Session 2368

Intuition, observations, and generalization in mechanics of materials

Madhukar Vable
Mechanical Engineering-Engineering Mechanics,
Michigan Technological University

Abstract

The introduction of advanced topics as means of modernizing engineering curriculum, the need for interdisciplinary research and education to meet societies challanges, the time constraint that engineering students graduate in four years while getting a modern-interdisciplinary-education, are some of the factors driving the evolution of basic engineering courses such as mechanics of materials. Generalization of principles in the basic engineering courses is one mechanism by which a greater amount of knowledge can be taught in a compact form. But intrinsic to any generalization is the increase in abstraction of concepts. This increase in abstraction may cause many engineering students to lose interest in the profession as they generally have a predisposition towards more practical and applied work. The challenge confronting the engineering education community is to present the subject material in such a way that the intuition, experimental observations and mathematical generalization complement each other and the students can see the practical applications of the general principles. In this paper a pedagogy of presentation of mechanics of materials concepts is described. Through a series of examples the pedagogy by which cultivation of intuition, experimental observations, and mathematical generalization can be presented in a complimentary manner is elaborated in context of two important concepts in mechanics of materials, namely: concept of stress and theory of one-dimensional structural elements. The practical application of general principles in context of design is presented in a separate paper.

1. Introduction

Near the beginning of twentieth century, courses using textbooks[1,2] with title 'resistance of materials' were significantly more applied than today's course called 'mechanics of materials'. As the emphasis on statics and the concept of stress and strains grew, the popular textbooks title[3,4,5] became 'strength of materials'. Today's textbooks[6-11] predominately presume that 'statics' and 'mechanics of materials' will be taught as independent courses.

Ecole Polytechnique —pioneer of modern engineering school, had a curriculum[12] in which the first two years were devoted exclusively to fundamental sciences. Engineering courses were taken in the third year. This teaching method differed dramatically from the prevalent practice of teaching engineering in an apprentice mode by practicing engineers with students taking no formal courses in mathematics and sciences. We see that since inception of formal engineering education, the engineering education community has struggled with the delineation of science, engineering, and vocational-technical education.

As elaborated in sections 3 and 5, the evolutionary forces in engineering education is toward greater generalization in basic engineering courses as it permits teaching of a greater amount of knowledge in a compact form. Without such generalization the problems of already overburdened engineering curriculum and the growing graduation time would become a lot worse. The abstraction that occurs with generality poses two different types of challanges to keep the students involved and motivated. The first challenge is the need to build a complimentary connection between intuition, experimental observations, and mathematical generalization— discussed in this paper. The second challenge is to show the practical relevance of the general principles— discussed in context of design in a separate paper[13].

2. Statics and Mechanics of Materials

Statics is primarily a skill and discipline developmental course. Starting with a structure or a machine and breaking it down into individual elements on which equilibrium equations can be used is an analysis skill that is honed in statics. Approaching the analysis problem in a methodical manner and drawing appropriate free body diagrams before invoking equilibrium equations is a discipline that is cultivated in statics.

Mechanics of materials is primarily a concept development course, but relies heavily on the analysis skill and discipline taught in statics. The concept of stress and strain are difficult concepts as these two variables are not directly measurable but must be inferred from other data. Understanding the theories of one-dimensional structure elements is another conceptual development.

One probable reason for development of statics and mechanics of materials as independent courses is that the teaching techniques for development of skills and disciplines are different from those for development of concepts. But there is another force that is equally important that is driving the developments in mechanics of materials.

In the past twenty-five years there has been tremendous growth in mechanics, material science, and in new applications of mechanics of materials. Twenty-five years ago, techniques such as the finite-element method and Moire' Interferometry, were research topics in mechanics, but today these techniques are routinely used in engineering design and analysis. Twenty-five years ago, wood and metal were the preferred materials in engineering design, but today machine components and structures may be made up of plastics, ceramics, polymer composites, and metal matrix composites. Twenty-five years ago, mechanics of materials was primarily used for structural analysis in aerospace, civil, and mechanical engineering, but today mechanics of materials is used in electronic packaging, medical implants, explanation of geological movements, and the manufacturing of wood products to meet specific strength requirements. Though the principles in mechanics of materials have not changed in the past twenty-five years, the presentation of these principles must evolve to provide the students with a foundation that will permit them to readily incorporate the growing body of knowledge as an extension of fundamental principles and not something added on, and vaguely connected to, what they already know. In other words, there must be greater generality in the presentation of the principles in mechanics of materials.

