# AC 2010-1017: INVESTIGATING ENGINEERING STUDENTS' MATHEMATICAL MODELING ABILITIES IN CAPSTONE DESIGN

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# Investigating engineering students' mathematical modeling abilities in capstone design

## Abstract

Engineering capstone design is a culminating experience that is intended to provide an opportunity for students to apply their previous engineering knowledge to develop solutions to open-ended problems. Capstone design problems are often analytically complex, and their solutions integrate several disciplinary fundamentals, as well as more general design process knowledge. Often, the expectation is that a thorough or rigorous solution to a capstone level problem would include some type of computational or mathematical analysis appropriate to that discipline. However, engineering students often struggle in recognizing when and how disciplinary knowledge (e.g. mathematical analysis inherent in many engineering fundamentals) applies to their particular design solutions.

This paper describes the strategy for and initial results of a study exploring how students use mathematical reasoning when developing design solutions. Specifically, we want to understand where students struggle in the development and implementation of a mathematical model. We conducted our study in a biomedical capstone (senior) design course. We presented students with a scenario based on a design problem in using phototherapy to treat jaundice, and asked specific questions relating to mathematical modeling in the solution to this problem.

We developed the scenario and corresponding assignments based on previous work that identified six steps for what mathematical modeling should include. We staged the activities over a four-week period such that students addressed two of these steps at each time interval, or assignment stage. This report analyzes results from the first two activities, which focused on identifying the real-world phenomenon and simplifying or idealizing it. We found that in an open-ended statement of the problem, no students proposed using a mathematical model to assist in designing the device. When we specifically asked for a mathematical model in a second activity, only five students of thrity-eight proposed a purely mathematical model, and another two proposed experiments that would lead to predictive equations. When asked to identify parameters that would be important to model, 37% of students chose ones that were part of the design requirements, and therefore fixed, and only 35% correctly chose parameters that could be adjusted to meet the design requirements. These results show a gap in using modeling skills in design, and suggest that educational interventions are needed to improve these capabilities.

## Introduction

Mathematical modeling is essential to engineering practice and a valuable tool for engineering design. Engineers who generate mathematical models or use mathematical and conceptual knowledge to reason, interpret, and communicate solutions have some level of "quantitative literacy." Dossey[1] defines quantitative literacy as "the ability to interpret and apply these aspects of mathematics to fruitfully understand, predict, and control relevant factors in a variety of contexts." By "these aspects," Dossey means "data representation and interpretation, number and operation sense, measurements, variables and relations, geometric shapes and spatial visualization, and chance." The education of future engineers must prepare them to approach situations with quantitative literacy, at least with the tools in Dossey's list, and ideally with higher level tools including the ability to frame problems in terms of appropriate mathematical models and finding solutions to those models. Modeling can be used in the design process in many ways: to avoid expensive and time-consuming tests of physical prototypes, to guide the range of physical models that should be tested, to rule out seemingly reasonable designs that are destined to fail, to avoid overdesign of components, to explore the likely range of performance of a device, and to estimate failure rates of a device composed of many elements. Thus, there are many reasons why engineering students should possess the capabilities to do modeling.

To further understand how to prepare students to have "quantitative literacy," we are investigating students' abilities at creating mathematical models in the context of design. Specifically, we seek to explore where students struggle in moving from routine use of mathematically-based disciplinary knowledge to a more flexible use of developing unique yet appropriate mathematical models in the process of design. We aim to use results from this research to inform how instruction may be improved, both in design courses as well as in traditional analysis-focused courses.

This study explores where students may struggle in the building of a mathematical model in the context of an innovation situation such as design, with a practical goal of using the results to gain insight into how instruction might be used to improve students' "quantitative literacy." Since our focus is on mathematical modeling in the context of engineering design, we used Gainsburg's[2] framework to structure our activities and data collection. (Other authors[3-5] have presented additional, similar frameworks for the creation of mathematical models.) Specifically, Gainsburg identified six steps for what mathematical modeling should include:

1. Identify the real-world phenomenon
2. Simplify or idealize the phenomenon
3. Express the idealized phenomenon mathematically (i.e., "mathematize")
4. Perform the mathematical manipulations (i.e., "solve" the model)
5. Interpret the mathematical solution in real-world terms
6. Test the interpretation against reality

She studied the use of mathematical models in the workplace, answering the question "What does adult mathematical modeling look like?" Her study involved observing structural engineers at different levels of experience at an engineering firm solving a problem on supports and compression forces, and drew insight mainly from one extended and detailed observation of the interaction between a junior and a senior engineer. This paper follows Gainsburg's six steps for the creation and use of mathematical models, and focuses on the way that these are employed in an educational setting.

**Research Method**

This project investigates students' abilities at generating models that they can use in the development of their design solutions. We seek to understand how students approach the creation, solution, and interpretation of a mathematical model, especially as it applies to creating a design. Our project is guided by the following research questions:

- What are students' abilities at creating mathematical models?
- What can we change in instruction to improve students' modeling abilities?

