

Inverse Kinematics of the 6DOF C12XL Robotic Arm

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This work was conducted under the supervision of Professor Lili Ma at New York City College of Technology during the 2021 STEM Research Academy Summer Internship sponsored by City University of New York (CUNY).

Dr. Lili Ma, New York City College of Technology

Professor Lili Ma received her Ph.D. in Electrical Engineering from Utah State University focusing on autonomous ground vehicles. After that she did three-year post-doctoral training at Virginia Tech working with autonomous aerial vehicles. Prior to joining the Computer Engineering Technology (CET) department at City Tech in fall 2016, she taught at Wentworth Institute of Technology for many years. Professor Ma's research areas include autonomous mobile robots, vision-based control, visual servoing, visual tracking, coordinated control, and sensing & perception techniques.

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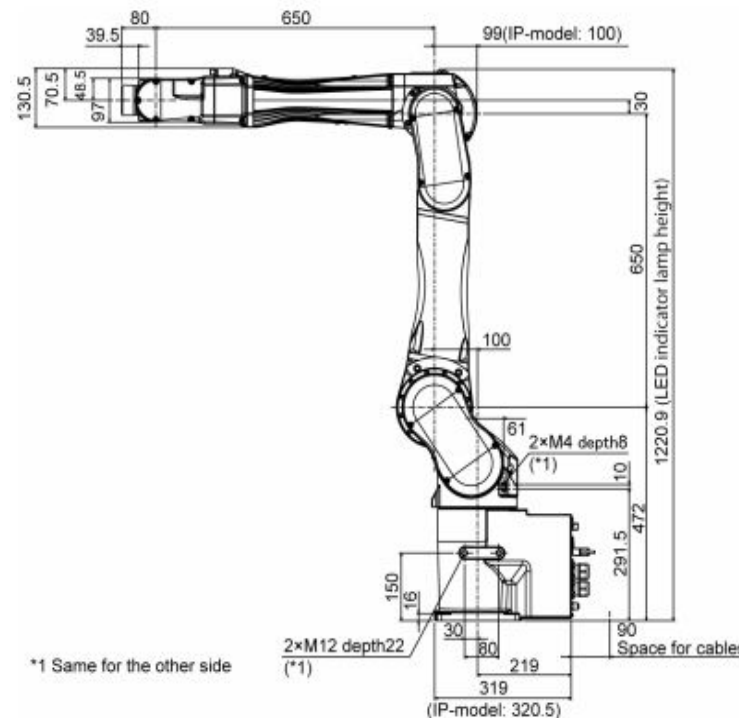
Introduction/Background Information

Robotic manipulators have been widely used in the industry performing repetitive tasks. Our work investigated the inverse engineering of the C12XL robot. The performed analyses and derivations help for the design of other robotic manipulators of similar kind.

Usage of DH convention to assign frames and create a DH table
 (Scaled down by a factor of 100)

Frames _i	(Joint Variable) θ_i	α_i	a_i	d_i
1	q_1	$\pi/2$	1	2.672
2	q_2	0	6.5	0
3	q_3	$-\pi/2$	0.3	0
4	q_4	$\pi/2$	0	-6.50
5	q_5	$-\pi/2$	0	0
6	q_6	π	0	-0.8

DH variables allow for joint axis to be related to one another, giving a simplified description of a robotic arm.



The inverse kinematics of a robotic arm is used to determine the joint variables that control the movement of each joint in a robotic arm. This makes it possible to command the robot's end effector to reach a desired position and orientation in space.

Dimensions of the C12XL Robotic Arm (Unit: mm)

Materials and Methods

Materials:

- Matlab R2021a
- Peter Corke's Matlab Robotic Toolbox
- Epson C12XL specification sheet

Desired orientation and position of the end effector

$$EOP = \begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$EO = \begin{bmatrix} nx & ox & ax \\ ny & oy & ay \\ nz & oz & az \end{bmatrix} \quad EP = \begin{bmatrix} px \\ py \\ pz \end{bmatrix}$$

$$EO^* [0 \ 0 \ 0.8]^T = [d_{x6} \ d_{y6} \ d_{z6}]^T$$

$$[p_x \ p_y \ p_z]^T - [d_{x6} \ d_{y6} \ d_{z6}]^T = [x \ y \ z]^T$$

To obtain the first three joint variables, the desired orientation of the end effector was multiplied with the final link's vector then the XYZ components of the modified link vector was subtracted from the desired position.