Often one hears arguments that seem to suggest that intuitive development comes at the cost of mathematical logic and rigor, or, the generalization of a mathematical approach comes at the expense of intuitive understanding. Yet, the icons in the field of mechanics of materials, such as Cauchy, Euler, and Saint-Venant, were individuals who successfully gave physical meaning to the mathematics they used. Accounting of shear stress in the bending of beams is a beautiful demonstration of how the combination of intuition and experimental observations can point the way when self-consistent logic does not. Intuitive understanding is a must—not only for creative engineering design but also for choosing the marching path of a mathematical development. By the same token, it is not the heuristic-based arguments of the older books, but the logical development of arguments and ideas that provide students with the skills and principles necessary to organize the deluge of information in modern engineering. Building a complimentary connection between intuition, experimental observations, and mathematical generalization is central to meeting the challenge of keeping students involved and motivated while studying general principles in mechanics of materials. This paper using examples from reference 11 shows how this may be achieved.

3. Logic in structural analysis

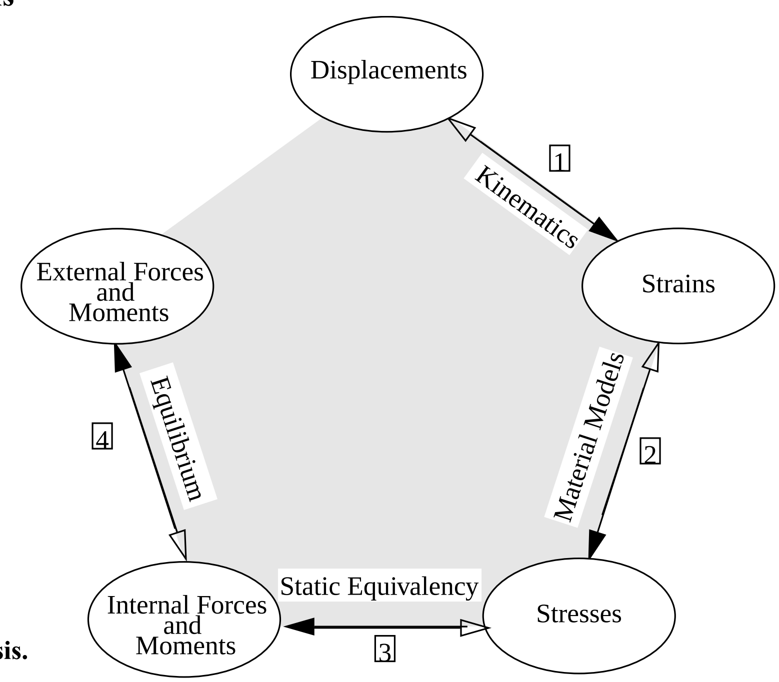


Figure 1. Logic in structural analysis.

Examination of the old textbooks[1,2] show that the derivation of the theory of one-dimensional structural members (axial members, torsion of shafts, and bending of beams) was done in a very heuristic manner. Today's textbooks show a significant use of deductive logic in derivation of the theories of one-dimensional structural members. This growth in use of logic is not surprising for mechanics of materials synthesizes the empirical relationships of materials into the logical framework of mechanics to produce formulas for use in the design of structures and other solid bodies.

This evolutionary growth in logic is the growth in generalization. The logic can be depicted as shown in Fig.1. The logic is used in deriving theories for the simplest structure elements studied in the sophomore/junior level mechanics of material course to graduate level courses on plates and shells with material and geometric non-linearities. The logic is intrinsically very modular—equations relating the fundamental variables are independent of each other, hence complexity can be added at any point without affecting the other equations. However, in an introductory course the central focus of the student must be to understand the simplest theory along with the limitations. To demonstrate how this may be achieved, consider example 1

EXAMPLE 1: A canoe on top of a car is tied down using rubber stretch cords as shown. The undeformed length of the stretch cord is L_o=40 inches. The initial diameter of the cord is d = 1/2 in and modulus of elasticity of the cord is E = 510 psi. Assume that the path of the stretch cord over the canoe can be approximated as shown below. Determine the approximate force exerted by the cord on the carrier of the car.

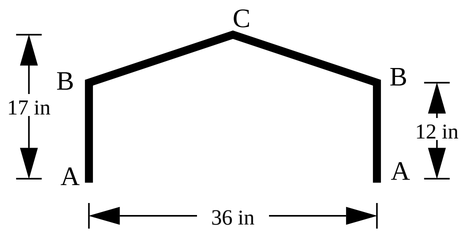


Figure 2. Approximation of stretch cord path on top of canoe.

Briefly stated, the solution proceeds as follows. From the given deformed geometry the length of stretched cord can be found and average normal strain determined. Using Hooke's law the average normal stress can be found. Knowing the diameter of the cord the area of cross-section can be found and the internal tension in the cord can be determined. By equilibrium the internal tension at point A is the force exerted by the cord on the carrier of the car. Thus the students see the solution as a straight forward application of the logic shown in Fig.1.