This paper focuses primarily on the first question. In order to investigate the first question we presented students with a design scenario and asked them to respond to questions designed to evaluate their abilities in Gainsburg's steps. There were four stages to the activity, each one focused on one or more of the steps, during the course of an academic quarter. We have used both quantitative and qualitative approaches to analyze student responses to document students' abilities (and inabilities) in each stage. As implied by Gainsburg's list, we consider a model to be more than a simple calculation. It needs to be a conception of a problem in a mathematical format that can yield different outputs depending on the values chosen for the inputs, initial conditions, and/or model parameters.

The activity used a scenario based on an actual project to design a phototherapy device to treat jaundice that is compatible with Kangaroo Mother Care (KMC) for premature infants in the developing world. Jaundice is easily treatable with phototherapy, a process in which light is shone on the infant's skin. Phototherapy lights are used in conjunction with incubators in the developed world, but incubators are too expensive for use in the developing world. However, a phototherapy device could be designed that is compatible with KMC. KMC utilizes a blanket-like device that wraps the baby and holds it close to the mother's chest, so that the skin-to-skin contact maintains the temperature of the baby, replacing the need for an incubator.

The activities were presented as questions about the way in which phototherapy could be built into a KMC device. Seniors in a required capstone biomedical engineering design course (n=38) were asked to assist the "Phototherapy Design Team" that was assigned the problem of jaundice in premature infants. They were given enough information in the scenario that they would not need to do additional literature searches. Students were also given the three main design requirements for the device as established by the American Association of Pediatrics (AAP)[6]: 1) blue light with a wavelength between 430 and 490nm is the optimum light for phototherapy, 2) the average surface area of the baby that should be illuminated is $0.24m^2$, or approximately 75% of the baby, and 3) the spectral irradiance, or strength of the light source, should be above 30 $\mu W/cm^2/nm$. Students were told that the light could be provided with blue LEDs that could be attached to a flexible substrate that could be put next to the baby inside the KMC, and the device would be run by battery power.

The scenario then instructed the students that the next step was to model the device. We chose a problem that was tractable but required many decisions and could be approached in different ways in order to approximate a realistic design scenario. However, because it was open-ended, we felt that analysis of their capabilities would be more feasible if we evaluated students on different aspects of modeling in different activities. For this reason, questions designed to address the six stages of the modeling process presented in Gainsburg's paper were presented in four parts, and each part was completed by the students in a different week.

Any solution to this problem requires that the designer decide how many LEDs would be required, how they would be distributed, and whether they would be touching the baby or positioned at some distance away, and this in turn requires some understanding of the light distribution provided by one LED. These are the elements that would lend themselves to mathematical modeling.

Students had all taken the physics course covering light, waves and optics, but few if any had used this type of information in an engineering context. They all had freshman design courses in which they had learned the design process and worked in teams on various types of designs. Few students had taken any additional design courses before their capstone course. In the capstone, they were learning further aspects of the design process, and were organized into teams to work on different problems posed by actual clients. None of the teams were working on KMC phototherapy. Students were not informed about our conception of the stages of modeling. They performed the activities for our study as regular activities for the course, for which they received credit. They were informed that their work on these activities would be helpful to their own design project and that they would also be analyzed as part of a research project.

Implementation Strategy

<u>Iteration 1</u>**:** This phase was designed to assess student ideas of what constitutes modeling, and covered Gainsburg's first two steps, 1) Identify the real-world phenomenon, and 2) Simplify or idealize the phenomenon. We planned this phase to be very open-ended. Students were asked to "tell the Phototherapy Design Team what you think should be modeled, how you would approach the modeling, and how you expect the model to eventually be helpful in the design." Ideally, we hoped that students would provide their conceptions of what modeling is, and not just list the steps in the overall design process. Note that we never said "mathematical model" or anything comparable.

This phase was completed in class in order to collect students' individual responses to this question. Students had 45 min to complete the task. Student responses were collected and were analyzed in terms of the first two Gainsburg steps.

<u>Iteration 2</u>**:** Just prior to this iteration, the instructor discussed responses to Iteration 1 (creating physical models, designing experiments, and mathematical models) and then indicated that what the Phototherapy Team needed was a mathematical model that will help them design the device, given certain specifications or design requirements.

This phase addressed steps one and two of the Gainsburg paper again, but with a guided approach. Specifically, in addition to highlighting "mathematical," students were asked to sketch the system they planned to model, list relevant parameters and variables, give reasons for why those parameters are important to the creation of the model, note any relationships between parameters or variables, note any judgments that would have to be made about the model components, describe possible geometries they may consider for the device, and propose possible mathematical approaches to the problem.

This strategy allowed us to return everyone to a common starting point, regardless of their performance level on Iteration 1, and it allowed us to probe deeper into their capabilities on these two steps. Again, this phase was completed in class so that students would have to answer the questions independently. Student responses were again interpreted in terms of the first two

Gainsburg steps. Student responses were compared to each other and to their own responses from Iteration 1.

