AC 2008-2227: INVESTIGATING IMPULSE LOADING USING MODEL ROCKETRY

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Investigating Impulse Loading using Model Rocketry

Abstract

A project is presented that uses experimentally determined thrust data for a commercial model rocket engine to investigate impulse loading relations. Certain model rocket engines approach impulse loading; completely burning in a fraction of a second. Using a fixture instrumented with strain gages and a high-speed National Instruments data acquisition system, the students experimentally collect the thrust verses time response of several Estes model rocket engines. The students formulate two flight models for a rocket of known mass loaded with the specific engine being investigated. The first model uses the measured thrust data directly as input to the governing differential equation for the rocket. The students are challenged to formulate an appropriate drag model through a literature search and must justify their choice. A second flight model is formulated using an equivalent impulse, based on the experimental thrust data, as loading. The students calculate flight trajectories based on both nonlinear models using numerical methods and critically compare/contrast the results. The project has been found to engage students and to effectively provide hands-on insight in the value, and limitations, of impulse loading methods.

Introduction & Motivation

A typical mechanical engineering program will address the concept of impulse loading at multiple points within the curriculum. The mathematical definition of impulse loading and the subsequent solution of the impulse response are ordinarily addressed within the differential equation course[1]. The physical nature of impulse and momentum are usually covered in a classical dynamics course[2,3]. Yet, even with this redundant coverage (and potentially more in specific programs) students often possess little intuition regarding impulse loading. How abrupt must the loading be to approach impulse loading? Can the dynamic response to real forcing, which is never a true mathematical impulse, be accurately modeled by an impulse assumption? What are the limitations of impulse response analysis?

The project presented herein challenges students to investigate and critically evaluate the response of a rigid body subject to rapidly time varying forces. While any rapidly changing force could be considered, a readily available source is commercial model rocket engines which represent a pseudo-impulse loading scenario. Figure 1 presents the thrust profile for an Estes B6-4 model rocket engine[4]. Note that the peak thrust is reached approximately 0.2 seconds after ignition, and the entire burn is completed in well under one second. This is in comparison to between 6 and 10 seconds required for most rockets sized for the B6-4 engine to reach apogee.

The students are tasked with answering the following simple question, “Can the dynamics of the model rocket be adequately simulated using an impulse model for the engine thrust?” The project is included within a junior-level course entitled Experimental Mechanics taught at Oklahoma Christian University. The course focuses on experimental investigations directly related to the theory learned in Strength of Materials and Dynamics (both prerequisites to this course).

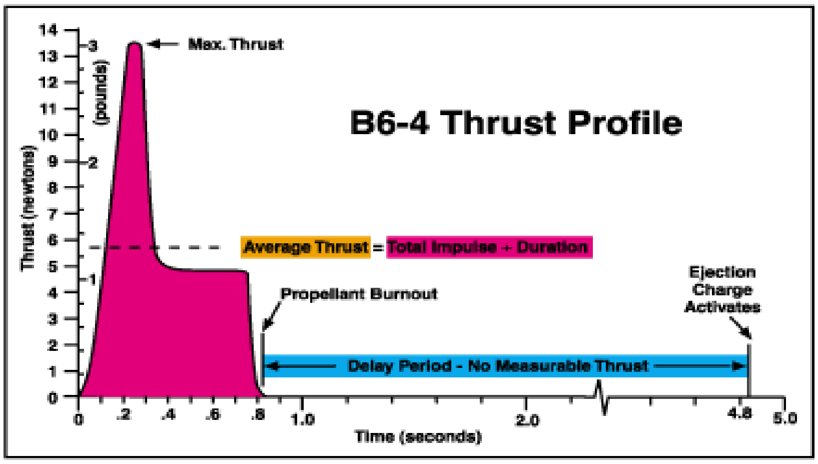

Figure 1 - Estes B6 Engine Thrust Curve[4]

It should be understood that the focus of this project is on comparing the impulse response solution and the solution obtained by direct integration for a non-trivial dynamic system. Model rocketry is simply a tool in reaching this desired goal. Many papers have been published discussing the use of model rocketry as an instructional vehicle within engineering. Boyer et.al. document using model rocketry within a team setting to effectively introduce both the fundamental concepts of experimentation and key topics related to aerospace engineering[5]. The Boyer paper does address the impulse generated by the rocket engine, yet stops short of using the impulse in the subsequent flight predictions. Martin Morris and David Zietlow present using model rocketry as the basis for an upper-level design competition intended to integrate experimental and analytical modeling techniques[6]. There work, which includes experimentally determining the thrust profile of the engine and the drag coefficient of the rocket, does not address impulse response predictions for the rocket dynamics. Suchora and Pierson address using model rocketry as a positive first semester project to introduce students to the nature of engineering while maintaining excitement for the material[7]. All of these papers provided valuable insight into using model rocketry within a classroom setting.

