

## **Introducing Stress Transformation and Mohr's Circle**

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**Introducing Stress Transformation and Mohr's Circle to  
Engineering Technology Students using an Interactive Example  
Problem**

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## Abstract

The concept of stress transformation using stress elements and differential areas is often a difficult concept to convey to these students. Because of this, they often struggle with relating stress at any angle to a Mohr's circle representation of stress. This paper discusses an interactive example problem used to illustrate Mohr's circle and the concept of stress transformation to engineering technology students. An axially loaded rectangular bar is presented and the normal stress is determined. The bar is then sliced at an angle and the internal reactions parallel and perpendicular to the cut surface are determined. The area of the inclined surface is determined so that the normal and shear stress can be determined for that angle. Pairs of students are then asked to perform the same task for a unique assigned angle ranging from -85 to 85 degrees. The students then enter their solutions into a spreadsheet which plots them as coordinates on a graph displayed on an overhead projector screen. At the end of the exercise, all of the entered coordinates plot Mohr's circle representing the stresses at a typical cross-section. Students clearly relate to what each point on the circle represents which, hopefully leads to a greater understanding of Mohr's circle's role in determining stress transformations.

## Introduction

Historically, engineering technology students entering the strength of materials stage of their education find elemental plane stress analysis challenging. At this point in their training, the 3-D visual skills are being developed and the introduction of stress analysis adds to the complexity of their growth. The presented interactive method has the potential to improve their understanding of stress analysis and the use of Mohr's circle. This presentation includes two semester's worth of student data. A population of students were introduced to Mohr's circle techniques with the interactive example and the second population were introduced to Mohr's circle techniques without the example.

## Interactive Example Problem Description

The interactive example problem consists of a structural bar axially loaded with a uniform cross section along its length. The bar is rectangular in cross section with sides 25.4 mm tall and 17.5 mm wide. Figure 1 below shows the bar with 9500 N load applied.

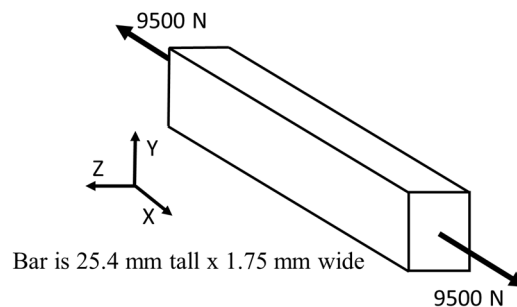


Figure 1: Axially Loaded Uniform Cross Section Bar

By definition, the average normal stress is determined as follows.

$$\text{Average Normal Stress} = \sigma = \frac{P}{A}$$

Where  $P$  = Internal Axial Reaction  
 $A$  = Cross-Sectional Area

Figure 2 shows where the bar is being cut perpendicular to the orientation of the load in order to determine the internal axial reaction force.

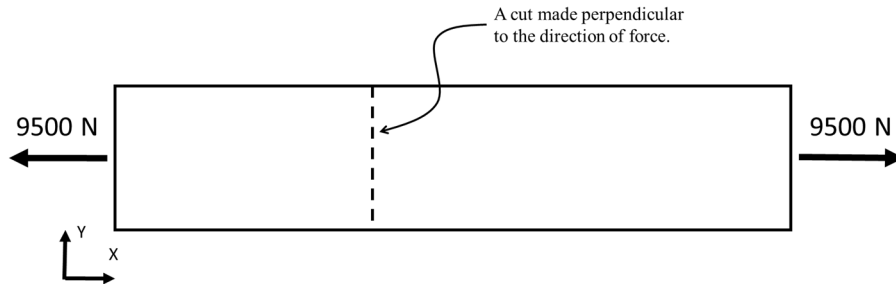


Figure 2: Perpendicular section with respect to axial load.

A free-body diagram is created and the internal axial reaction is found to be 9500 N. The resulting stress is determined by dividing the internal axial reaction by the cross-sectional area of the bar. In this case, the average normal stress is determined to be 21.37 MPa. Students are shown the resulting stress distribution which is shown in Figure 3.

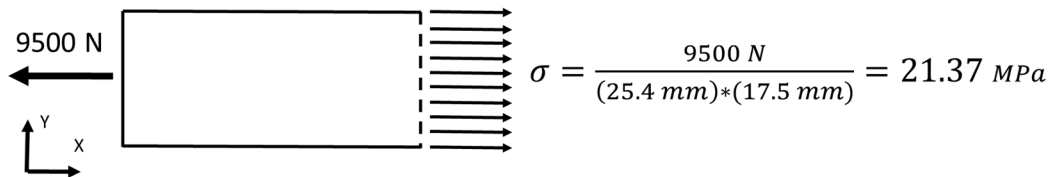


Figure 3: Average Normal Stress at Perpendicular Cut

Now, consider the same axially loaded bar sliced at some angle,  $\phi$ , from the original cut. An example of the typical cut is shown in Figure 4.

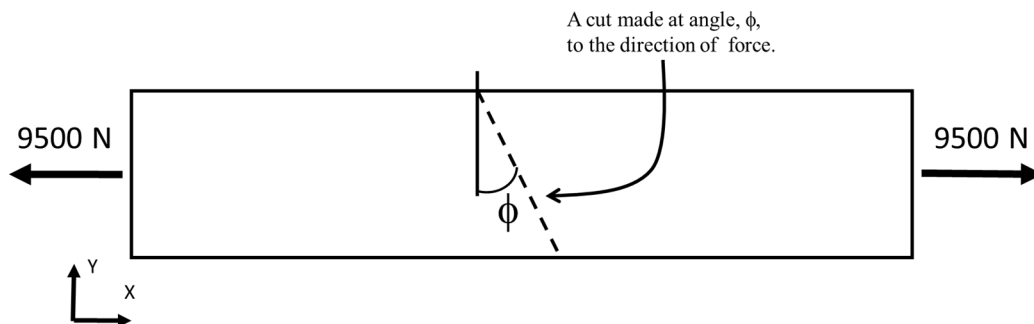


Figure 4: Structural bar sliced at angle,  $\phi$

The free-body diagram for the bar cut at an angle is shown in Figure 5. In this diagram, the internal axial reaction has been resolved into components, which are parallel and normal to the cut. The X' axis is drawn perpendicular to the cut surface and the Y' axis is parallel to the cut forming a right-hand coordinate system. The typical force triangle is shown to illustrate the resolution of the axial reaction force into its component forces.

