Introducing Chaotic Circuits in Analog Systems Course

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Abstract

For decades, the engineering undergraduate education in the area of systems design has been mainly focused in linear models. Today, it is important for students to be exposed to chaos phenomena which exist in electrical, mechanical, biological and other systems. In this paper, we focus in electrical chaotic systems with the understanding that similar systems exist in other fields of study. We present two autonomous and two nonautonomous circuits that behave chaotically. We have selected simple circuits, such as the Colpitts oscillator, that are actually taught to students in electronics courses. Multisim is used to simulate circuits and show the presence of chaos. Complete circuits implementation and oscilloscope plots are all given. Student's first time experience with these phenomena is discussed.

Introduction

One of the reasons that engineering undergraduate education in the area of circuit and design has been mainly focused in linear models is that linear system theory has been thoroughly developed, and mathematical tools are available to analyze such systems. This philosophy has led many scientists and experimentalists to disregard many observed phenomena because linear system theory can not explain them. In the last decade, there is a strong interest in exploring systems that display unusual complicated waveforms, commonly known as strange attractors. These attractors have been increasingly observed in several nonlinear deterministic systems.

Therefore, it is important for today's students to be exposed to these complex chaos phenomena. From the educational aspect, students need to learn not only how to control and avoid chaos but also how to design chaotic circuits and develop applications, which explore these phenomena. Realizing the educational value to introduce undergraduate students to the phenomena of chaos, Lonngren [1] describes an interesting electronics experiment to illustrate the existence of chaos. The described laboratory experiment with the accompanying theory is a good start for the student to grasp and understand chaos. As a continuation of this goal and to enhance students understanding of chaos, Hamill [2] presented a collection of ten chaotic circuits simulated using PSpice. These circuits, some are quite simple, illustrate how chaos can be generated.

This paper can be viewed as a logical follow up on the progress made to understand chaos. It is a continuation to strengthen the student understanding not only on how chaos is simulated but also how to implement it. It is worthwhile to mention that some of the presented circuits, such as the Colpitts oscillator, are taught to students in electronics courses. Yet, there is no mention that chaotic behavior may occur. We have chosen to stay away from circuits that require a degree of mathematical sophistication beyond the undergraduate level. Although the paper uses electronics circuits to illustrate the existence of chaotic behavior, and will benefit instructors in the electrical/electronics field, it can also be used in other fields of study to present similar chaotic system such as the “pendulum.” For the reader, who would like to experiment for more complex circuits, references are given [3-9]

In this paper, several chaotic circuits are presented for the student and the practicing engineer to study and experiment with. We have selected to use Multisim [10] to simulate circuits since it provides an interface as close as to the real implementation environment. In addition, complete circuits implementation and oscilloscope graphical plots are all presented.

Examples of autonomous chaotic circuits

Chua's circuit: The first example we present is the Chua's circuit (a third-order autonomous, dissipative electrical circuit). It has been investigated thoroughly at the experimental, numerical and analytical levels [11-15]. This circuit, known for its rich repertoire of nonlinear dynamical phenomena, has become a universal paradigm for chaos. Fig. 1(a) shows the Chua's circuit which includes two capacitors, a resistor, an inductor and a nonlinear resistor NR. Applying KCL and KVL, the Chua's circuit is described by three differential equations:

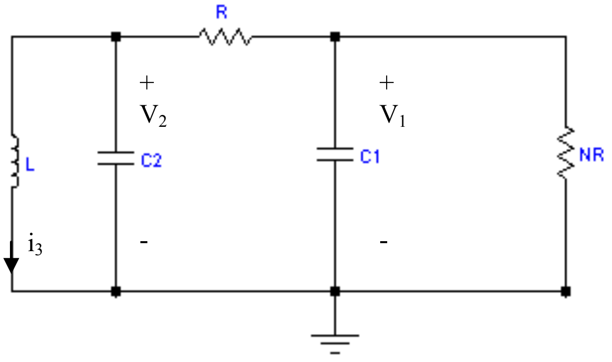

Fig 1. (a) The unfolded Chua’s circuit

$$
\begin{aligned}
C_1 \frac{dv_1}{dt} &= \frac{1}{R}(v_1 - v_2) - f(v_1) \\
C_2 \frac{dv_2}{dt} &= \frac{1}{R}(v_1 - v_2) + i_3 \\
L \frac{di_3}{dt} &= -v_2
\end{aligned}
\tag{1}
$$

Where the nonlinear Chua's function, shown in Fig. 1(b), is described by

$$f(v_R) = m_o v_R + \frac{1}{2}(m_1 - m_o)\left\{ \left| v_R + B_p \right| - \left| v_R - B_p \right| \right\} \tag{2}$$