Iteration 3**:** Just before this iteration, the instructor discussed problem areas from Iteration 2 and provided an example of a good response (sketches, parameters, simplifications, and approach) but did not discuss the geometry or possible layout of the LEDs, or their light distribution. Thus, again the students were brought to a common point of departure for Iteration 3. The activity associated with this phase was designed to address steps three and four of Gainsburg's list: 3) Express the idealized phenomenon mathematically (i.e., "mathematize"), and 4) Perform mathematical manipulations.

Students were asked to find the equations and list the assumptions that would be useful to create the model. Then students were asked to manipulate the equations. Because students would need additional time outside of class to look up or develop the mathematical equations, this part of Iteration 3 was given as homework.

In the following class, students were asked to submit comments on some assumptions that could be made in their models. Students were asked about the error associated with the assumptions and whether they thought the model's predictions would be accurate for a real situation. To begin to address Gainsburg's step 4, students were asked to solve for one parameter of the model.

For the purpose of simplifying the analysis of students' abilities at "mathematizing" the phenomena, we gave the students a few key simplifications in the hopes that their equations would be similar (i.e. a baby wrapped in a thin blanket could be approximated as two concentric cylinders, in which the diameter of the inner cylinder approached the diameter of the outer cylinder leading to essentially parallel plates when looking at a small patch of the circumference. This indicated that spherical coordinates were not necessary).

Iteration 4**:** As part of the last iteration, students were provided with a particular model, equations, model outputs, and real experimental data generated by a design team that had worked on the phototherapy device in a prior year. This allowed us to start the students with common information again, and attempt to isolate their capabilities in Gainsburg's steps five and six: 5) Interpret the mathematical solution in real-world terms, and 6) Test the interpretation against reality.

Students were asked to explain the model outputs in their own words, to decide whether the experimental data provided verified the model, and to provide a recommendation for the design of the Phototherapy device based on the model outputs. This phase was also completed as a homework assignment.

Expert responses, to which student work can be compared, will be obtained from several post-doctoral and grad students in BME with experience in optical design and analysis.

## Data Analysis

The first two iterations were both scored with the same rubric, in order to facilitate comparison between responses to the problem posed in an open-ended way (Iteration 1) and a guided way (Iteration 2). Figure 1 shows the scoring rubric, which was developed after reading a few responses. Student responses were read thoroughly and their modeling approach was captured in the rubric. Parameters and variables are categorized and recorded in the top section. The middle section is intended to record the type of model proposed. First, we tried to determine if a mathematical approach was considered at all; this led to the creation of two “yes” categories. The first was the pure mathematical model, and the second was a proposal for a physical model of the device in which variables could be altered and tested, and the output measured, recorded, and used to create a mathematical relationship. These relationships were most often in the form of plots or simple equations that could then be used as the “math model” for the purposes of aiding in the design of their device. Credit for creating a mathematical model was given even if the model had flaws. The bottom section of this rubric was intended to record students’ attempts at simplifying and mathematizing the problem.

We found that the rubric worked well in allowing us to capture almost all of what students discussed in their answers. This allowed us to categorize responses without attempting to reduce their answers to specific scores. Two of the authors independently assessed papers from six of the 38 subjects, and had over 85% agreement on how to categorize responses for these subjects. Where they differed, it was generally in how to characterize student responses that were off-target. Following this preliminary analysis, the rest of the data were analyzed by one author.