Materials and Methods

Solving for Positioning

modd4 is the hypotenuse of a right triangle with sides a3 and d4. This is used instead of d4 because of the a3 offset

$$\text{modd4} = \sqrt{a_3^2 + d_4^2}$$

$$D = \sqrt{D_{xy}^2 + (z-d1)^2}$$

Law of Cosine: $c^2 = a^2 + b^2 - 2ab\cos(C)$

$$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$$

$$-\cos(C) = \frac{c^2 - a^2 - b^2}{2ab}$$

$$-\cos(e) = \frac{D^2 - \text{modd4}^2 - a_2^2}{2a_2 \text{modd4}}$$

$$\sin(e) = 1 - (-\cos(e))^2$$

$$-\cos(e) = \cos(q3)$$

$$\sin(e) = \sin(q3)$$

$$q3 = \pi/2 + \tan^{-1}(\sin(e)/-\cos(e)) - \sin(a3/\text{modd4})^*$$

$$B = \tan^{-1}((\text{modd4} \sin(q3))/(a_2 + \text{modd4} \cos(q3)))$$

$$q2 = \tan^{-1}(z-d1/D_{xy}) - \tan^{-1}((\text{modd4} \sin(q3))/(a_2 + \text{modd4} \cos(q3)))$$

$$q2 = \tan^{-1}(z-d1/D_{xy}) - B$$

To solve for positioning, the law of cosine was used to find joint variables 2 and 3. Joint variable 1 was solved by using the pythagorean theorem

Second Set of Solutions for Positioning of End Effector

$$q1 = \tan^{-1}(y/x)$$

Law of Cosine: $c^2 = a^2 + b^2 - 2ab\cos(C)$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(e) = \frac{\text{modd4}^2 + a_2^2 - D^2}{2a_2 \text{modd4}}$$

$$\sin(e) = 1 - \cos(e)^2$$

$$q3 = \pi - \tan^{-1}(\sin(e)/\cos(e)) - \sin(a3/\text{modd4}) + \pi/2^*$$

$$q2 = \tan^{-1}(z-d1/D_{xy}) - B$$

Finding End Effector Orientation

$$Ry(RA) = \begin{bmatrix} \cos RA & 0 & \sin RA \\ 1 & 1 & 1 \\ -\sin RA & 0 & \cos RA \end{bmatrix}$$

$$Rz(mq1) = \begin{bmatrix} \cos(mq1) & -\sin(mq1) & 0 \\ \sin(mq1) & \cos(mq1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(RA) * \cos(mq1) & \cos(RA) * -\sin(mq1) & \sin(RA) \\ \sin(mq1) & \cos(mq1) & 0 \\ -\sin(RA) * \cos(mq1) & -\sin(RA) * -\sin(mq1) & \cos(RA) \end{bmatrix} = {}^0_3R$$

$$\begin{bmatrix} \cos(RA) * \cos(mq1) & \cos(RA) * -\sin(mq1) & \sin(RA) \\ \sin(mq1) & \cos(mq1) & 0 \\ -\sin(RA) * \cos(mq1) & -\sin(RA) * -\sin(mq1) & \cos(RA) \end{bmatrix}^T = \text{Inverse}R$$

$$\begin{bmatrix} \cos(Yrot) * \cos(Zrot) & \cos(Yrot) * -\sin(Zrot) & \sin(Yrot) \\ \sin(Zrot) & \cos(Zrot) & 0 \\ -\sin(Yrot) * \cos(Zrot) & -\sin(Yrot) * -\sin(Zrot) & \cos(Yrot) \end{bmatrix} = Ry(Yrot) Rz(Zrot)$$

$${}^0_5R = Ry(Yrot) Rz(Zrot) {}^0_3R$$

$$\begin{aligned} RA &= -(q3+q2) - \pi \\ mq1 &= -q1 + \pi \\ Zrot &= q4 \\ Yrot &= -q5 \end{aligned}$$

$$\begin{aligned} EO_{F5} &= {}^0_5R * EO \\ q6 &= -\tan^{-1}(EO_{F521}/EO_{F511}) \end{aligned}$$