Project Theory

The Quest PayloaderONE model rocket was selected as the flight vehicle for the project[8]. The flight dynamics of the rocket are modeled using the expression below. The students are expected to formulate this expression themselves.

$$m\ddot{y}(t) + \frac{1}{2}\rho A C_d \dot{y}(t)|\dot{y}(t)| = F(t) - mg \qquad where \qquad y(0) = 0 \;\; and \;\; \dot{y}(0) = 0$$

In the above expression, m denotes the mass of the flight vehicle, A denotes the drag area (frontal area), ρ denotes the density of the atmosphere, C_d denotes the drag coefficient, and g is the gravitational constant. F(t) represents the engine thrust profile. It is assumed that the rocket will fly vertically, characterized completely by the vertical displacement variable y(t). This implies a vertical launch angle and the presence of no wind. For simplicity, a constant C_d value is used.

The prior expression allows the actual engine thrust profile to be used during flight predictions. An alternative is to assume an impulse load for the engine. This assumption implies a step change in the velocity of the rocket at launch.

$$m\ddot{y}(t) + \frac{1}{2}\rho A C_d \dot{y}(t)|\dot{y}(t)| = -mg \qquad where \qquad y(0) = 0 \;\; and \;\; \dot{y}(0) = \frac{\tilde{F}}{m}$$

In this second governing relation, $\tilde{F}$ denotes the total impulse from the rocket engine which is equal to the shaded area below the thrust profile in Figure 1. It should be noted that both of the above governing equations are valid only until the time at which the rocket reaches apogee. At apogee the rocket will begin to tumble and the engine ejection charge will deploy the recovery device (i.e. streamer or parachute). Both of these events radically alter the drag model and invalidate the prior equations.

Experimental and Computational Implementation

The experimental component of the project consists of characterizing the thrust profile of the selected rocket engine. It must be noted that a model generated from the published manufacturer's data could be used instead of experimental results. Experience teaching the project over two semesters has shown, however, that the hands-on component of experimentally measuring the thrust profile greatly increases student engagement. Requiring experimental characterization of the engine thrust also enhances the richness of the project by introducing added complexity (the experimental data will not be as clean as the published curves).

To measure the engine thrust, the test stand shown in Figure 2 is used. Alternative methods of measuring the engine thrust have been proposed by various authors and may be substituted if desired[5,6,7,9].

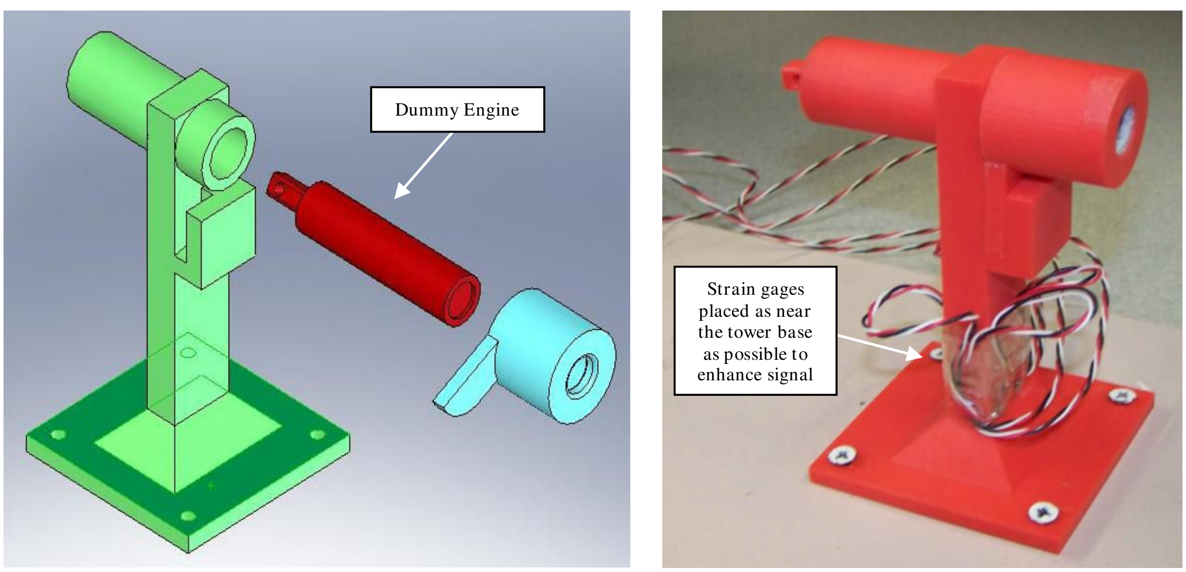

Figure 2 – Rocket engine test stand. Left: 3D solid model exploded view. Right: Actual test stand equipped with four strain gages at the base (full bridge configuration).