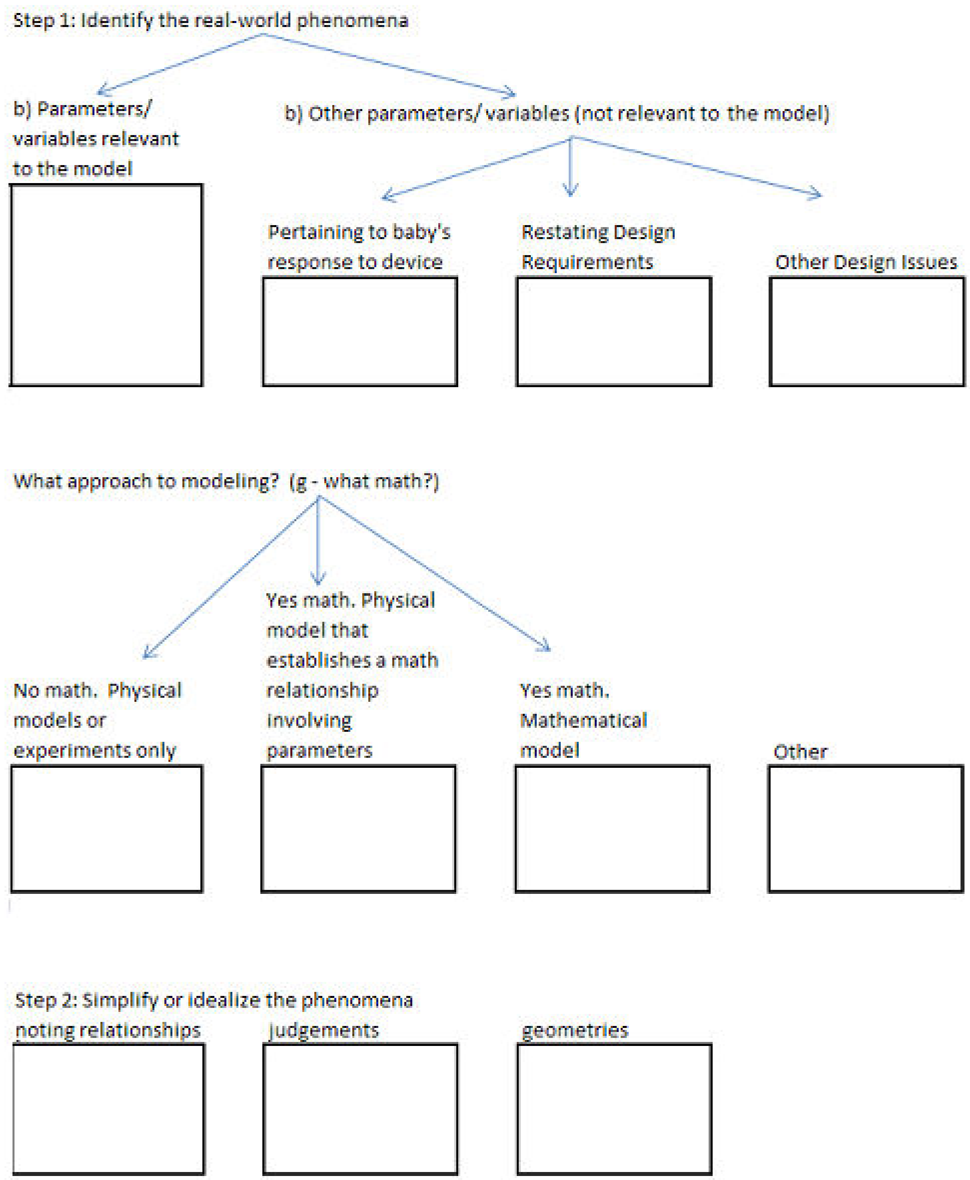


Figure 1: Scoring rubric for recording student responses to Iterations 1 and 2

## Results

Due to the large amount of very revealing data collected in the first two iterations of this project, this results section will be limited to an initial analysis of the first two iterations, which provide insight into students' conceptions of modeling related to design.

Following the Gainsburg steps, we first analyzed whether students were able to "Identify the real-world phenomena." Reading through student responses to the open-ended iteration (Iteration 1) we noted the student responses for what parameters they would be modeling. We followed the same procedure for student responses from Iteration 2 (with the guiding questions) that was completed by students one week later. Tallying up the student responses we identified

37 items that students felt should be modeled. Of those 37 "parameters" we found several things worthy of note.

Figure 2 shows the frequency with which students identified different parameters that should be modeled. Students frequently listed the Design Requirements of the device given in the scenario statement as parameters to be varied in their models (three left bars in Figure 2 for each iteration). In the first iteration, two of the three design requirements (the best wavelength and the baby's surface area covered by the light) were listed by over 26% of the students as parameters that could be varied in their model. We regard these as inappropriate choices of parameters to be modeled, because the scenario statement gave a set value to be achieved.

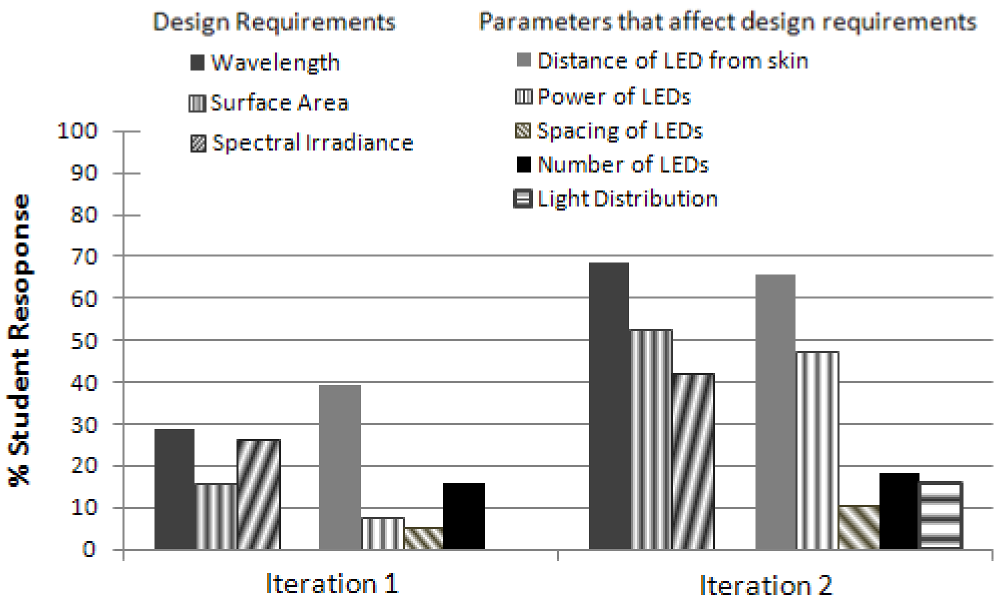


Figure 2: Frequency of students stating the parameters that should be modeled in response to Iterations 1 and 2 of the Phototherapy Modeling Scenario