To find the orientation of $[d_{x6} \ d_{y6} \ d_{z6}]^T$ (last link) relative to the 3rd frame.

$$[d_{x6F3} \ d_{y6F3} \ d_{z6F3}]^T = {}^0_3R * [d_{x6} \ d_{y6} \ d_{z6}]^T$$

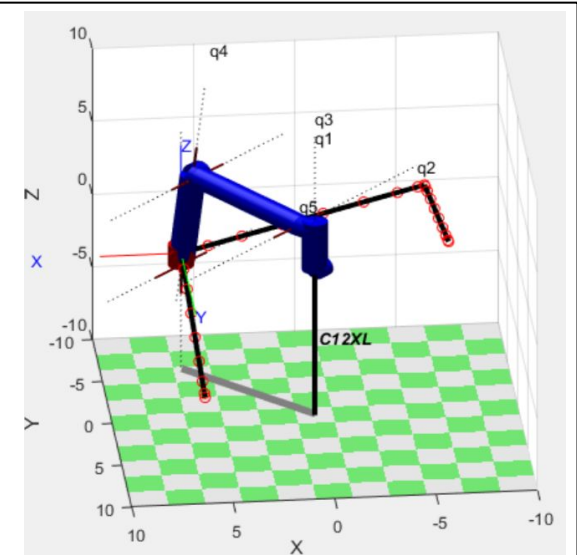
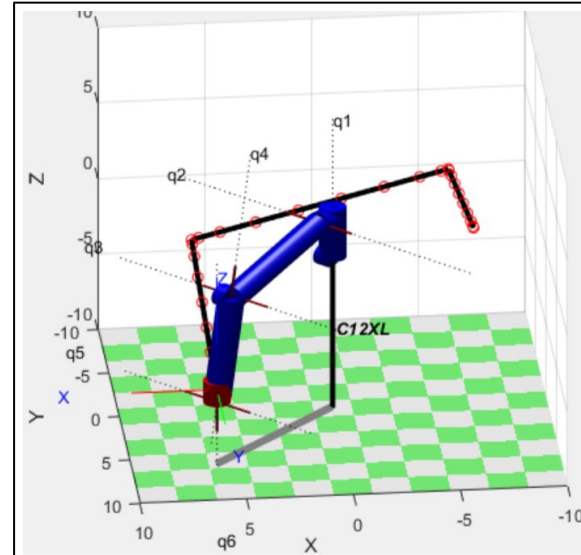
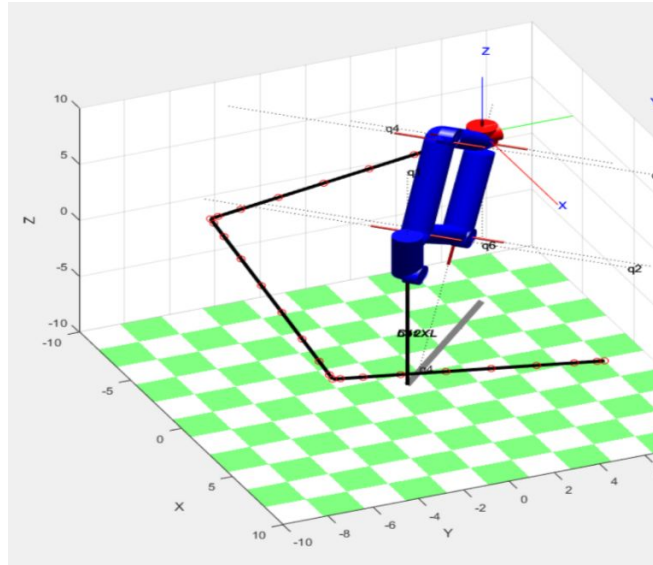
$$d_{xyF3} = \sqrt{d_{x6F3}^2 + d_{y6F3}^2}$$

Because the z axis of the final frame is aligned with vector $[d_{x6} \ d_{y6} \ d_{z6}]^T$,