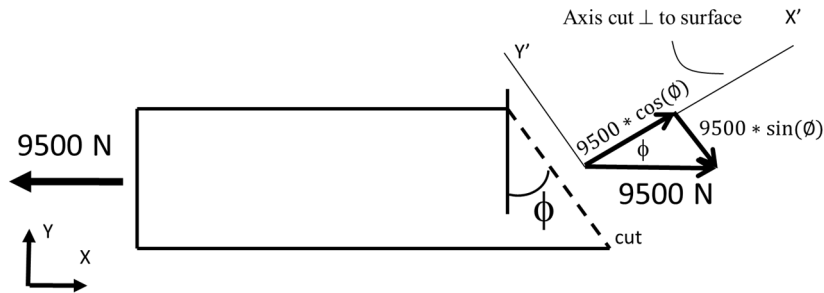


Figure 5: Resolution of Internal Forces at Cut Angle,  $\phi$ .

The area of the cut surface is a function of the angle,  $\phi$ , and can be computed using trigonometry. As the angle changes, the cross-sectional area of the cut will change per this general equation:

$$\text{Cross-sectional Area @ cut} = A_{\phi} = \frac{25.4 \text{ mm}}{\cos(\phi)} * 17.5 \text{ mm}$$

For example, a cut angle of  $35^{\circ}$  results in a cross-sectional area of  $542.6 \text{ mm}^2$ .

The force acting perpendicular to the cut results in normal stress acting on the face. Likewise, the force acting parallel to the cut results in shear stress acting on the face. The stresses resulting from the angled cut are defined as average normal stress,  $\sigma'$ , and direct average shear stress,  $\tau'$ , in the X'-Y' coordinate system. Figure 6 depicts the average normal stress and direct average shear stress.

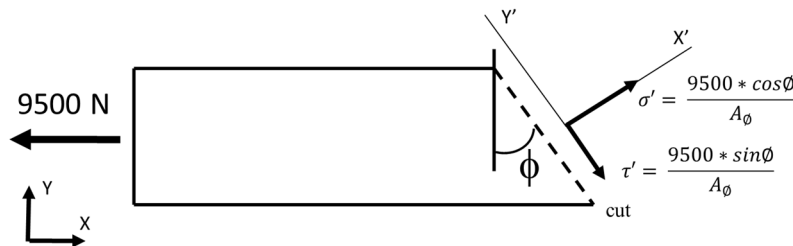


Figure 6: Average Normal and Direct Average Shear Stresses

NOTE: By definition, both stresses have uniform stress distributions across the cut surface.

As the cut angle,  $\phi$ , is varied from  $0^{\circ}$  to  $90^{\circ}$  in  $5^{\circ}$  increments, the normal stress,  $\sigma'$ , and the shear stress,  $\tau'$ , can be calculated. The solution for the cut at each angle represents a point on the circle developed by Christian Otto Mohr in the late 1800's.

At this point in the classroom, the students are separated into teams of two. Each team is assigned a specific angle, in increments of  $5^{\circ}$ . The teams calculate their respective cross-

sectional area, normal stress, and shear stress. The answers are added to a spreadsheet where the points can be plotted and projected to a classroom screen.

Also at this time, the students are introduced to two possible sign conventions in constructing Mohr's circle. Firstly, and for both sign conventions, the positive normal stresses are plotted to the right. On the other hand, shear stress plotting can have different sign conventions. One option is to plot shear stresses tending to rotate a stress element clockwise upwards on the shear stress axis<sup>[1]</sup>. The other sign convention for shear stress plotting is upwards on the shear stress axis when it acts in the negative Y' direction<sup>[2]</sup>. Both sign conventions are acceptable.

Figure 7 shows the plot of normal vs. shear stresses.

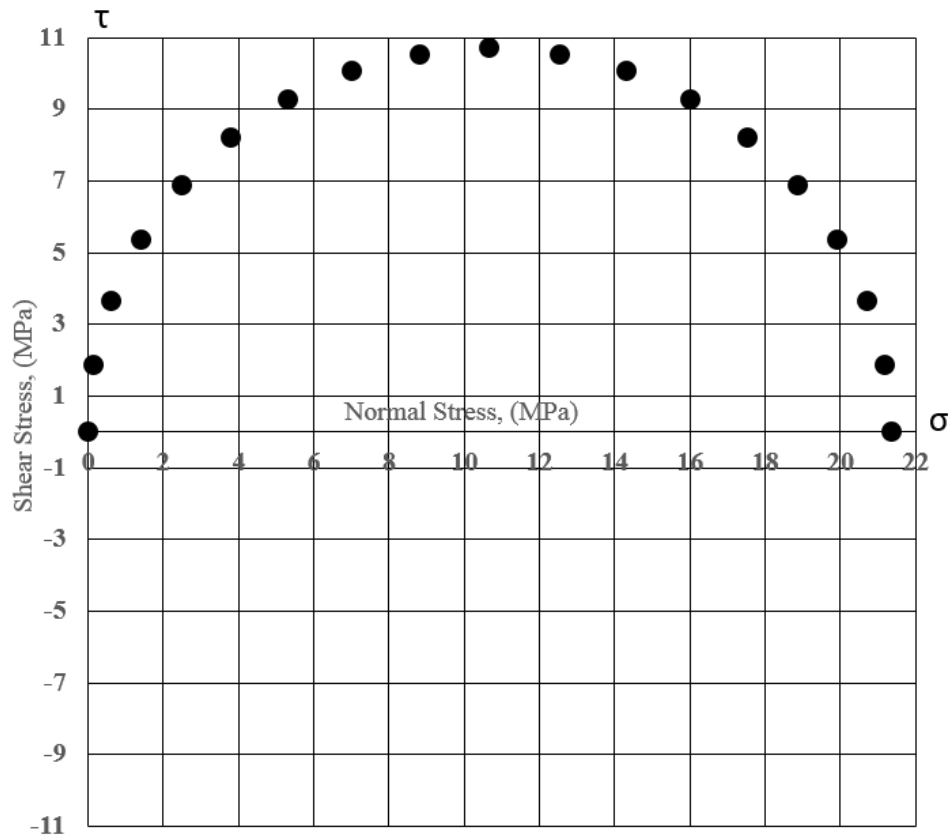


Figure 7: One-half of Mohr's Circle with Shear Stress plotted Upwards

This process is then completed for the complement of angles resulting in a complete 360° circle. Figure 8 shows the same axially loaded bar with the cut angle,  $\phi$ , on the other side of the perpendicular cut.