However, parameters that affect the ability to achieve those requirements were appropriate parameters to model. On the right hand side of the table legend are the parameters that affect the design requirements and are the ones that are adjustable, both in the model and in the physical design. With one exception, students listed these parameters less frequently than they listed the design requirements. Of these five main parameters that students listed, they were more likely to state that the distance of the phototherapy treatment light from the skin was a key parameter to be modeled. What we felt were equally important but not as frequently listed, were the spacing of the lights in the KMC blanket, the power of the bulbs, and the light distribution from the source, projecting onto the skin. Less than 20% of students listed these three parameters in Iteration 1.

During Iteration 2, in which the problem statement was more specific about mathematical modeling, students' ability to identify what they believed to be relevant parameters nearly doubled with respect to Iteration 1. While over 40% of students restated all three design requirements, at least 45% of students were able to identify two of the key parameters that were appropriate to model because they affected the design requirements. These two parameters were the distance of the phototherapy treatment light from the skin and the power at which the bulbs are operated.

Indeed, Figure 3 shows that in Iteration 1, the design requirements averaged 27% of a student's list of parameters, while the key parameters that affect the design requirements averaged 26% of the list. In Iteration 2, these averages increased to 37% and 35% for the design requirements and key parameters, respectively. Therefore, with more prompting about modeling, in Iteration 2, students did identify the correct parameters more frequently, but they also identified the design requirements themselves more frequently. We note that the identification of design requirements may be specific to this study, since we provided the design requirements in the problem statement. We did this to prevent students from spending too much time looking through the literature for the design requirements, allowing them to focus solely on the creation of the model. It is unclear whether students would have stated the design requirements if the problem statement had not included them.

It is also interesting to note that students listed several design issues that could be important to the overall design, but were not relevant to the specific question we asked about a mathematical model of phototherapy light. The "other design issues" most often given were the duration of exposure to phototherapy light, materials to be used in the device, LED and battery lifetime, the temperature of LEDs, and bilirubin concentrations in the infant (or the amount of jaundice). We did see that from Iteration 1 to Iteration 2, students increased their lists of key parameters, while the list of other design issues decreased.

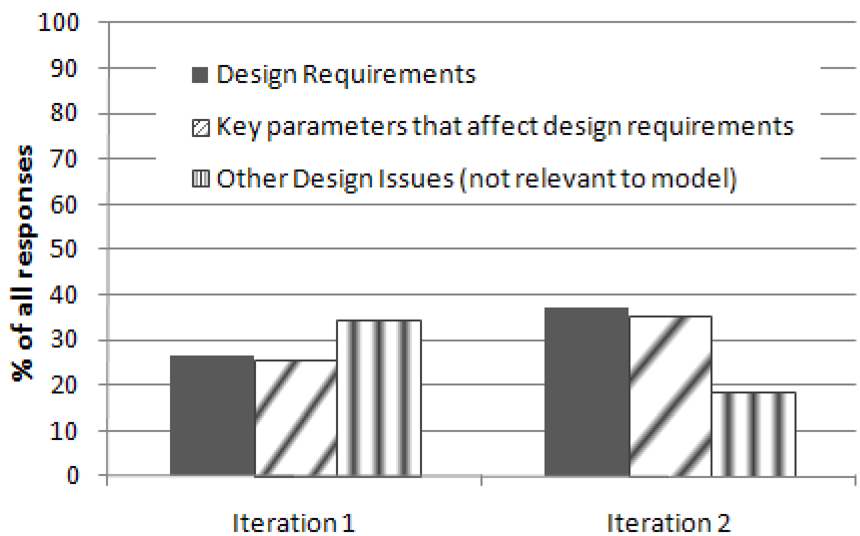


Figure 3: Frequency that the design requirements, key parameters, and other design issues appeared on the average student's list of parameters to be modeled.

For Gainsburg's second step, "Simplify or idealize the phenomenon" we were interested in how students articulated the details necessary for creating a model. Reading through student responses from Iteration 1 (open-ended), we first evaluated whether or not they were using a mathematical approach. Then we placed their responses into one or more of 4 categories:
1) Mathematical model mentioned (i.e. a mathematical model was explicitly mentioned), 2) Use of a physical model or experiments that could establish a mathematical relationship that could be

used in a predictive way, 3) No mathematical model (i.e. a physical model or experiments only), and 4) Other. The category of "Other" contained student responses in which no model at all was proposed, researching the literature was proposed, or there was no plan for using a proposed physical model or device. Figure 4 illustrates the percentage of students falling into those categories.