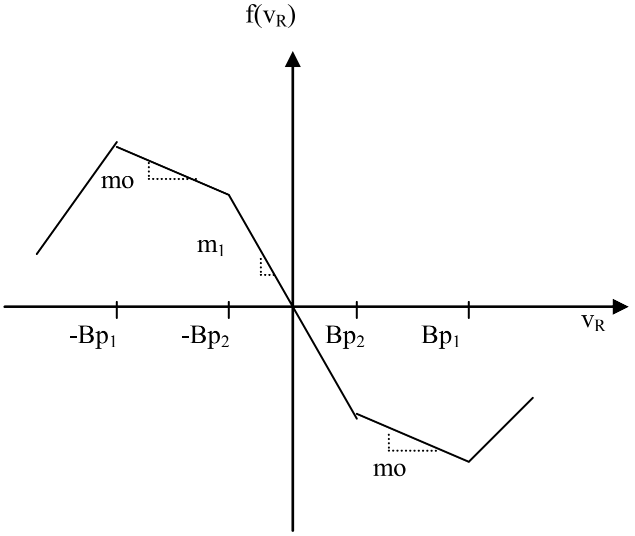

Fig.1(b) Chua's nonlinear function

The realization of the Chua's circuit is shown in Fig 1.(c). The constant m_o , m_1, and B_p can be easily computed.

$$\left(m_1 = -\frac{R_2}{R_1R_3} - \frac{R_5}{R_4R_6},\ m_o = -\frac{R_2}{R_1R_3} + \frac{1}{R_4},\ B_{p_1} = \frac{R_3}{R_2 + R_3} E_{sat},\ B_{p_2} = \frac{R_6}{R_5 + R_6} \right)$$

Where, E_{sat} is the saturation voltage of the operational amplifier. The complete implementation of the Chua's circuit is shown in Fig. 1(c).

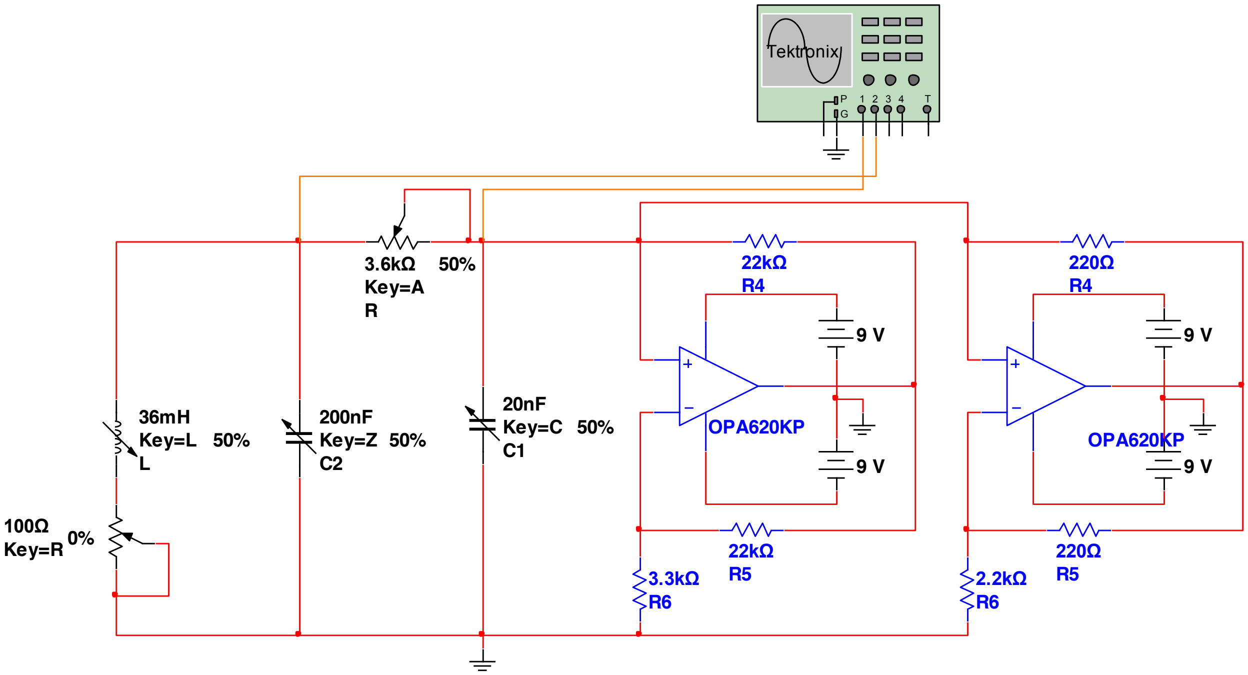

Fig 1.(c) The realization of the Chua's circuit [1]

The results of Multisim simulation show the phase portrait of the probed signals in fig. 1(d).