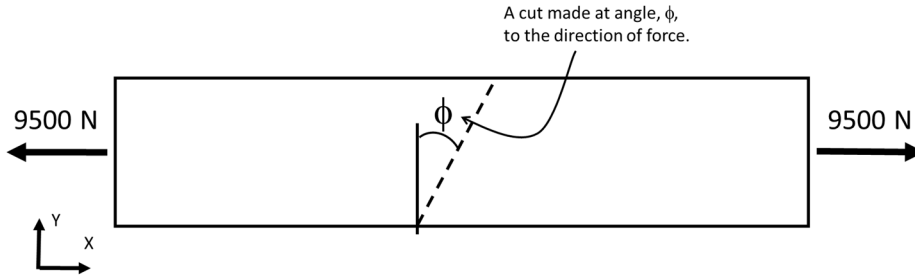


Figure 8: Axially Loaded Bar with opposite Cut Angle

Figures 9 and 10 show the derivation for the other half of Mohr's circle.

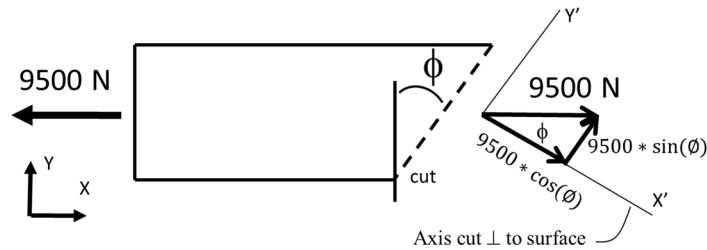


Figure 9: Resolution of Internal Forces at Cut Angle,  $\phi$ .

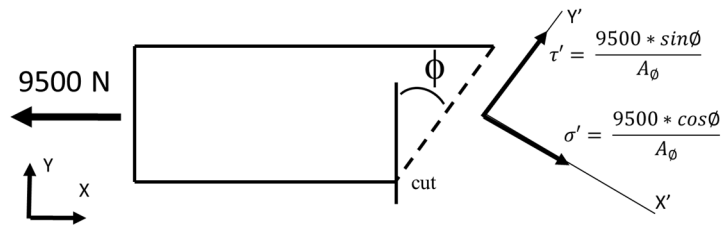


Figure 10: Average Normal and Direct Average Shear Stresses

With this model, the second half of Otto Mohr's circle can be plotted as shown in Figure 11. The second half of Mohr's circle uses positive shear stresses. Per the sign convention, these shear stresses are plotted downward on the  $\tau$  axis of Mohr's circle.

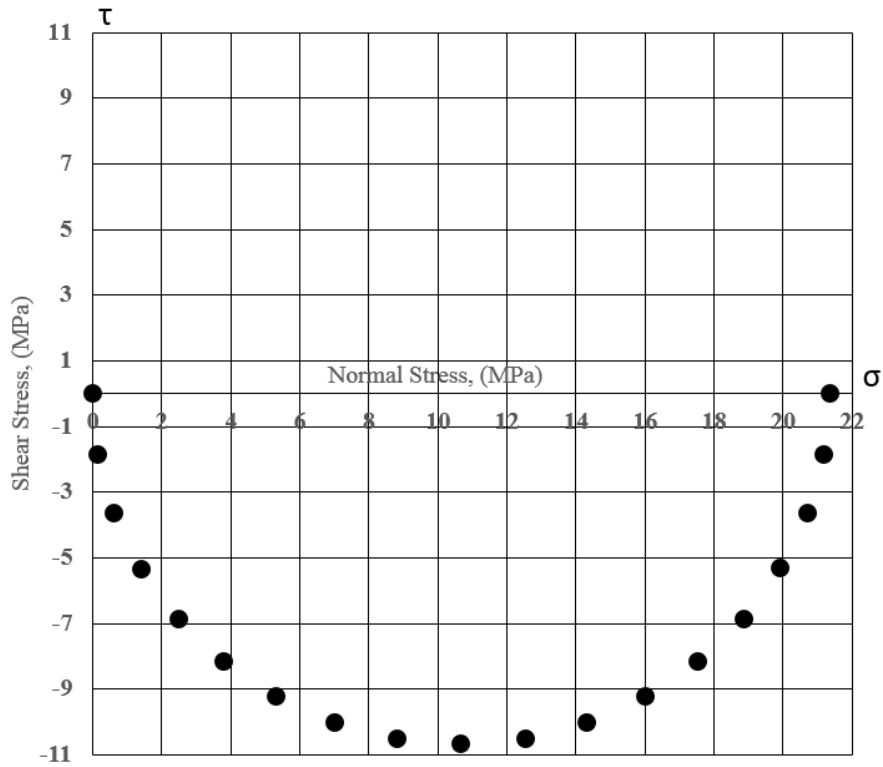


Figure 11: Second Half of Mohr's Circle using Shear Stresses plotted Downwards

And finally, Figures 7 and 11 can be combined to plot the complete, 360° Mohr's circle. Figure 12 shows the final result of this example exercise.