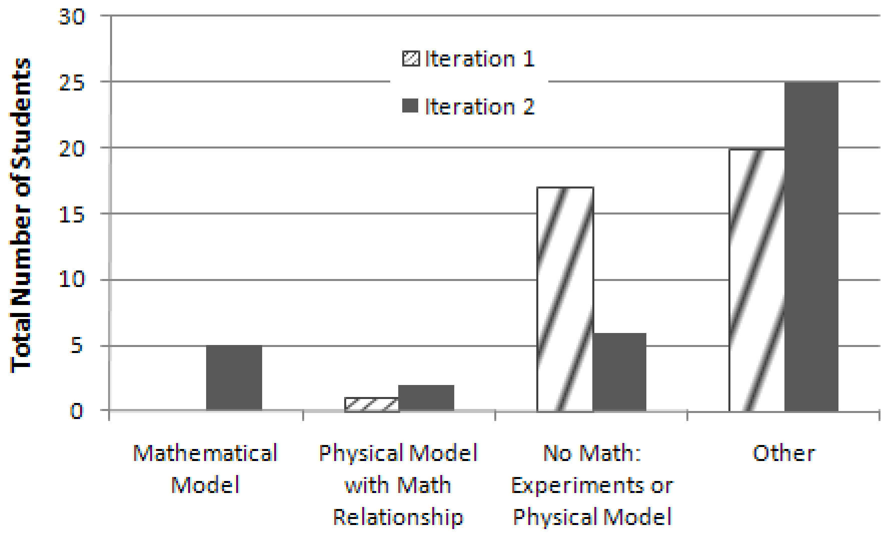


Figure 4: Number of students' proposed models that fell into the four categories on Iterations 1 and 2

In the first Iteration no students proposed an entirely mathematical modeling approach. One of the thirty-eight students proposed a physical model that they could use to develop some mathematical relationship. For example, this student proposed a model of light where the light would be shining through the blanket material which would be in direct contact with the skin:

> One would need to test the irradiance and amount of light emitted at all areas of the surface of the blanket from one LED bulb buried in the blanket. Once the irradiance as a function of distance from the buried bulb is determined, this could be extrapolated to determine the required spacing of the bulbs to allow at least the minimum irradiance to be present at all areas on the surface of the blanket.

This student proposes creating a physical device to make some measurements that would lead to a mathematical relationship between distance (a key parameter) and irradiance (a design requirement). A further calculation was proposed by the student to estimate the spacing of the bulbs (another key parameter) while maintaining the desired irradiance.

However, these types of insights were relatively rare. Seventeen of the thirty-eight students proposed models that ended up in the third category, no math – physical model or experiments only. And twenty of the responses were categorized as "Other." The most common response categorized as "Other" was solving for power using an incorrect interpretation of the units of spectral irradiance (power = spectral irradiance × surface area × wavelength). Thus, even though the question specifically asked the students to help the phototherapy team create a model, and used the word "model" or "modeling" three times in that question, only one student gave a response that involved either mathematical or physical modeling.

We analyzed the student responses from Iteration 2 (guided questions) in the same way, placing student responses into the same four categories, and these are also shown in Figure 4. Five of thirty-eight students proposed a mathematical model and two proposed a physical model to generate a mathematical relationship. That is, adding the words "mathematical modeling" to the question did not prompt many students to provide a description of a mathematical model. A particularly good response was:

> [Student's sketch shows overlapping circles (of radius, r) around three LEDs, arranged a certain distance, y, apart.] r = radius of light emitted per bulb that contains 10-15$\mu$W/cm$^2$/nm of light at the surface of the blanket (assuming all light on surface of blanket makes it to the baby when in direct contact with the baby.) This needs to be determined so that the spacing of the LED lights is such that enough light reaches the skin of the baby. $r = y \cos \theta$. A more accurate model would calculate the intensity of light from each bulb at variable distances so that light from the parts of the blanket not in contact with the baby can be taken into account. Without using that the radius of 10-15$\mu$W/cm$^2$/nm was used because the model drawn shows that light overlapping from 2-3 bulbs to sum to be greater than or equal to 30$\mu$W/cm$^2$/nm.

In iteration 2, the number of responses falling into the "Other" category increased to twenty-five. This may have been due to the wording of the question, "propose possible mathematical approaches to the problem" in Iteration 2. Many students perhaps narrowly interpreted this to mean performing calculations to determine a mathematical value to this question. The calculation they most commonly performed was a simple power calculation, multiplying the required irradiance of µW/cm$^2$-nm by a given area of coverage and particular wavelength of light, (This is a misunderstanding of the units of irradiance, but that was understandable.)

Lastly, we note that in all cases in these two iterations we did not grade the models for correctness, only sorted them into the categories regardless of whether they were using the correct or incorrect parameters in their model.

We hypothesized that drawing a sketch of the situation to be modeled would help the students visualize the problem and help them set up a model, so we specifically asked them to create sketches in Iteration 2. Students drew a range of sketches, from ones showing light rays reaching a surface, or arrangements of LEDs in space, which would assist in eventually modeling the situation, to ones that showed a baby in an enclosure, which would not be very helpful to a modeling effort. Some of the sketches of light, which could be very useful in getting students to realize that they should consider each LED as a point source, and remind them of the inverse square law relating distance to irradiance, showed instead misconceptions about how light rays would travel through space. This relates back to difficulty with step 1 of Gainsburg's list, the critical ability to first identify the physical situation being considered.