After the presentation of the solution, the various approximations can be highlighed and the student shown how the accuracy of the solution can be improved by addition of one complexity at a time. This is described briefly below.

(i) The cord should follow the contour of the canoe from the point of contact leading to non-uniform distribution of the strain along the cord. Suppose marks are made on the cord every 2 in before the cord is stretched. Strain can be found in each of the 2 in segment and the calculations for finding internal force can be repeated as before. Attention could be drawn to problem example 1 in appendix A and could be given as a bonus homework problem.

(ii) The stress-strain curve of the rubber cord is non-linear. Thus, as the strain changes along the length, so does the modulus of elasticity E. The variation of E must be accounted in the calculation of stress. The tangent modulus of elasticity can be used in Hooke's law for each segment and more accurate stresses in each segment can be obtained. The internal force in each segment can be found as before. Now the attention can be drawn to problem example 2 in appendix A and could be given as a bonus homework problem.

(iii) The area of cross-section for rubber will change significantly with strain and must be accounted for in the calculation of the internal tension. Rubber has a Poisson's ratio of 1/2. Knowing the longitudinal strain for each segment, the transverse strain in each segment can be found, from which the diameter of cord in the stretched position in each segment can be determined. This will give a more accurate area of cross-section, and hence a more accurate value of internal tension in the cord. Now the attention can be drawn to problem example 3 in appendix A and that could be given as a bonus homework problem.

The above example describes the process by which the student focus is kept on learning the logic shown in Fig.1 which is fundamental to mechanics of materials, but the student can appreciate how complexities can be added to simplified analysis, even if no bonus problems are solved. This type of process can also used in developing the simplified theories of axial members, torsion of shafts, and bending of beams with assumptions clearly identified and associated bonus problem in which the assumption is violated can be identified (see reference 11 for details). The logic shown in Fig.1 is used four different times in the course with same type of assumptions made for axial members, torsion of shafts, and bending of beams. This repetition is one reason to expect that the students will leave the course with a firm grasp of the logic and a good appreciation of the limitations (assumptions) of the theory and where the complexities are added.

4. Prelude to theory

Compact organization of information seems like an abstract reason for learning theory to some engineering students. Some students have difficulty visualizing a continuum as an assembly of infinitesimal elements whose behavior can be approximated and / or deduced. As faculty members we all know that the students are more attentive when a numerical problem is being solved then when the theory is being derived in the class. A survey of reading habits of students in my class showed that only 76% of the students read text but 100% read the numerical examples. Thus if the ideas that are used in the derivation of the theory are developed as a numerical problem then one may expect that these ideas will be retained by the student more easily—this is the gist of prelude to theory. Consider the example below.

EXAMPLE 2: Two thin bars of hard rubber (Shear Modulus G = 280MPa) have a cross-sectional area of 20 mm^2. The bars are attached to a rigid disc of radius 20 mm as shown in Fig.3(a). Due to the applied torque T_{ext}, the rigid disc is observed to rotate by an angle of 0.04 rads about the axis of the disc. Determine the external torque T_{ext}.

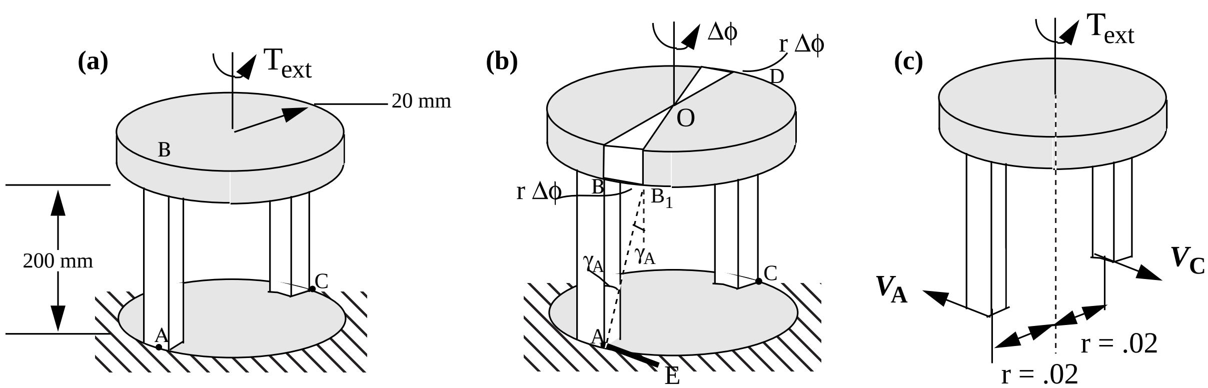


Figure 3. (a) Geometry in example 2 (b) Deformed geometry. (c) Free body diagram

The solution to the problem is briefly described below.