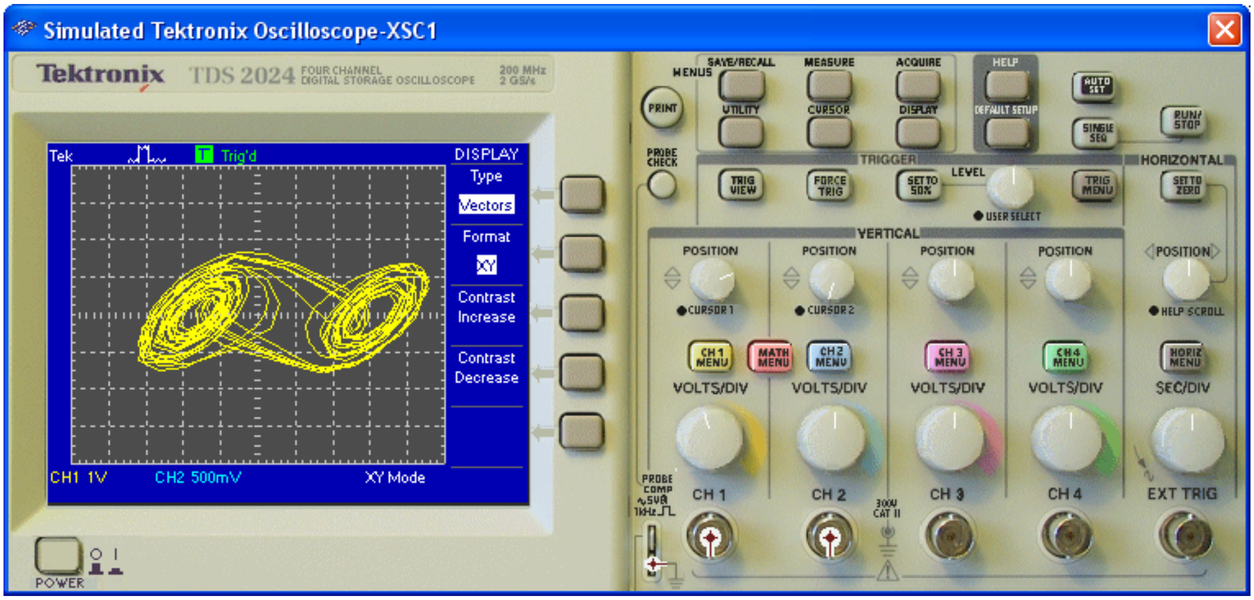

Fig. 1(d). Phase portrait Vc_2 versus Vc_1.

The Chua's circuit was implemented using the TL082 operational amplifier. The double scroll is shown in fig. 1(e) and the voltage $v_{c1}(t)$ as well as its spectrum are shown in fig.1(f). The reader is encouraged to experiment with this circuit by connecting a variable resistor in series with the inductor L.