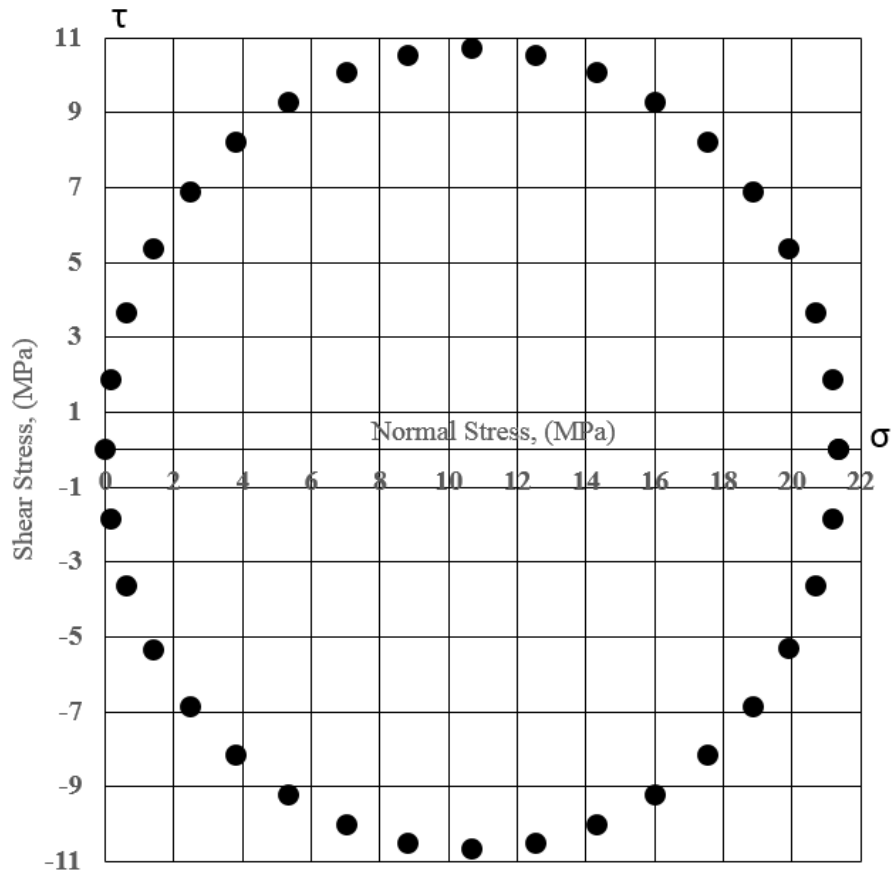


Figure 12: Complete Plot of Normal and Shear Stresses (i.e. Mohr's Circle) in Increments of 5°

### Strength of Materials Course Demographics

During a traditional 15-week long semester, a sophomore level Strength of Materials class meets for 50 minutes three times per week. To verify the effectiveness of this teaching approach, a total of 116 students were asked to participate in the activity over two semesters. The 2015 spring semester consisted of two groups with 49 total students and the fall semester of 2015 also had two groups with 67 total students. Table 1 below shows the participating student population.

Table 1: Strength of Materials Class Size

2015 Semester	Strength of Materials Course Group #	Student Population
Spring	1	26
	2	23
Fall	3	33
	4	34
<b>Total</b>		<b>116</b>

## Evaluation of Method

In order to evaluate the effectiveness of the interactive example, there were 2 approaches used in presenting Mohr's circle. The first approach uses the interactive example after discussing the theory of stress transformation. The second approach presents the theory of stress transformation and Mohr's circle without the use of the interactive example. Table 3 below summarizes the two approaches over the span of three (3) 50-minute classes.

Table 3: Two Instructional Approaches to Stress Transformation and Mohr's Circle

<b>Strength of Materials Course Section</b>	<b>Instructional Approach</b>
Group 1 and 3	<u>1<sup>st</sup> 50 minute class</u> Introduce Stress Transformation Equations <u>2<sup>nd</sup> 50 minute class</u> Briefly Introduce Mohr's Circle <b>Conduct team exercise</b> <u>3<sup>rd</sup> 50 minute class</u> Review Mohr's Circle Principles & do examples
Group 2 and 4	<u>1<sup>st</sup> 50 minute class</u> Introduce Stress Transformation Equations <u>2<sup>nd</sup> 50 minute class</u> Introduce Mohr's Circle <u>3<sup>rd</sup> 50 minute class</u> Review Mohr's Circle principles & do examples

After the three class periods, a 15 question multiple choice quiz (Appendix I) was administered. The student's level of understanding was determined based upon quiz grades.

### Interactive Example Verification Results

Figure 13 below shows the overall average and median quiz scores for students exposed to the interactive example and those not exposed to the interactive example.

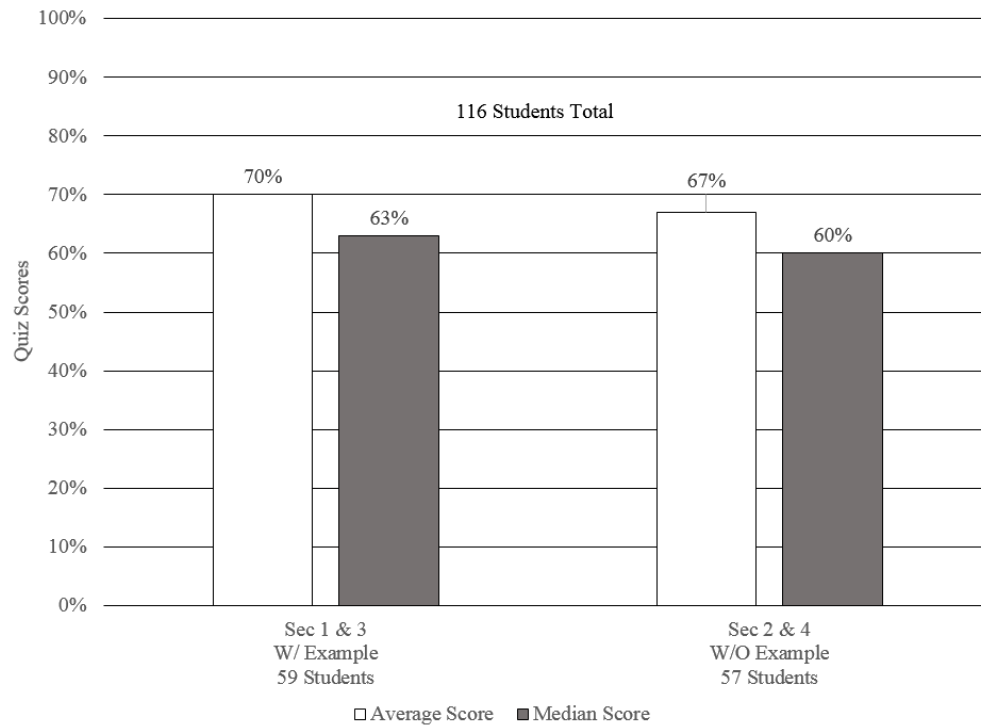


Figure 13: Stress Transformation and Mohr’s Circle Quiz Scores

Using the total student population of 116 students, a quiz score histogram can be constructed. Figure 14 below depicts the total population quiz scores in histogram format.