1. Strain Calculations: The deformed shape of the two bars is drawn as shown in Fig.3(b) and the shear strain calculated as:

$$\tan\gamma_A \approx \gamma_A = (BB_1)/(AB) = (0.02)\Delta\phi/0.2 = 0.004 \qquad and \qquad \gamma_c = \gamma_A = 0.004$$

2. Stress Calculations: From Hooke's Law the shear stresses are:

$$\tau_C = \tau_A = G_A\gamma_A = 280(10^6)(0.004) = 1.12(10^6)\ \ N/m^2$$

3. Internal Forces: Assuming uniform shear stresses across the cross-section, the shear forces can be obtained as

$$\boldsymbol{V_A} = A_A\tau_A = 22.4\ \ N \qquad \boldsymbol{V_C} = A_C\tau_C = 22.4\ \ N$$

4. External Torque: By equilibrium of moment in the free body diagram shown in Fig.3(c) the external torque can be found as:

$$T_{ext} = (r)(\boldsymbol{V}_A) + (r)(\boldsymbol{V_C}) = 0.896\ \ N-m\,.$$

The kinematics of obtaining the shear strain is similar to that of a circular shaft in torsion. As the geometry is made of discrete elements rather than a shaft continuum, the students can visualize the deformed geometry of Fig.3(b) more easily. The steps executed in solving the problem are the same as in development of the theory of torsion of circular shafts. As the number of bars increase the discrete system becomes a shaft continuum. The impact of the increase in number of bars can be explained as follows. The summation in the expression of external torque can be rewritten as $\sum_{i=1}^{2} r\tau\Delta A_i$, where τ is the shear stress acting at the radius r, and ΔA_i is the cross-sectional area of the of the i^{th} bar. If there were n bars attached to the disc at the same radius, then the total torque would be given by $\sum_{i=1}^{n} r\tau\Delta A_i$. As the number of bars n increases to infinity, the cross-sectional area ΔA_i will tend to zero (infinitesimal area written as dA) —resulting in a continuous body with the summation replaced by an integral.

In a similar manner, using other numerical examples additional ideas that are needed in the theory can be developed. Assignment problems of similar nature then consolidates these ideas. The presentation of the theory is now formalizing of the ideas the students have learned through numerical examples. The advantage of this approach is that as the ideas are being developed for discrete systems first which the students can visualize, that is, understand the ideas intuitively. Mathematical development of theory then can quickly consolidate these ideas understood intuitively. In a similar manner discrete bars welded to a rigid plate subjected to displacements that simulate the behavior of the cross-section in bending can be used to develop the ideas in the theory of bending for symmetric beams as described in detail in reference 11.

Experimental evidence can now be used to convince the students that the rigid plate subjected to the rotation in Fig.3(a) is simulating the behavior of a cross-section of a circular shaft in torsion. Fig.4 shows the photographs a a rubber shaft deforming under torsion. The students can see with their own eyes that the vertical lines representing the cross-sections remain plane but rotate about the axis of the shaft just like the rigid disc in Fig.3(a).

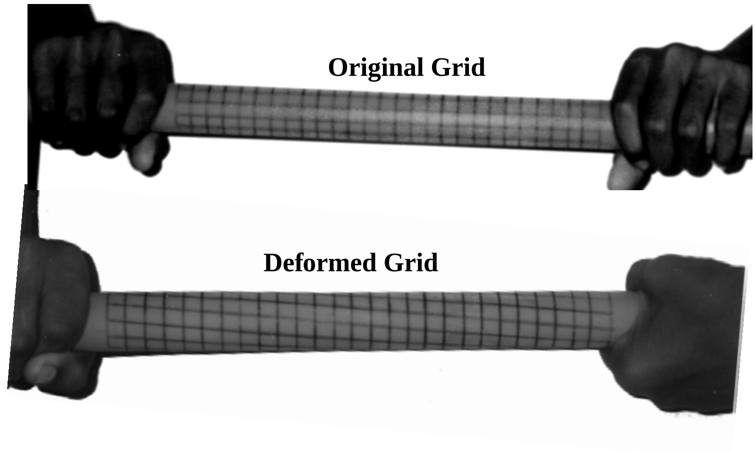


Figure 4. Torsional Deformation.
Courtesy of Professor J.B. Ligon.

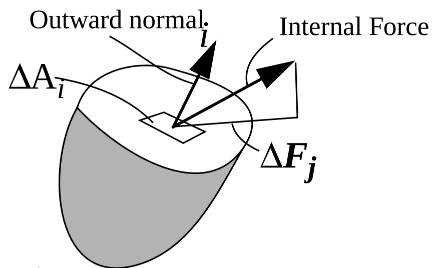
We thus see that the logic in Fig.1 is the mathematical generalization, the prelude to theory is the intuitive development, and the photographs in Fig.4 are the experimental evidence.