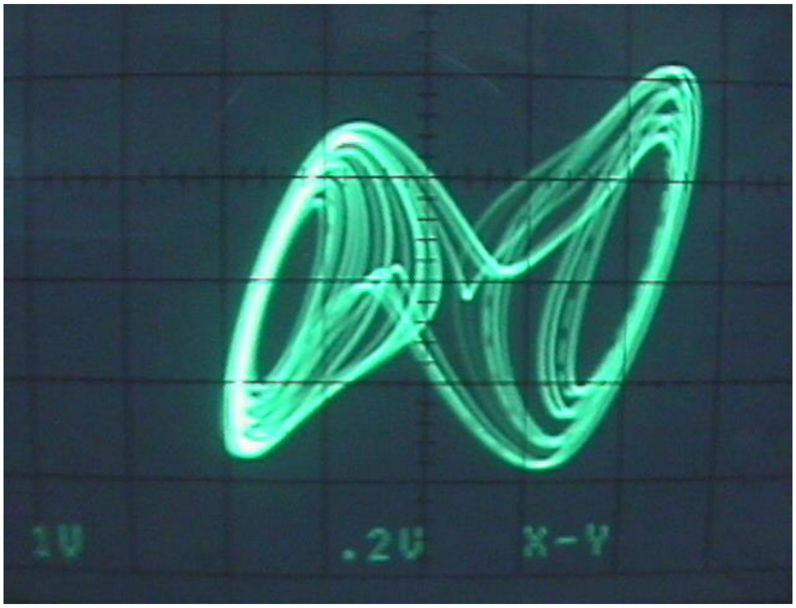

Fig.1(e). double scroll Vc2 versus Vc1 for R=1.53KΩ

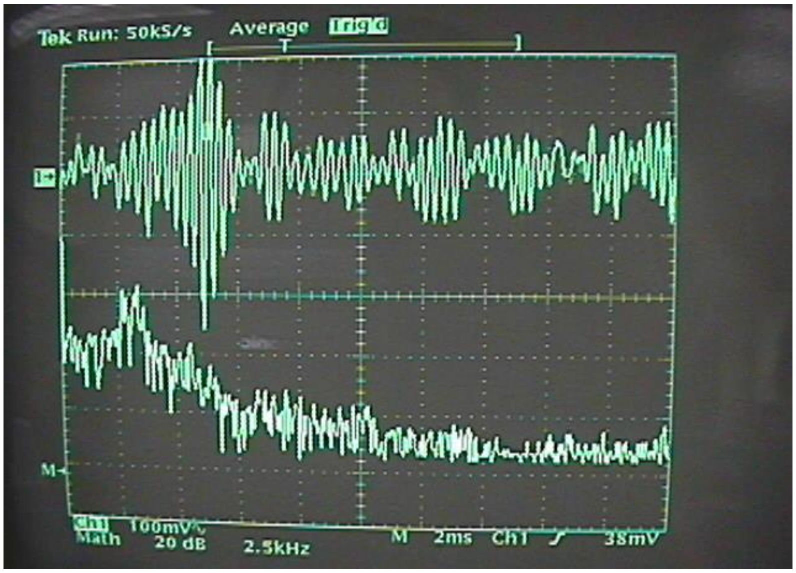

Fig.1(f). Top: $V_{c1}(t)$. Bottom: Its corresponding Spectrum

Colpitts oscillator: The second example is the Colpitts oscillator [16] shown in Fig. 2(a).

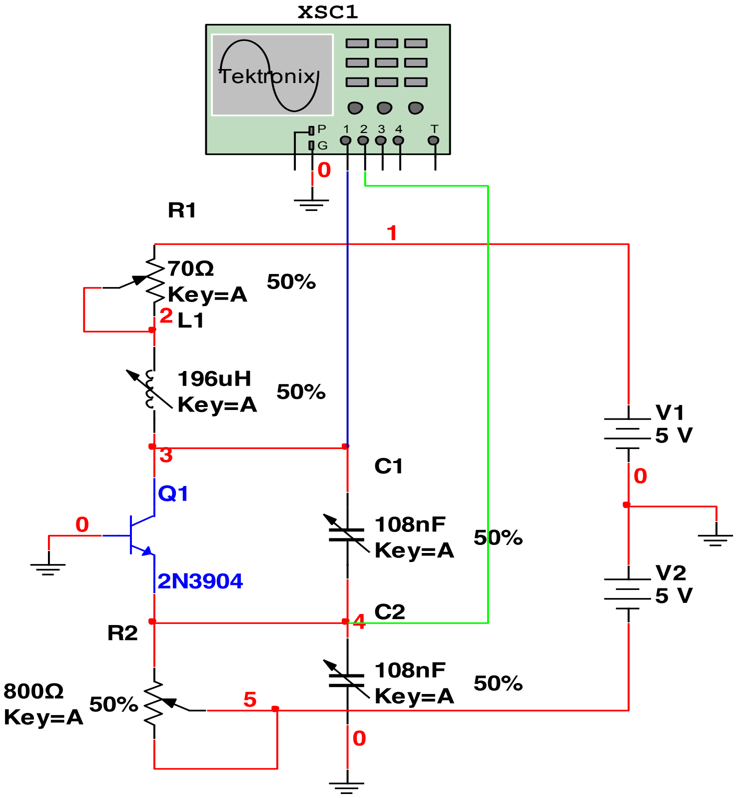

Fig. 2(a). Colpitts oscillator.

It is described by a system of three state equations:

$$\begin{aligned} C_1 \frac{dV_{CE}}{dt} &= i_L - i_C \\ C_2 \frac{dV_{BE}}{dt} &= -\frac{V_{EE}+V_{BE}}{R_{EE}} - i_L - i_B \\ L\frac{di_L}{dt} &= V_{CC} - V_{CE} + V_{BE} - i_L R_L \end{aligned} \tag{3}$$

The transistor is modeled as follows

$$i_B = \begin{cases} 0 & \text{if} \quad V_{BE} \le V_{TH} \\ \dfrac{V_{BE}-V_{TH}}{R_{ON}} & \text{if} \quad V_{BE} > V_{TH} \end{cases}$$

$$i_C = \beta_F i_B$$

Where, V_{TH} is the threshold voltage (≈0.75V), R_{ON} is the small signal on-resistance of the base-emitter junction, and β_F is the forward current gain of the device. In most undergraduate electronics books, this circuit is shown to oscillate but yet it can be driven to chaos. Fig. 2(b) shows the chaotic phase portrait. By changing the value of resistance R_L we obtain different trajectories.

Fig. 2(b). Phase portrait of V_C versus V_E

The Colpitts oscillator was implemented and its phase portrait scroll is shown in Fig. 2(c). The voltage $V_C(t)$ and its spectrum are shown in Fig. 2(d)