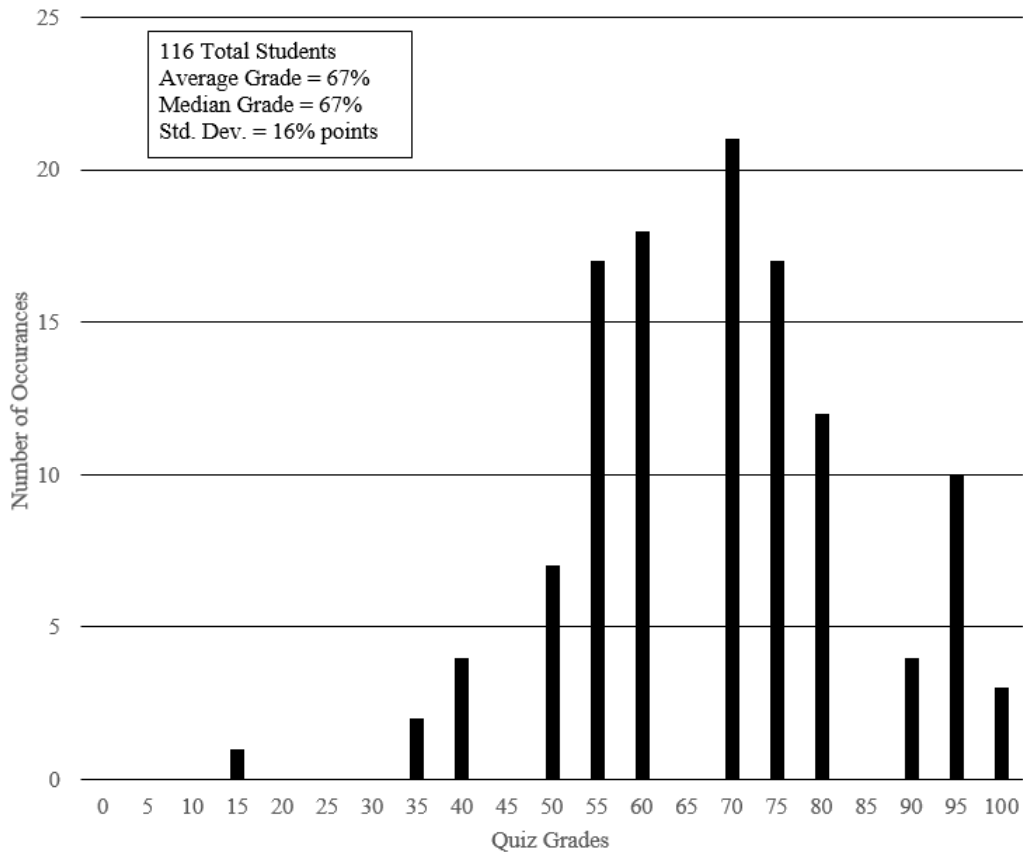


Figure 14: Total Student Population Quiz Scores

Overall, the average and median scores are lower than expected. However, the distribution shape is reasonable. Figure 15 below shows a similar type histogram result for the students exposed to the interactive example problem.

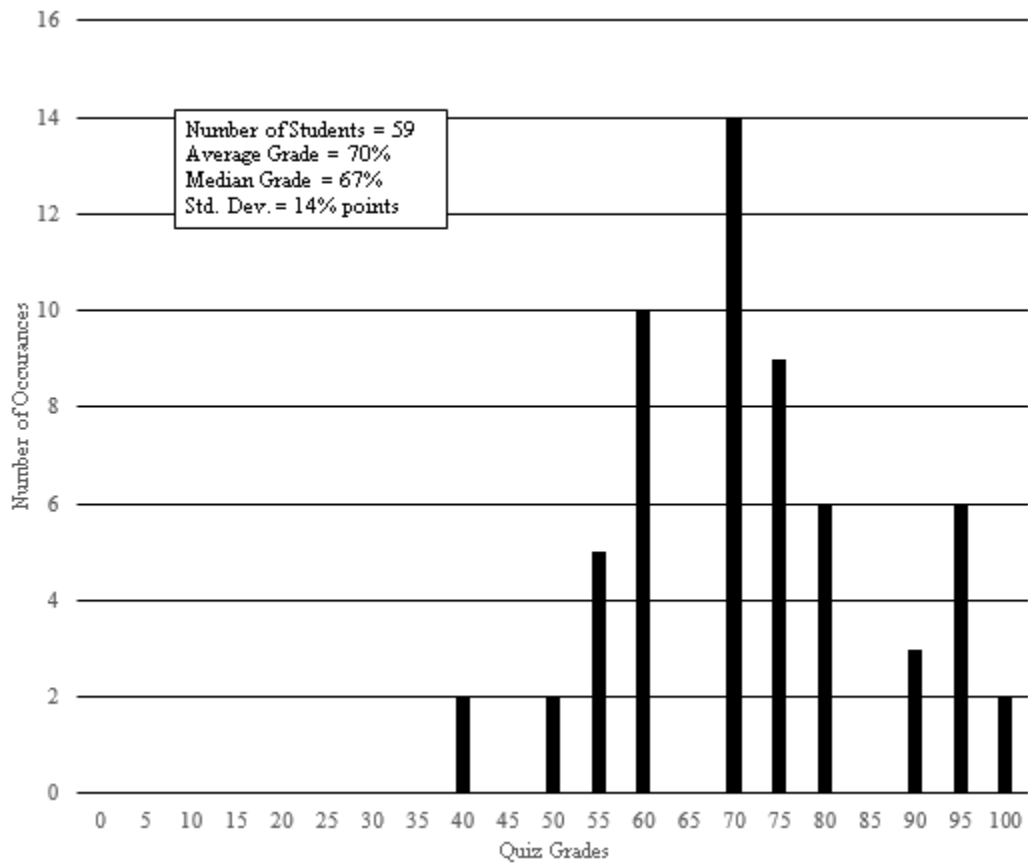


Figure 15: Student Quiz Scores exposed to Interactive Example Problem

The quiz grade average of 70% is a slight increase while the median remained at 67%. On the other hand, the histogram of quiz grades for students not exposed to the interactive example is also plotted below in Figure 16.