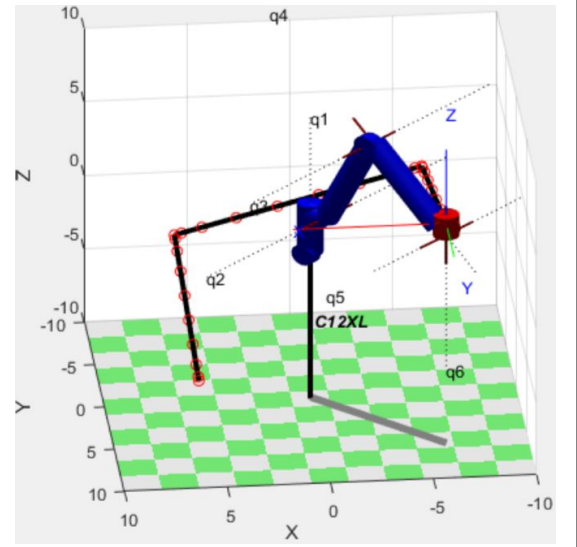
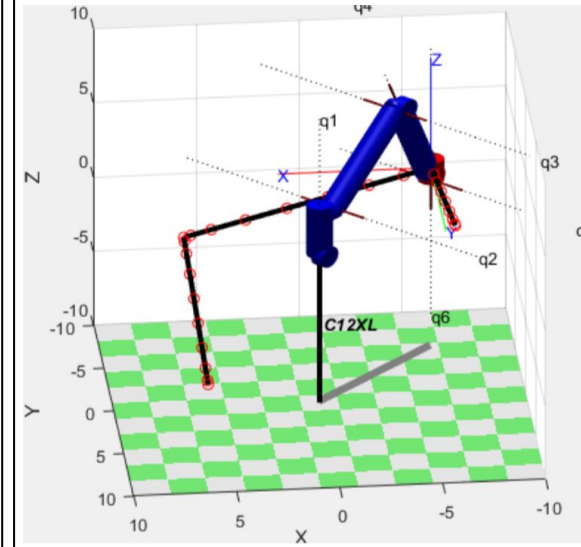
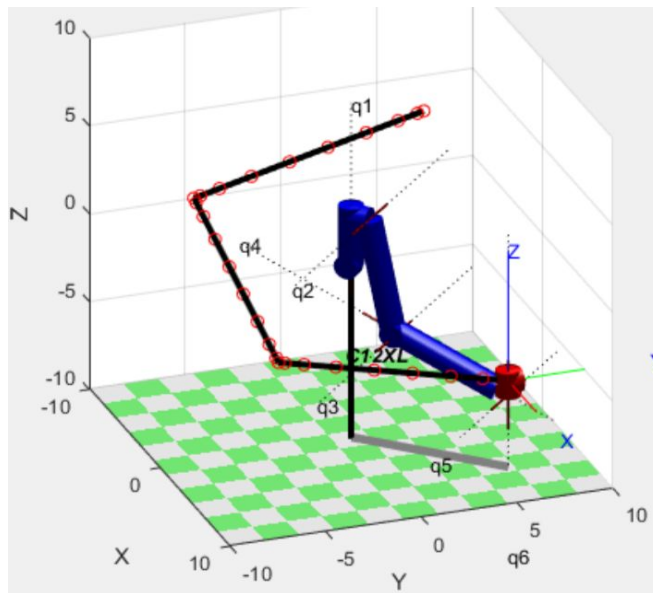
$$q5 = \tan^{-1}(d_{xyF3}/d_{z6F3})$$

$$q4 = -\tan^{-1}(d_{y6}/d_{x6})$$

Two sets of solutions of joint variables for the same positioning



Same end effector position and orientation as the images to the left but using a different set of solutions. In this case the links are likely bumping into one another or the actual robot can't rotate this far down which is why many sets of solutions are necessary for positioning and orienting the end effector



Given the desired position and orientation of the end effector, (in this case the desired orientation is a 3x3 identity matrix) the inverse kinematics can find the joint variables that allow the arm to position and orient the end effector to what is desired.

Conclusion

The Inverse kinematic of the C12XL Robotic arm is found by using a combination of trigonometry and rotation matrices. There are also two sets of solutions found for the robotic arm throughout this research. There is likely still many more sets of solutions for the C12XL robotic arm.

Citations

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