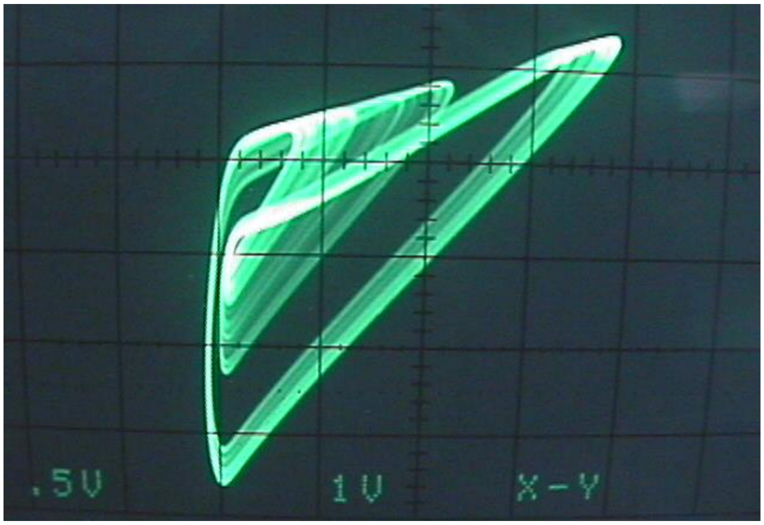

Fig.2(c). Phase portrait of V_C versus V_E for R_{EE}=466Ω

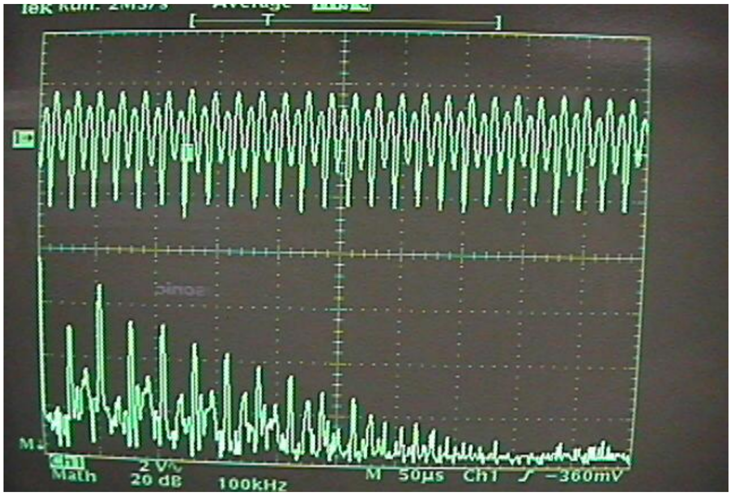

Fig.2(d). Top: $V_C(t)$. Bottom: its corresponding spectrum

Examples of nonautonomous chaotic circuits

RL-Diode circuit: The first example of nonautonomous chaotic circuit is the driven RL-diode circuit [17-18] shown in Fig 3(a).

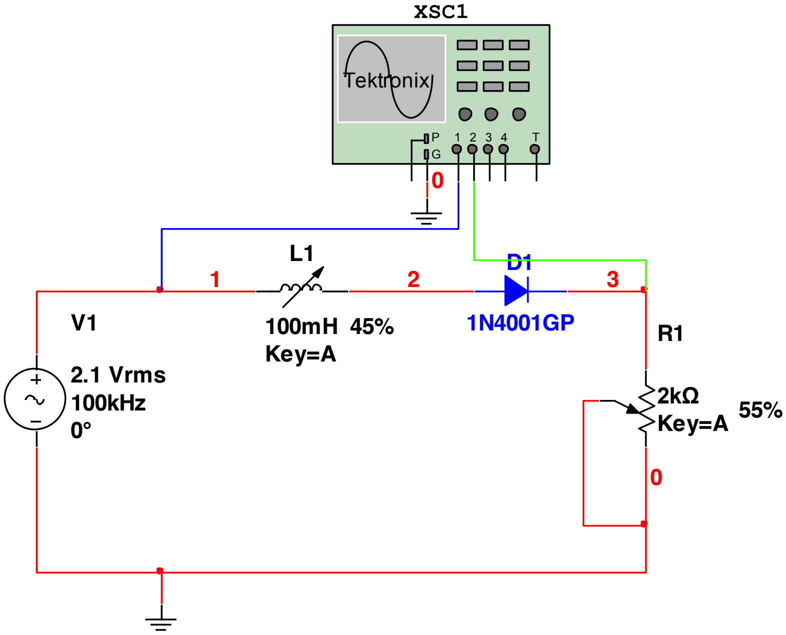

Fig. 3(a). RL-Diode chaotic circuit

It consists of a series connection of an ac-voltage source, a linear resistor, a linear inductor and a diode which is the only nonlinear circuit element. The state equations describing this circuit are

$$
\begin{aligned}
V_{in} &= Ri + L\frac{di}{dt} + V_D \\
i &= I_S\left(e^{\frac{V_D}{nV_T}} - 1\right)
\end{aligned}
$$

Where V_D is the voltage across the diode, I_S is the diode saturation current, n is a constant which has a value between 1 and 2 depending on the material and the physical structure of the diode, and V_T is the thermal voltage. The i-v characteristic of a diode is shown in Fig. 3(b).

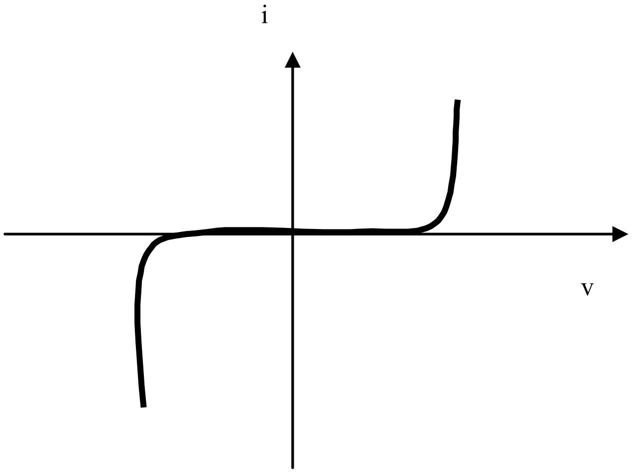

Fig. 3(b). I-V diode characteristic.