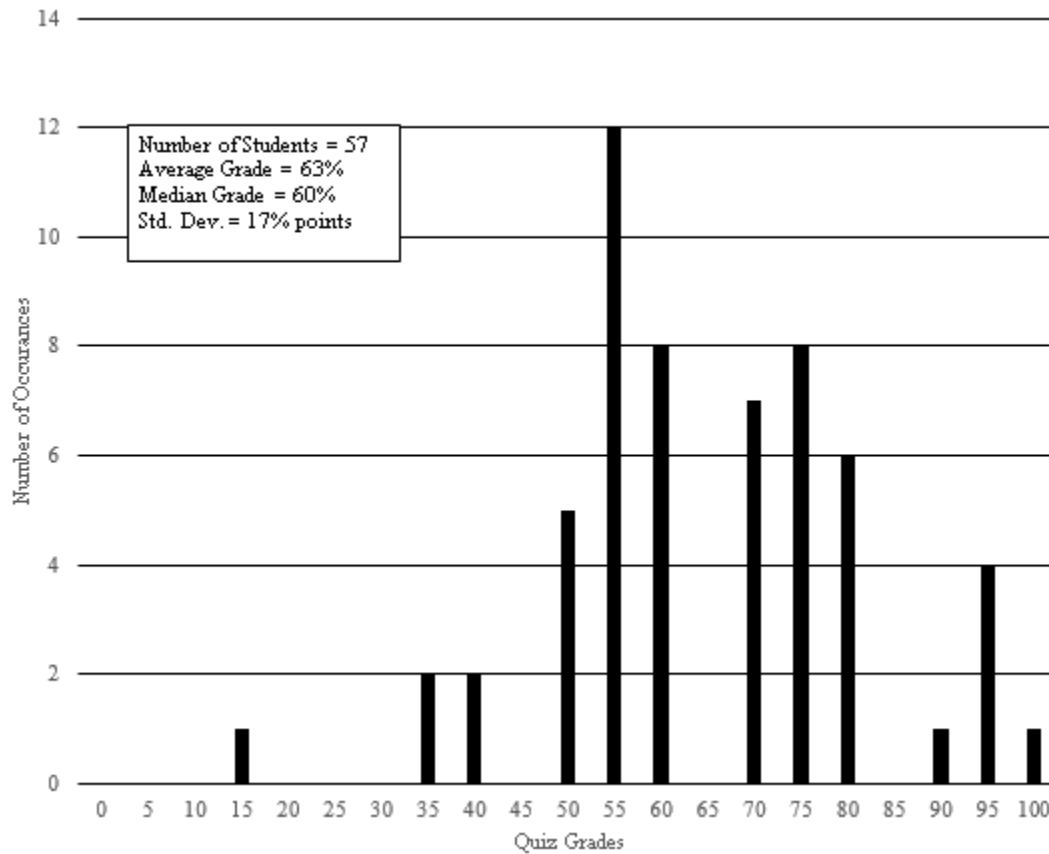


Figure 16: Student Quiz Scores not exposed to Interactive Example Problem

And finally, overlaying the standard deviation envelope over the average and median quiz scores shows a different perspective. Figure 17 below shows the standard deviation envelope “sketched” into the quiz score comparison.

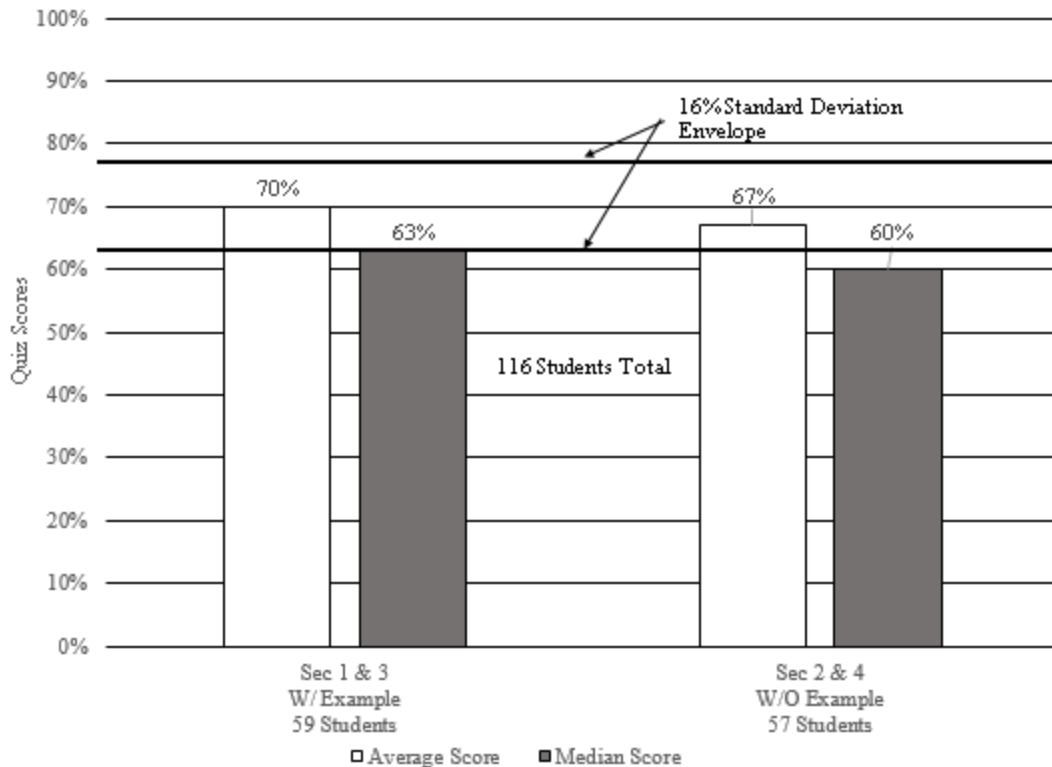


Figure 17: Standard Deviation Envelope “sketched” over Quiz Score Average and Median

With the addition of standard deviation, confidence in the improved quiz scores is decreased. Both average and one mean scores are within one standard deviation.