We performed a simple analysis of these sketches, sorting them by the components they contained. There were four main categories into which the sketches were sorted: sketch of a general phototherapy system interacting with a baby, a sketch of a general phototherapy system device incorporating LEDs, a relevant sketch of LED spacing, and a relevant sketch of light distribution. These "relevant" sketches were considered useful, or relevant, to the creation of a mathematical model, and were accompanied with labels (often they noted the presence of key parameters relevant to the model.)

We noted that in Iteration 1 only eight of 38 students submitted any sketches with their response. Those eight students provided ten sketches; four were general systems interacting with a baby and six were general systems incorporating LEDs. No student submitted a simplified sketch in Iteration 1. In Iteration 2, students were explicitly asked to provide a sketch. The distribution of sketch types can be seen in Figure 5.

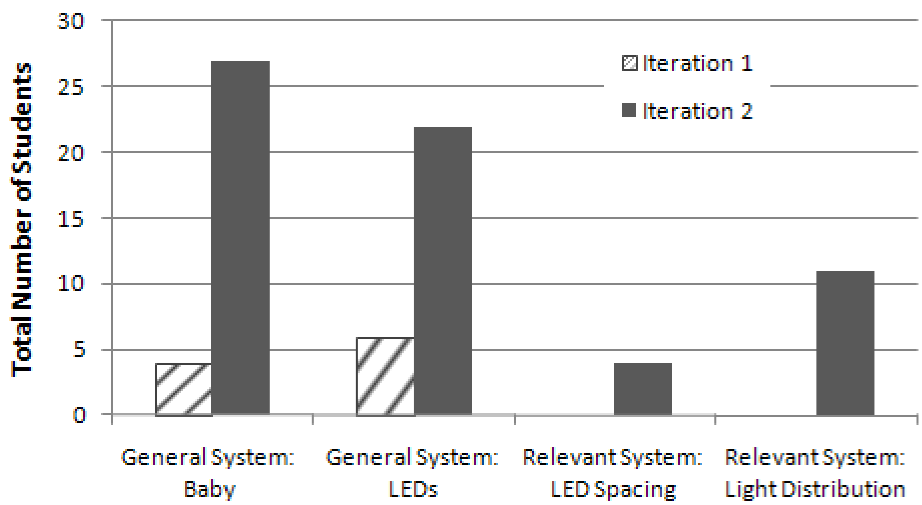


Figure 5: Sketches provided by students in Iterations 1 and 2, categorized by the components within the sketch.

All students in Iteration 2 provided a sketch, and many students drew several sketches. In fact, every student that submitted a sketch that fell into one of the "Relevant" categories had additional sketches in the "General" categories as well. To determine whether the category of sketch impacted the type of model the student created, we matched the proposed models with the sketches provided by students. Figure 6 shows that students with only a general system sketch did not create a mathematical model or a physical model with a mathematical relationship. Only students who generated simplified sketches developed appropriate mathematical relationships or models.

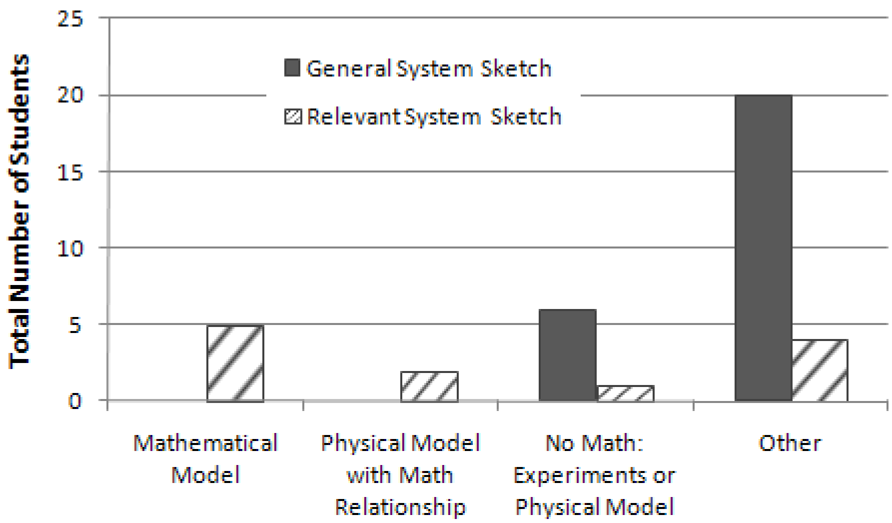


Figure 6: Matching students' sketching to development of mathematical models. Total number of students developing mathematical relationships for two categories of sketch: General system and Relevant system.