An important feature of this circuit is that the current i (or the voltage across the resistor R) can be chaotic although the input voltage V_{in} is nonchaotic. The results of the Multisim simulation, are shown in Fig. 3(c) for R=1kΩ.


Simulated Tektronix Oscilloscope-XSC1

DISPLAY
Type: Vectors
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CH1 1.21V CH2 99.1mV XY Mode

Fig. 3(c). Phase portrait of Vin versus V_R

The RL-diode was implemented and its phase portrait is shown in fig. 3(d). The voltage across the resistance R and its spectrum are shown in fig. 3(e). The reader is encouraged to experiment with different values of R to obtain different phase portraits.

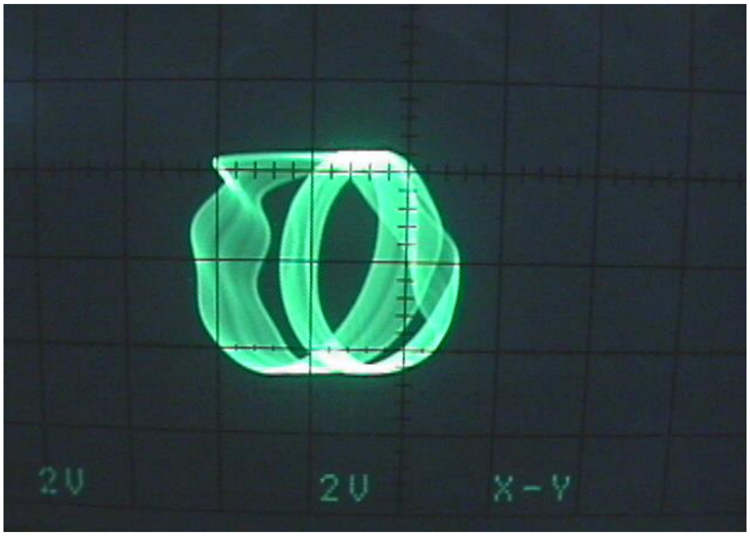


Fig.3(d) Plot of Vin versus VR for the input frequency = 130KHz Vin peak-peak=6.5V and R=26KΩ.

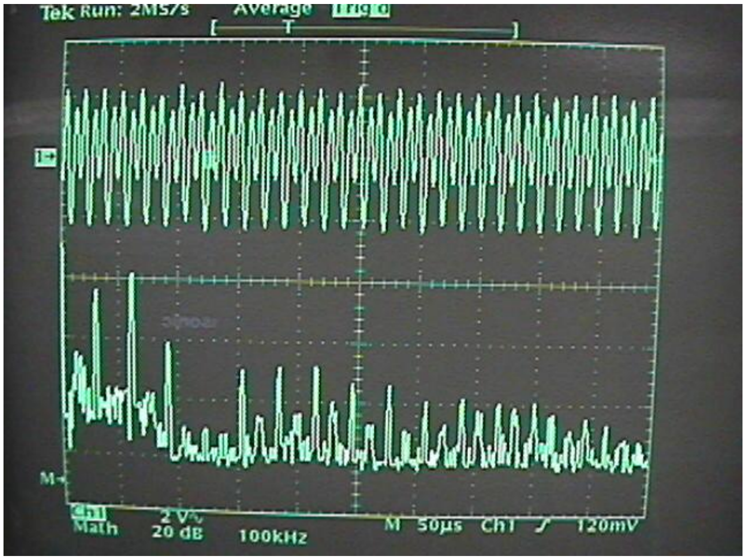


Fig.3(f). Top:$V_R(t)$. Bottom: Its corresponding spectrum

Nonautonomous Chua's family circuit: The second example is a simple sinusoidal driven circuit shown in Fig.4(a) [19].