The test stand, fabricated of ABS plastic using a Dimension SST 768 Series 3D printer, has four strain gages in a full bridge configuration at the base of the tower[10]. The rocket engine is enclosed within the upper cylindrical portion of the stand. When the engine is ignited, the thrust generates strain at the base of the vertical tower that is sensed using the strain gage bridge. Prior to testing an actual engine, the stand must be calibrated. The dummy engine (shown in Figure 2) is equipped with a stem that allows calibrated laboratory weights to apply a known load to the test stand. Figure 3 provides representative calibration results for the test stand.

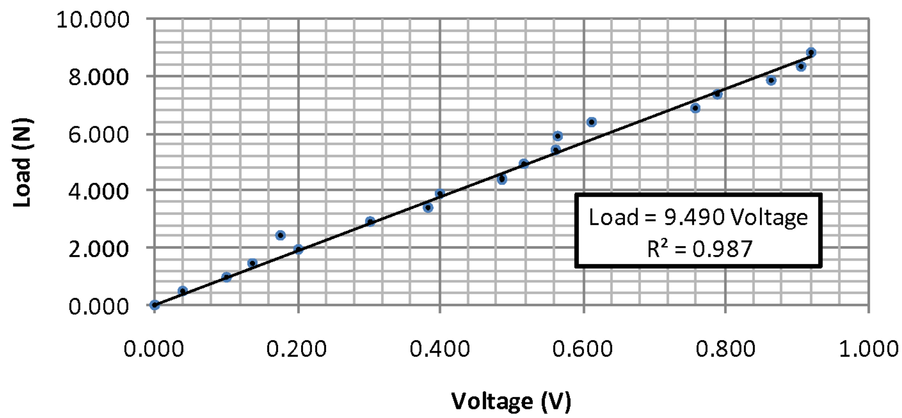

Figure 3 – Representative engine test stand load calibration curve

Figure 4 shows a typical measured engine thrust profile. As the entire engine burn is less than one second, computer data acquisition is a must. The results below have been collected using Labview in combination with a National Instruments NI USB-6009 data acquisition module[11]. The students are required to test multiple engines (note that the engines cost less than $2US each) and to generate an average thrust profile for use in subsequent flight calculations. The average profile is generated by first computing the mean thrust value for small time intervals between 0 seconds (ignition) and 1.5 seconds for each engine. The mean thrust results for the time interval are then averaged across the various engines tested (also shown in Figure 4).

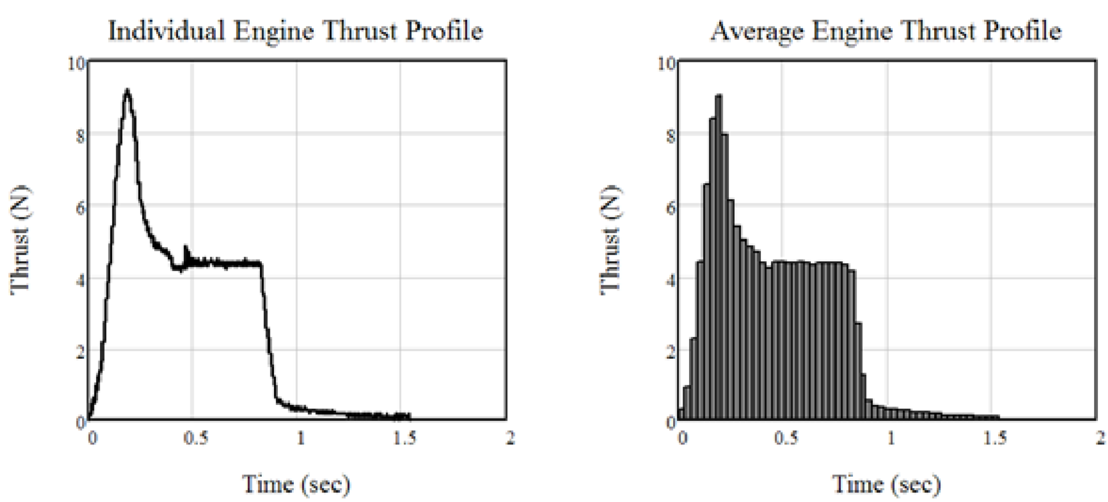

Figure 4 – Individual and averaged measured engine thrust profiles. Only three thrust profiles are averaged in this example.