5. Double subscripts

Textbooks[1,2] at the beginning of twentieth century did not use any subscripts on stresses and strains. Textbooks[9,10] today generally use a single subscript for normal stress and strain components and double subscripts for shear stress and shear strain components in order to better explain the character of stress and strain. The next step in this evolution of subject material is to use double subscripts and explain the concept of stress and strain in a consistent manner[11]. There are three distinct advantages to using double subscripts: (i) It provides students with a procedural way to compute the direction of a stress component which they calculate from a stress formula. This procedural determination of direction of a stress component on a surface can help many students overcome the shortcomings in intuitive ability. (ii) Computer programs such as finite element method or those that reduce full-field experimental data, produce stress and strain values in a specific coordinate system that must be properly interpreted, which is possible if the students know how to use subscripts in determining the direction of stress on a surface. (iii) It is consistent with what the student will see in more advanced courses such as those on composites where the material behavior can challenge many intuitive expectations.

The use of double subscripts should be to compliment not substitute intuitive determination of stress direction. Internalizing the concept of stress needs the cultivation of intuitive determination of stress direction on a surface. If the students are able to draw the stress cube in a procedural manner and check the answer by inspection then the benefit of both approaches can be realized. The use of double subscript is described briefly first to elaborate this teaching approach.

Through numerical examples the need to specify the orientation of the internal surface and the direction of internal force in describing a stress component can be established. Then the following mathematical definition for a stress component can be introduced.



$$\sigma_{ij} = \lim_{\Delta A_i \to 0}\left(\frac{\Delta F_j}{\Delta A_i}\right)$$

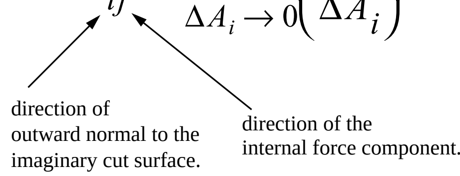


Figure 5. Stress at a point.

where, ΔF_j is the component of the internal force in the j-direction, and ΔA_i is the differential area on the imaginary cut surface that has an outward normal in the i-direction. Now the sign convention is that ΔA_i will be considered positive if the outward normal of the imaginary cut surface is in the positive i (coordinate) direction. The students do not have difficulty understanding that for a stress component to be positive the numerator and denominator must have the same sign otherwise the stress component will be negative. This concept can be consolidated by assigning problems like that in Example 3.

EXAMPLE 3: Show the following stress components on the surfaces A, B, and C of the two cubes shown in different coordinate systems in Fig.6.

$$\begin{bmatrix} \sigma_{xx} = 80MPa(T) & \tau_{xy} = 30MPa & \tau_{xz} = -70MPa \\ \tau_{yx} = 30MPa & & \\ \tau_{zx} = -70MPa & & \sigma_{zz} = 40MPa(C) \end{bmatrix}$$

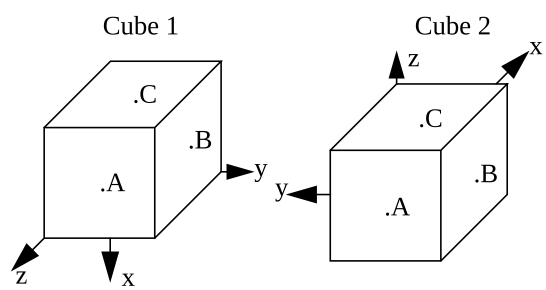


Figure 6. Cubes in different coordinate systems.

In the past 12 years I have given the above type of problem as a short exam problem and have observed that over 90% of the students get it correct, which is not surprising as students love procedures.

Now consider the torsional shear stress formula in which the shear stress in polar coordinate is shown with the subscripts as: $\tau_{x\theta} = \boldsymbol{T}\rho/J$. If the internal torque $\boldsymbol{T}$ is drawn on the free body diagram as per the sign convention (counter-clockwise with regard to the outward normal) then moment equilibrium will result in a positive or a negative value for the internal torque. The students already know how to draw a positive or negative stress component using subscripts in a procedural manner. Thus the determination and depiction of torsional shear stress on a stress cube can be done in a very procedural manner. Consider now the following example.