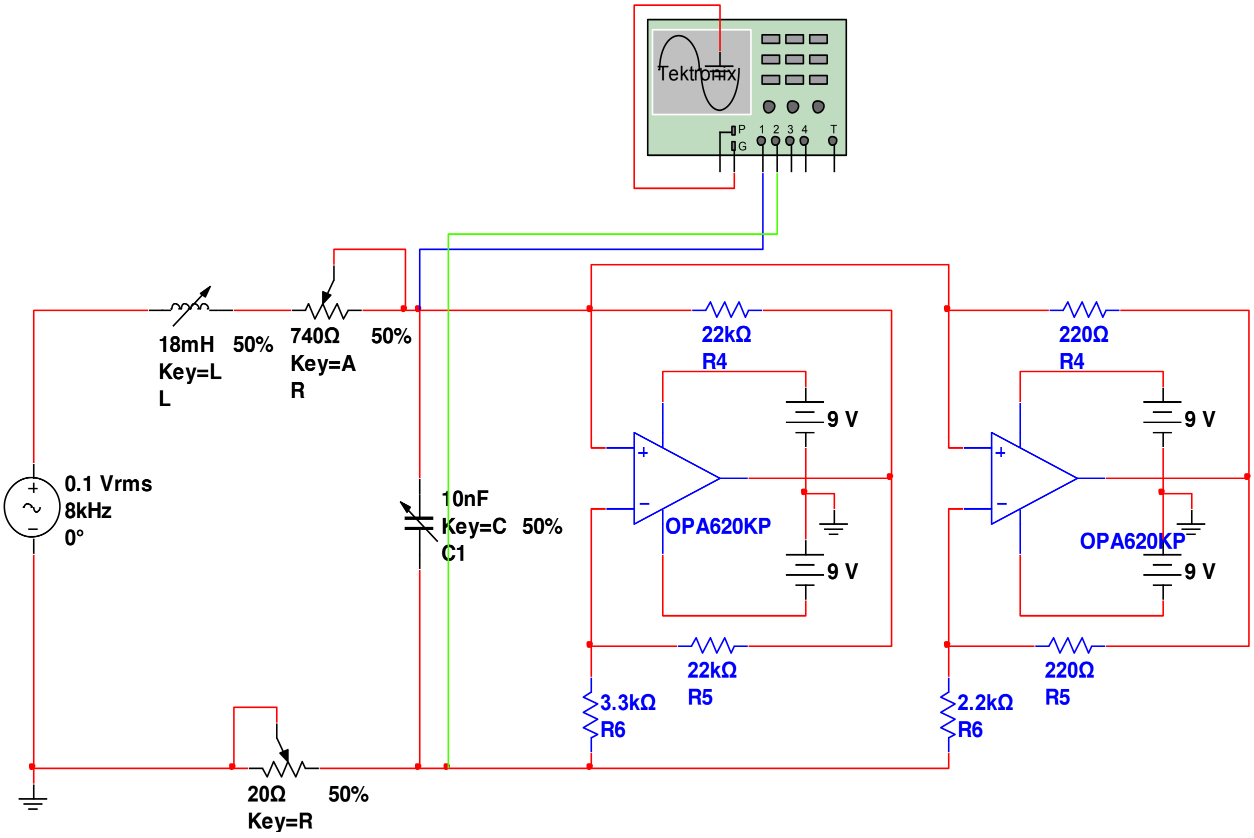
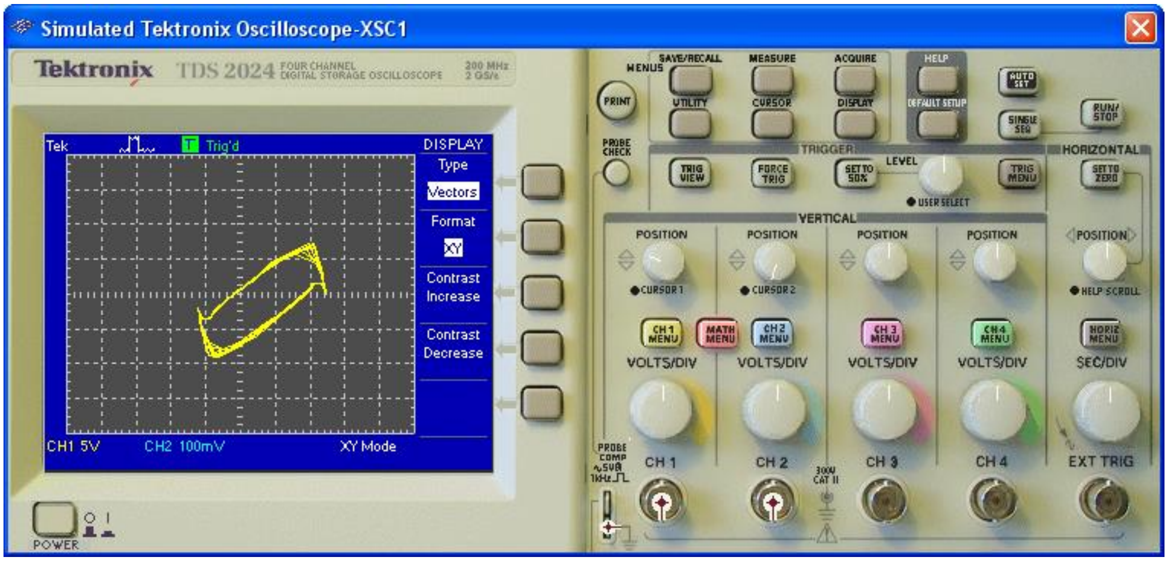


Fig. 4(a). Dissipative nautonomous chaotic circuit

It consists of an external periodic source, a linear resistor, an inductor, a capacitor and the nonlinear Chua's function NR. By applying Kirchoff's laws, the resultant state equations are:

$$\begin{aligned} C\frac{dv_1}{dt} &= i_L - f(v_1) \\ L\frac{di_L}{dt} &= -Ri_L - v_1 + a\sin(\omega t) \end{aligned}$$

Where, the nonlinear Chua's function, shown previously in Fig. 1(b), is described by equation (2). In addition to the implementation of the Chua's function, a small resistor, connected in series with the external source, is added to the circuit in order to probe the current i_L. The results of Multisim simulation show the phase portrait of the probed signal in Fig.4(b).