**Discussion**

Our project was motivated by several factors. Specifically, there is general recognition of the importance of computational expertise in engineering. In the research described here, we were specifically interested in how students approach mathematical modeling in the context of design. In addition, our work is motivated by our previous experience teaching design, and noticing the persistent and common struggles students have in recognizing when engineering knowledge applies in the process of design. Moreover, students find it particularly challenging to generate mathematical models or use mathematical and conceptual knowledge to reason, interpret, and communicate solutions, in other words demonstrate "quantitative literacy" as described by Dossey. Many would argue that this kind of quantitative literacy is a core competency of engineers. Therefore, our work seeks to understand how and why students struggle with developing this type of quantitative fluency.

This paper reports an initial analysis of students' conceptions and abilities in the first two steps of the mathematical modeling process, as identified by Gainsburg. We found that when students were asked to identify the real-world phenomena to be modeled (Gainsburg's first step), they were very likely to repeat the design requirements back as parameters. Figures 2 and 3 illustrated that nearly 30% of the parameters for modeling that students listed were the design requirements that had predetermined values based on medical recommendations. Students listed key parameters relevant to the modeling of phototherapy light only 26% and 35% of the time (Iterations 1 and 2, respectively). This indicates that students may be struggling with how to determine the key parameters, regardless of how much prompting they receive.

When looking at how students were approaching the mathematical model (Gainsburg's second step, "Simplify or idealize the phenomenon") we found that in the first Iteration no students were proposing entirely mathematical modeling approaches. Only one student of thirty-eight proposed a physical model that they could use to develop some mathematical relationship, seventeen proposed physical model or experiments without mathematical relationships, and twenty did not propose any model or experiments.

In the second Iteration we saw some improvement in the incorporation of mathematics. Five students proposed a mathematical model, two proposed a physical model to generate a mathematical relationship, six proposed physical model or experiments without mathematic relationships, and twenty-five were "Other." The increase in "Other" may have been due to the wording of the question "propose possible mathematical approaches to the problem." Had this question been worded differently, more students may have created or suggested a mathematical model rather than performing a simple calculation.

Students who sketched a system that showed some aspect of the light distribution (called relevant sketches in the analysis) were more likely to generate mathematical relationships. Students who failed to develop a mathematical model, failed to break down the problem into the basic elements of one (or more) LED(s) shining on a surface during the early stages of model development. Those who succeeded better in developing the mathematical relationships saw this as important. We do not yet know whether the sketching contributed to their ability to generate a model, but our initial analysis suggests exploring this in more detail.

It is possible that the phototherapy problem was too difficult as a first attempt at modeling. We chose phototherapy because students all had some background in previous coursework related to light, and because it was unlikely to give an advantage to any students as a problem in mechanical or electrical modeling might have. We would have given students credit for a mathematical approach, however, even if it were entirely incorrect, and we were not especially harsh in our analysis of their answers, so the difficulty of the subject should not have substantially influenced their performance on these first two iterations. For example, a student with a flawed conception of light distribution would still have been given credit for creating a mathematical model even though his or her equations were incorrect. In practice, a designer would usually become adept with a restricted range of models[2]. He or she usually does not have to create a model from scratch, but only choose the parameters, or the ranges of parameters to be modeled. It is possible that our students would have done better if we had provided the model and asked them to identify the parameters, but this would not have tested their abilities on the first steps in modeling, and we speculate that abilities in those steps might be among the factors that distinguish the practicing designers.

When students proposed creating physical models or experimental systems with no mathematics involved, they were often thinking of ways that they could justify the design requirements. In the capstone design course we often emphasize that students should not take the client's ideas for granted, or assume that the client's proposed solution is really solving the important problem. Students are instructed to evaluate the situation and work with the client to get to the root of the need. However, this mindset may have misled them in the phototherapy problem. Here, the problem identification had already been done and a set of conditions, in terms of the required wavelength, coverage, and irradiance, exists. These conditions were given to the students in the problem statement. Their job was to be a designer meeting those requirements. Instead, a number of students wanted to perform tests on babies to determine whether light meeting the design requirements would actually cure the jaundice, and this may have distracted them from creating an appropriate design.

The fact that many students did not propose anything that could be called a mathematical model even on the second iteration, after the questions explicitly referred to "mathematical models," indicates that modeling of the type that practicing or professional engineers use is nuanced and complex, and is disconnected from how students approach solving problems. We did see a small increase in the number of students proposing a mathematical approach to their scenario on the second iteration. This may indicate that a guided approach to the use of models in design would be beneficial to novice modelers, although our Iteration 2 still did not provide enough of that type of guidance. It is not too surprising that students would need help with this, since, in general, an engineering curriculum tends to teach the disciplinary fundamentals in an abstract way, with routine and well-posed problems, which are often disconnected from how one would reason through a solution to a non-routine, "real-life" situation. While we have used the scenario here primarily as a research tool, it can be adapted in the future into a teaching tool to aid students in modeling. Additional research would be needed to create the best guiding questions that would be beneficial to students.

Due to the large amount of information gathered from the first two iterations of this study, we decided to limit this report to the analysis of those data. We will present the results of the other two iterations of this project in a subsequent paper. We also have further plans to compare our students' responses to those of experts, and to study students' abilities in mathematical modeling by repeating this process for capstone courses in other disciplines and in the freshman design courses.

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