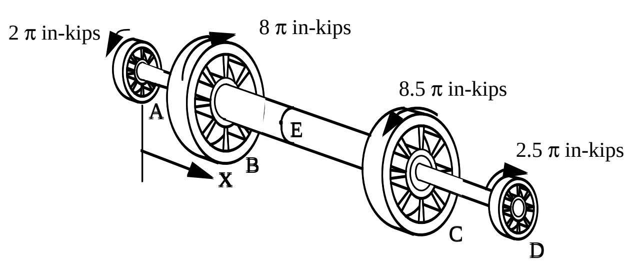


Figure 7. Shaft and loading in example 4

EXAMPLE 4: A solid circular steel (G_s = 12,000 ksi) shaft of variable diameter is acted upon by torques as shown. The diameter of the shaft between wheels A and B, and between wheels C and D is 2 inches, and the diameter of shaft between wheels B and C is 4 inches. Determine the shear stress at point E and show it on a stress cube.

The use of torsional shear stress formula yields $\tau_{x\theta} = +1.5\ ksi$. The two methods of determining the direction of shear stress may be described briefly as below.

Shear stress direction using subscripts: In the Fig.8(a), we note that $\tau_{x\theta}$ is +1.5 ksi. The outward normal is in the positive x-direction, the force has to be pointed in the positive θ-direction (tangent direction) which at point E is downward.

Shear stress direction determined intuitively: In Fig.8(b) a schematic of section BC is drawn. Consider an imaginary section through E in section BC. The section BE tends to rotate clockwise with respect to section EC. The shear stress will oppose the imaginary clockwise motion of section BE, hence the direction will be counter-clock-wise as shown.

The stress cube Fig.8(c) is drawn using the fact that pair of symmetric shear stresses either points towards the corner or away from the corner.

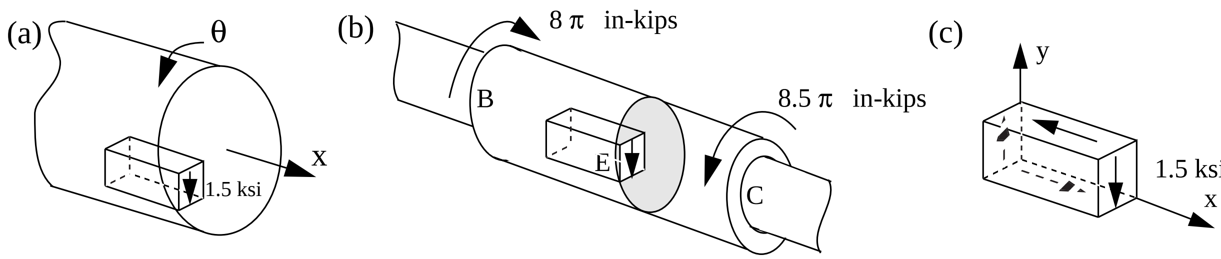


Figure 8. Direction of shear stress (a) using subscripts. (b) by inspection. (c) stress element.

The procedural determination of stress direction from a formula requires drawing of internal forces and moments as per the sign convention. If as a teacher we rely the cultivation of intuition only as a check to answers then some students may simply ignore it. If both ideas are considered important then problems have to be designed that emphasize the sign conventions and problems that emphasize determination of stress direction by inspection. Consider the following example:

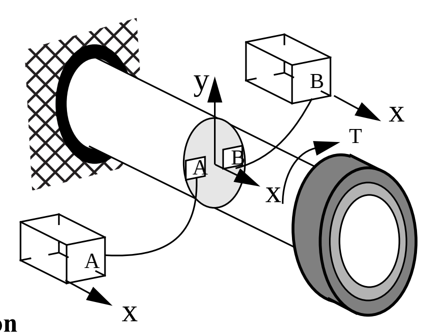


Figure 9. Torsional shear stress direction

EXAMPLE 5: Determine the direction of shear stress at points A and B (a) by inspection, and (b) by using the sign convention for internal torque and the subscripts. Report your answer as a positive or negative τ_{xy}.

The last part of reporting the answer as positive or negative τ_{xy} is important because equations of stress and strain transformation that students will use later in the course relate stresses and strains in cartesian coordinates to normal and tangential coordinates.

In similar manner the determination of bending stresses by subscripts require a sign convention for internal shear force and bending moment that can be emphasized with problems such as shown in example 6.

EXAMPLE 6: A beam and loading in three different coordinate system is shown. Determine the internal shear force and bending moment at the section containing point A for the three cases shown using the sign convention described in class.

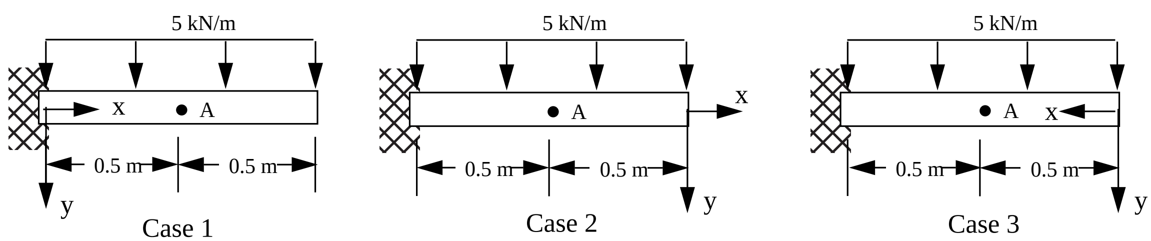


Figure 10. Example on sign convention.