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CH1 5V CH2 100mV XY Mode

Fig. 4(b). Phase portrait of the probed signals Vc$_1$ versus Vr

This circuit was implemented and the phase portrait is shown in fig. 4(c). The voltage v_{C1}(t) and its spectrum are shown in fig. 4(d). The reader is encouraged to experiment with this circuit by changing the amplitude of the external source as well as the frequency.

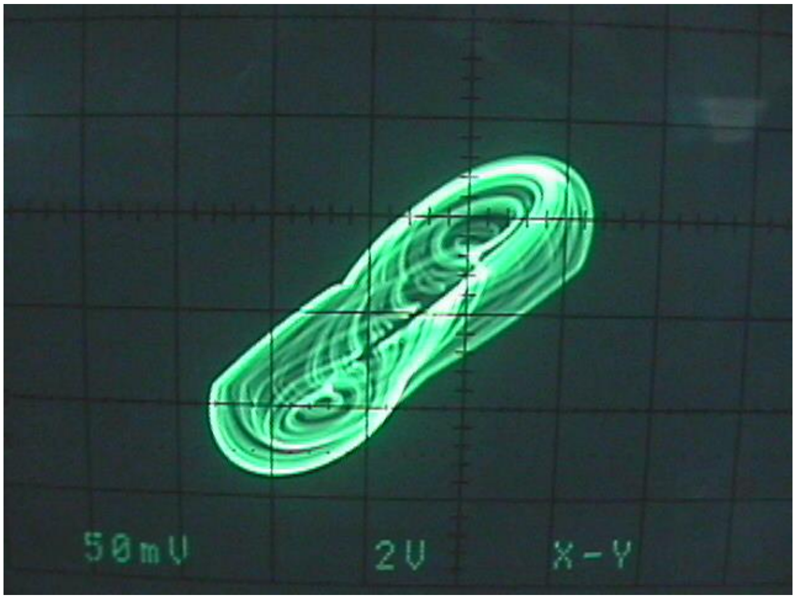

Fig.4(c): V_{C1} versus V_r for R=740Ω. For frequency = 15KHz
and Vin-peak-peak=2V

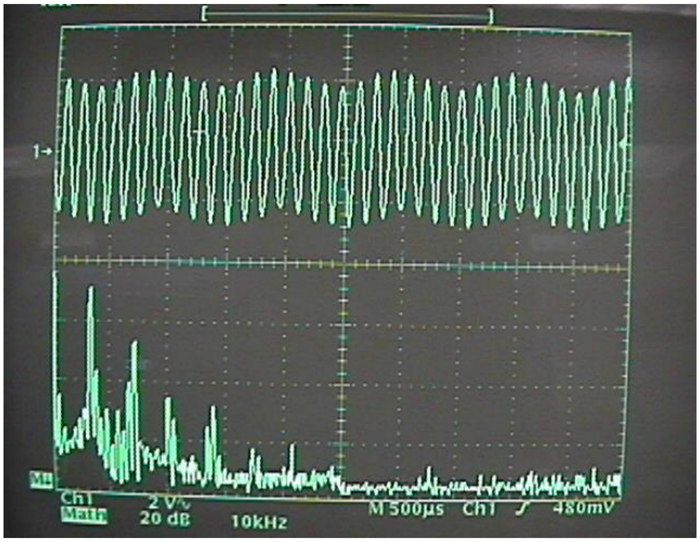

Fig.4(d): Top: $V_{C1}(t)$. Bottom: its corresponding spectrum

Conclusions and Discussions

Introducing the phenomena of chaos to students and practicing engineers is very important not only to investigate its existence even in simple nonlinear circuits but also to explore it and build some sophisticated applications. The circuits presented here are selected because of their simplicity from the mathematical point of view. Multisim simulation provides a virtual electronic lab environment to experiment with chaos. This will enhance the learning process by being able to make all circuit changes before purchasing any component to implement actual circuits. Complete circuits implementations are presented to show the existence of chaos behavior.

At the end of semester, students were definitely fascinated by the existence of this chaos phenomenon. They understood that the linear model that describes the device is a simplified version of a complex real one. They spent hours in trying to find other parameters to obtain other attractors. Finally some of their questions that remained unanswered are (1) how to set the parameter to guarantee the display of attractors and (2) has anyone developed a commercialized product that takes advantage of this phenomenon. It is our hope that this paper will entice the reader to experiment with this nonlinear phenomenon.

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