### Conclusions

Having used the interactive example in classes over two semesters, the following conclusions are presented.

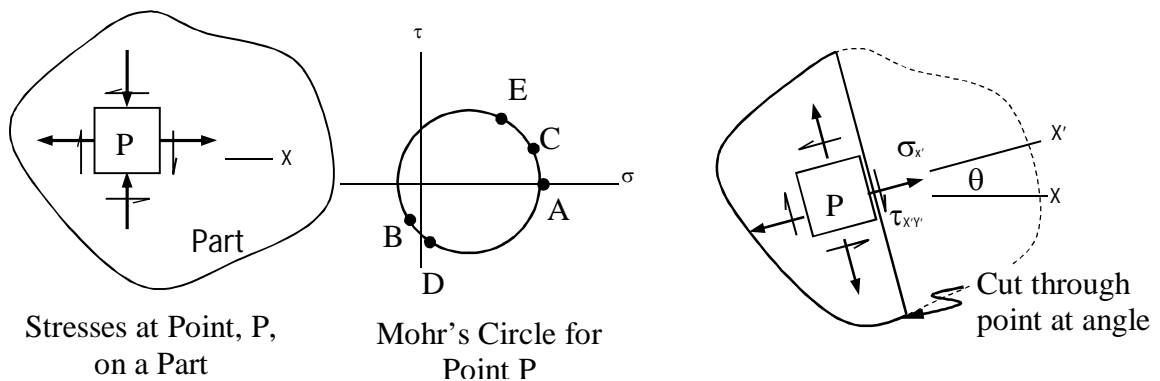
- The average and mean quiz scores were 3% higher for students involved with the interactive students when compared with students who did not participate. This value is lower than expected and it suggests that the interactive example is not effective.
- Anecdotal evidence obtained through regular conversations with students suggests that there is better understanding among those who participated. In retrospect, the authors are not convinced that the quiz used in the evaluation process actually measures student understanding. The quiz will be reconsidered and will likely be modified to measure future trials more accurately.
- Engineering technology students are generally very eager to participate in hands-on activities and this one is no exception. This example will continue to be used since the students have fun with it and it is at least as good as conveying the material as a direct lecture.

## Discussion of the Assessment Quiz

The assessment quiz, which is included in Appendix A, consists of 15 multiple-choice questions, mostly related to the rules of stress transformation and Mohr's circle. It is lacking in questions that directly ask about stress transformations and how the stresses relate to points on the representative Mohr's circle.

Better results may be obtained by having students answer questions about stress elements and Mohr's circle related to different plane stress situations. Some initial thoughts on more appropriate questions would include:

- Given an initial stress element for a point on a part and Mohr's circle describing the stresses at that point; the student would indicate which point on Mohr's circle provides the stresses on the face of a cut through the point. Figure 18 illustrates this concept.

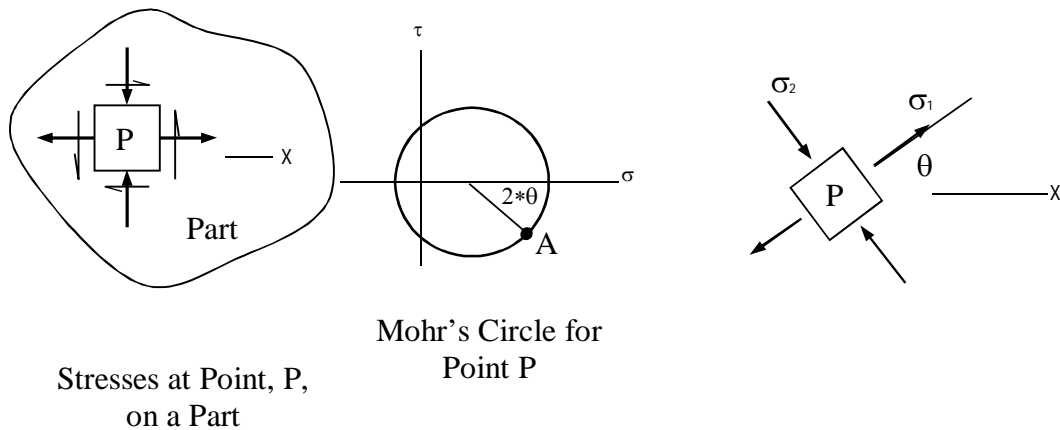


Q: Which point on Mohr's Circle (A, B, C, D, E) indicates the values of  $\sigma_{x'}$  and  $\tau_{x'y'}$ ?

Figure 18: Sample Question #1



- Given an initial stress element for a point on a part and Mohr's circle describing the stresses at that point; the student would indicate what the proper orientation for the principle stress element. Figure 19 illustrates this concept.



Q: Draw the properly oriented principle stress element for point, P?

Figure 19: Sample Question #2

Using “application” type questions as shown above, the quiz may more clearly assess the students understanding of elemental plane stress and the use of Mohr's circle. For the upcoming semester, a revised quiz will be implemented and comparison analysis will be conducted for groups exposed/not exposed to the interactive example.

## Bibliography

Evans, Edward R. Jr. (2013). Advanced Strength of Materials Lecture Notes “Stress in Axially Loaded Bar,” Mechanical Engineering Technology, Penn State Erie - The Behrend College

Jones, Philip A. (2014). Strength of Materials Lecture Notes “Stress Transformation and Mohr's Circle,” Mechanical Engineering Technology, Penn State Erie - The Behrend College

## References

<sup>[1]</sup>Mott, Robert L. (2008). Applied Strength of Materials, 5<sup>th</sup> Edition. Upper Saddle River, NJ: Pearson Education Inc.

<sup>[2]</sup>Hibbler, R.C. (2014). Mechanics of Materials, 9<sup>th</sup> Edition. Upper Saddle River, NJ: Pearson Education Inc.

Appendix I

**MCHT 213 Fall 2015 Quiz #10: Stress Transformation & Mohr's Circle Quiz(15 pts)**

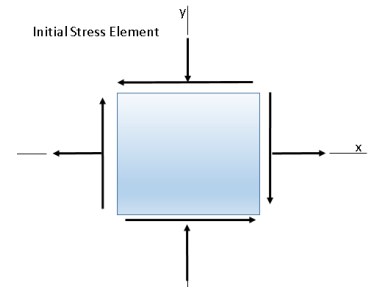
Name: \_\_\_\_\_

Date: \_\_\_\_\_

Please Note: This is a closed book, closed notes quiz. Student collaboration is not permitted.