The cultivation of intuitive determination of bending normal stress can be emphasized with problem such as shown in example 7, where the deformed shape of the beam shown in Fig.11(b) must be drawn or visualized.

EXAMPLE 7: By inspection determine whether the bending normal stress is tensile or compressive at points A and B for the beam and loading shown in Fig.11(a).

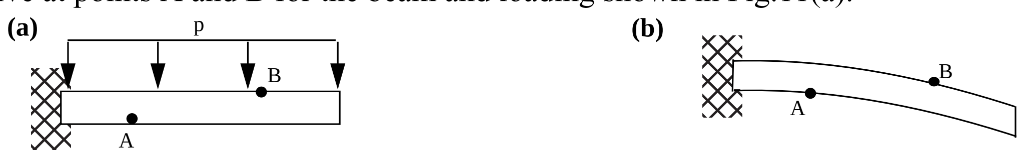


Figure 11. Example on intuitive development for bending normal stress (a) beam and loading. (b) deformed shape.

The cultivation of intuitive determination of bending shear stress (shear flow) can be emphasized with problem such as shown in example 7, where using the symmetry about the y-axis and force equilibrium in y and z direction require the direction of shear flow shown in Fig.12(b).

EXAMPLE 8: Assuming a positive shear force in the y-direction, sketch the direction of the shear flow along the center-line on the thin cross-sections shown in Fig.12(a).

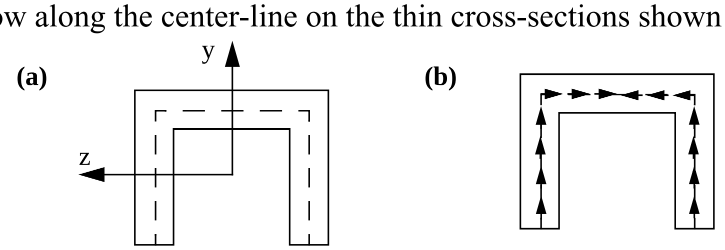


Figure 12. Example on intuitive development for bending shear stress. (a) thin cross-section. (b) shear flow.

One of the difficulty students face with combined loading is visualizing the stress components due to various loading. The procedural approach of determining stress components using subscripts that is developed for individual loading can be extended to combined loading as elaborated in reference 11. But once more it is equally important to be able to determine by inspection the manner in which stress components superpose. This can be facilitated by example of the type shown below.

EXAMPLE 9: By inspection determine and show the total stresses at points A and B on stress cubes using the following notation for the *magnitude* of stress components:
σ_{axial} —axial normal stress;
τ_{tor}—torsional shear stress;
$\sigma_{bend\text{-}y}$ —normal stress due to bending about y-axis;
$\tau_{bend\text{-}y}$ —shear stress due to bending about y-axis;
$\sigma_{bend\text{-}z}$ —normal stress due to bending about z-axis;
$\tau_{bend\text{-}z}$ —shear stress due to bending about z-axis.
The solution is as shown below.

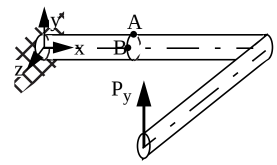


Figure 13. Pipe in combined loading

The normal stresses due to bending about the z-axis are compressive at point A and zero at the neutral axis at point B. The shear stress due to torsion is shown in Fig.14(a) and (b). Fig.14(c) and (d) show the stress cubes at points A and B, respectively.

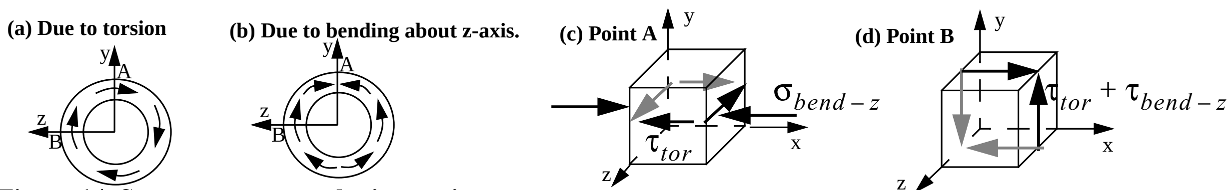


Figure 14. Stress components by inspection.

