical report will be required which will include the design study for both projects 4 and 5.

Appendix II

Mass Exchanger Modeling

Craig W. Somerton
7/29/05

In the mass exchanger component of a dialysis machine we have two fluid streams blood (B) and dialysate (D) with a mass transfer between them of several different species. The primary species of this mass transfer is urea (U). An overall urea balance on the mass exchanger is

$$\dot{m}_U = \dot{m}_B(m_{U,B,in} - m_{U,B,out}) \quad (1)$$

$$\dot{m}_U = \dot{m}_D(m_{U,D,out} - m_{U,D,in}) \quad (2)$$

where

$\dot{m}_i$: mass flow rate (kg/s) of fluid or species i

m_i: mass fraction of species I (kg of species i/kg of mixture)

For a counterflow mass exchanger the mass transfer of urea between the two fluid streams can be shown to be

$$\dot{m}_U = K_U A_{sur} (\Delta m_U)_{lm} \quad (3)$$

where

K_i: overall mass transfer coefficient for species i

A_{sur}: mass transfer surface area

$(\Delta m_i)_{lm}$: log mean mass fraction difference for species i

For the counterflow mass exchanger of urea, we can write

$$(\Delta m_U)_{lm} = \frac{(m_{U,B,in} - m_{U,D,out}) - (m_{U,B,out} - m_{U,D,in})}{\ln\left\{\dfrac{m_{U,B,in} - m_{U,D,out}}{m_{U,B,out} - m_{U,D,in}}\right\}} \quad (4)$$

We can now use Eqs.(1)-(4) to solve for the exit mass fractions of urea in the two fluid streams. Staring with Eqs.(1) and (2), we have

$$\dot{m}_D(m_{U,D,out} - m_{U,D,in}) = \dot{m}_B(m_{U,B,in} - m_{U,B,out}) \quad (5)$$

Using the simplification that

$$m_{U,D,in} = 0 \quad (6)$$

and defining

$$\gamma = \frac{\dot{m}_B}{\dot{m}_D} \qquad (7)$$

We can then solve for the exit mass fraction of urea in the dialysate as

$$m_{U,D,out} = \gamma(m_{U,B,in} - m_{U,B,out}) \qquad (8)$$

Next we wish to solve for the exit mass fraction of urea in the blood. Substituting Eqs. (1) and (4) into Eq. (3), we have

$$\dot{m}_B(m_{U,B,in} - m_{U,B,out}) = K_U A_{sur} \frac{(m_{U,B,in} - m_{U,D,out}) - (m_{U,B,out} - m_{U,D,in})}{\ln\left\{\frac{m_{U,B,in} - m_{U,D,out}}{m_{U,B,out} - m_{U,D,in}}\right\}} \qquad (9)$$

Eliminating $m_{U,D,out}$ from this equation using Eq. (8) gives

$$\dot{m}_B(m_{U,B,in} - m_{U,B,out}) = K_U A_{sur} \frac{(m_{U,B,in} - \gamma(m_{U,B,in} - m_{U,B,out})) - (m_{U,B,out})}{\ln\left\{\frac{m_{U,B,in} - \gamma(m_{U,B,in} - m_{U,B,out})}{m_{U,B,out}}\right\}} \qquad (10)$$

Rearranging this equation we can write

$$\dot{m}_B(m_{U,B,in} - m_{U,B,out}) = K_U A_{sur} \frac{(1-\gamma)(m_{U,B,in} - m_{U,B,out})}{\ln\left\{\frac{(1-\gamma)m_{U,B,in} + \gamma m_{U,B,out}}{m_{U,B,out}}\right\}} \qquad (11)$$

Note that ($m_{U,B.in}$-$m_{U,B,out}$) can now be divided out. Doing so and bring the log term over to the other side of the equation yields

$$\ln\left\{\frac{(1-\gamma)m_{U,B,in} + \gamma m_{U,B,out}}{m_{U,B,out}}\right\} = \frac{(1-\gamma)K_U A_{sur}}{\dot{m}_B} \qquad (12)$$

Taking the exponential of both sides

$$\frac{(1-\gamma)m_{U,B,in} + \gamma m_{U,B,out}}{m_{U,B,out}} = \exp\left\{\frac{(1-\gamma)K_U A_{sur}}{\dot{m}_B}\right\} \qquad (13)$$

Finally solving for the exit mass fraction of urea in the blood flow

$$m_{U,B,out} = \frac{(1-\gamma)m_{U,B,in}}{\exp\left\{\frac{(1-\gamma)K_U A_{sur}}{\dot{m}_B}\right\} - \gamma} \quad (14)$$