\_\_\_\_\_ 1. For the initial stress element shown below, what is the proper designation for normal stress on the x and y axis?

- a.  $\tau_{xy}$  and  $\tau_{yx}$ , respectively
- b.  $\sigma_x$  and  $\sigma_y$ , respectively
- c.  $\sigma_{xy}$  and  $\sigma_{yx}$ , respectively
- d.  $\tau_x$  and  $\tau_y$ , respectively
- e.  $\sigma_z$  and  $\sigma_y$ , respectively

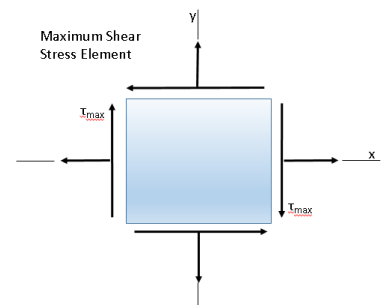


\_\_\_\_\_ 2. In this class, Mohr's Circle is used to calculate stresses at any angular orientation for the following state of stress.

- a. Plane stress
- b. 3-D state of stress
- c. Mohr's Circle cannot be used to calculate stress at any angle
- d. Plane strain
- e. Both a and d

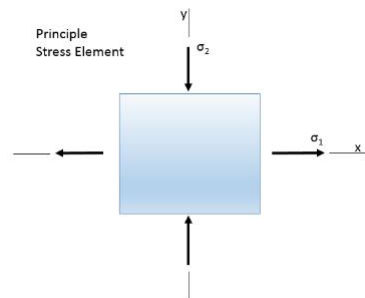
\_\_\_\_\_ 3. What is the magnitude of normal stress shown on the maximum shear stress element shown below?

- a. Zero
- b.  $\tau_{max}$
- c.  $\sigma_{max}$
- d.  $\sigma_{avg}$
- e.  $\tau_{avg}$



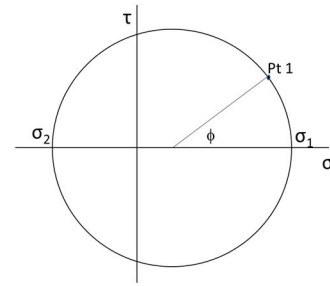
\_\_\_\_\_ 4. On the stress element on which the principle stresses act, the shear stress is:

- a. Maximum
- b. Equal to the normal stress
- c. Minimum
- d. Zero
- e. Equal and opposite the normal stress



\_\_\_\_\_ 5. The principle plane angular orientation,  $\phi$ , on Mohr's Circle is \_\_\_\_\_ the angle when compared to the stress on the element.

- a. Equal to
- b. 1/2 of
- c. 2 times
- d. Not comparable to
- e. Graphically equivalent to



\_\_\_\_\_ 6. The magnitude of shear stress present on the principle stress plane is

- a. Equal to the maximum principle stress
- b. Zero
- c. The maximum shear stress
- d. Equal to  $\frac{1}{2}$  of the maximum principle stress
- e. The minimum shear stress

\_\_\_\_\_ 7. The plane stress transformation equations are in the form of a:

- a. Trapezoid
- b. Circle
- c. Ellipse
- d. Polygon
- e. Rectangle

\_\_\_\_\_ 8. On Mohr's Circle, the principle stress plane and the maximum shear stress plane are oriented \_\_\_\_\_ degrees apart.

- a. 0
- b. 45
- c. 90
- d. 180
- e. 60

\_\_\_\_\_ 9. On the resulting stress element, the principle stress plane and the maximum shear stress plane are oriented \_\_\_\_\_ degrees apart.

- a. 0
- b. 45
- c. 90
- d. 180
- e. 60

\_\_\_\_\_ 10. In translating the principle stresses from Mohr's Circle back to the stress element, the direction of rotation from point 1 to the principle plane is \_\_\_\_\_ to the direction taken on Mohr's Circle.

- opposite
- 180 degrees from the initial point
- the same
- 45 degrees from the principle plane
- Is equivalent to the maximum shear stress plane

\_\_\_\_\_ 11. In translating the maximum shear stress from Mohr's Circle back to the stress element, the direction of rotation from point 1 to the maximum shear stress plane is:

- In the opposite direction
- 180 degrees from the characteristic point
- In the same direction
- 45 degrees from the principle plane
- Is equivalent to the maximum shear stress plane

\_\_\_\_\_ 12. For Mohr's Circle, the average normal stress is calculated by:

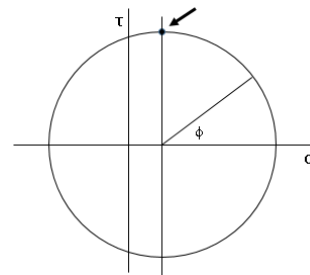
- $\sigma_{avg} = \frac{1}{2} * (\sigma_x + \sigma_y)$
- $\sigma_{avg} = \frac{1}{4} * (\sigma_x + \sigma_y)$
- $\sigma_{avg} = \frac{1}{2} * (\sigma_x - \sigma_y)$
- $\sigma_{avg} = \frac{1}{2} * (\tau_x + \tau_y)$
- $\sigma_{avg} = \frac{1}{4} * (\sigma_x - \sigma_y)$

\_\_\_\_\_ 13. For Mohr's Circle, point 1 is defined as:

- $(\sigma_y, \tau_{xy})$
- $(\sigma_x, \tau_{yx})$
- $(\sigma_x, \sigma_{xy})$
- $(\sigma_y, \sigma_{xy})$
- $(\sigma_x, \tau_{xy})$

\_\_\_\_\_ 14. The point shown on Mohr's Circle below is defined as:

- $\tau_{min}$
- $\tau_{max}$
- $\sigma_1$
- $\sigma_2$
- $\sigma_x$



\_\_\_\_\_ 15. For the Mohr's Circle shown below, what is the magnitude of the angle,  $\phi$ , shown on the principle stress element?

- a. 35 degrees, CW
- b. 70 degrees, CW
- c. 17.5 degrees, CCW
- d. 17.5 degrees, CW
- e. 70 degrees, CCW