For simple problems such as in example 9 if the students can see how various stress components superpose then they will have an easier time tackling more complex combined loading that arise in design of simple structures described in reference 13. Bent pipes with simple loading can be used for cultivation of intuition for combined loading.

The above examples demonstrate the generalization of double subscript in stresses and the development of intuition can be done in a complimentary manner, thus enhancing the student understanding of the concept of stress.

6. Conclusions

In the past one hundred years the presentation of concept of stress has moved from no subscripts on stress and strain symbols to single subscript on normal stresses and strains and double subscripts on shear stresses and strains. Thus the next evolutionary generalization is the use of double subscripts on stresses and strains as elaborated in this paper. In the past hundred years, the derivation of theories for axial members, torsion of shafts, and bending of beam has moved from heuristic arguments to increase use of deductive logic. Thus the logic in Fig.1 represents the next evolutionary generalization as elaborated in this paper. Generalization permit teaching of greater amount of knowledge in compact form. It does not require structural changes in the curriculum as the changes can be made in the presentation of the subject matter of mechanics of materials as demonstrated in this paper. Generalization caters to the interdisciplinary needs of research and education. The difficulties that arise from the abstraction that is intrinsic in any generalization can be overcome by complimentary presentation of experimental observations and problems that cultivate intuitive development as demonstrated by several examples in this paper. Design can be used to show the practical relevance of these generalized concepts as presented briefly in reference 13 and in detail in reference 11. Thus this paper shows one way by which the presentation of mechanics of material concepts can alleviate some of the pressures from burgeoning curriculum and increasing graduation time.

7. References

1. Wood, D. V., “A treatise on the resistance of materials,” J. Wiley & Sons,1888.
2. Seely, F. B., “Resistance of materials,” J. Wiley & Sons, 1925.
3. Timoshenko, S., “Strength of materials,” D. Van Nostrand company inc., 1930.
4. Trathen, R. H., “Statics and strength of materials, an integrated presentation,” J. Wiley & Sons, 1954.
5. Case, J., and Chilver, A. H., “Strength of materials; an introduction to the analysis of stress and strain,” E. Arnold, 1959.
6. Bedford, A. and Fowler, W.L., “Statics,” Prentice Hall PTR,2001.
7. Beer, F.P. and Johnston E.R, “Statics”, McGraw-Hill Co., 1995.
8. Hibbeler, R.C., “Statics”, Prentice Hall PTR, 2000.
9. Gere J.M. and Timoshenko S.P., “Mechanics of Materials,” PWS publishing Co., 1997.
10. Riley W.F., Sturges L.D., and Morris D.H., “Mechanics of Materials,” John Wiley & Sons, Inc.,1999.
11. Vable, M., “Mechanics of Materials,” Oxford University Press, 2001.
12. Timoshenko, S.P., “History of strength of materials,” Dover publications, Inc., 1983.
13. Vable, M., “Enhancing understanding of concepts in mechanics of materials using design,” Submitted to Frontiers in Education conference, Boulder, 2003.

8. Biographical Information

Madhukar Vable, Associate Professor, has research interest in computational mechanics. He is a Fellow of Wessex Institute of Great Britain. He was named MTU Distinguished Teacher in 1998 and Distinguished Faculty Member from the Michigan State in 1999. He is author of ‘Mechanics of Materials’ textbook published by Oxford University Press. He is developing a stress analyzer called BEAMUP, details of which can be found at his webpage.

Appendix A: Bonus problems

example 1—example 3Marks were made on the cord used for tying the canoe on top of the car in example 1 These marks were made every two inches to produce a total of 20 segments. The stretch cord is symmetric with respect to the top of the canoe. The starting point of the first segment is on the carrier rail of the car and the end point of the tenth segment is on the top of the canoe. The measured length of each segment is as shown in the table below. Using this information and the data given in each of the problems example 1 through example 3, determine: (a) the tension in the cord of each segment. (b) the force exerted by the cord on the carrier of the car.

A.1 Use Modulus of Elasticity of E = 510 psi and the diameter of the stretch cord as (1/2) inch.

A.2 Use the diameter of the stretch cord as (1/2) inch and the following equation for stress-strain curve:

$$\sigma = \begin{cases} (1020\varepsilon - 1020\varepsilon^2)\ psi & \varepsilon < 0.5 \\ 255\ psi & \varepsilon \geq 0.5 \end{cases}$$

A.3 Use Poisson's ratio of ν=1/2 and initial diameter of 1/2 inch and calculate the diameter in deformed position for each segment. Use stress-strain relationship given in problem example 2.

Segment Number	1	2	3	4	5	6	7	8	9	10
Deformed Length (inches)	3.4	3.4	3.4	3.4	3.4	3.4	3.1	2.7	2.3	2.2