Flight Calculations

The nonlinear nature of both the fluid drag model and the engine thrust profile mandate a numerical solution of the governing equation. Selection of an appropriate software application or computer language is strongly dependent upon the educational goals of the instructor. The goals of the course at Oklahoma Christian University do not include crafting unique numerical algorithms. Students are therefore encouraged to use the numerical solvers available in either Matlab or Mathcad[12,13]. Both applications provide commands to convert the measured thrust profile into a functional form (e.g. linterp in Mathcad) and to numerically integrate/solve the nonlinear differential equation (e.g. odesolve in Mathcad).

Figure 5 shows results using the experimentally measured thrust profile directly in the governing equation (left graph) and an impulse model for the engine (right graph).

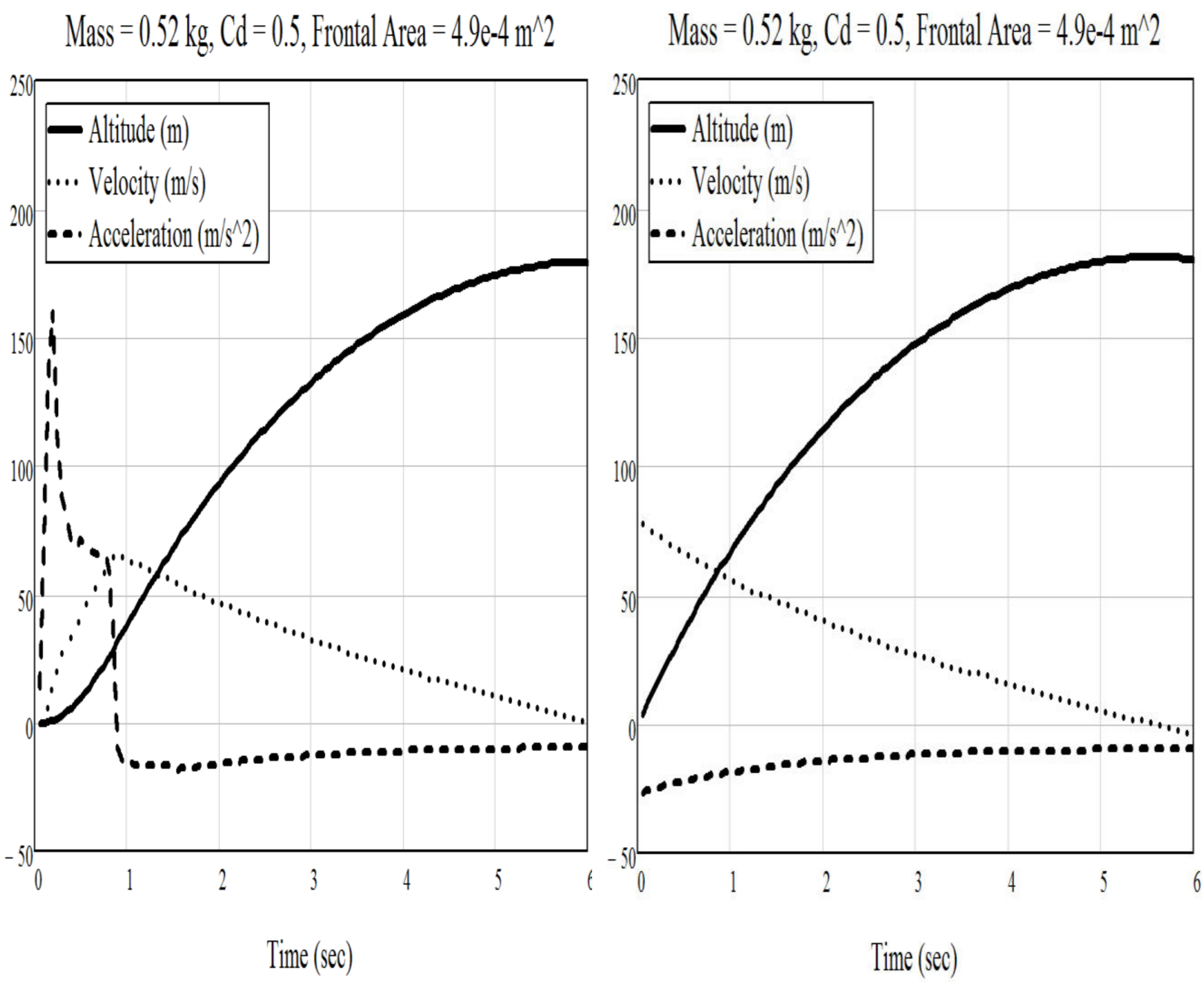

Figure 5 – Simulated flight results. The left plot considers the actual engine thrust profile in the solution. The right plot uses an impulse model for the engine.

Note that the altitude predictions for the two models are very similar. In fact, the simulation using the measured engine thrust directly as loading predicts an apogee altitude of 179 meters while the model with impulse loading predicts 181 meters (a difference of 1.1%). Figure 6 shows the altitude results superimposed.

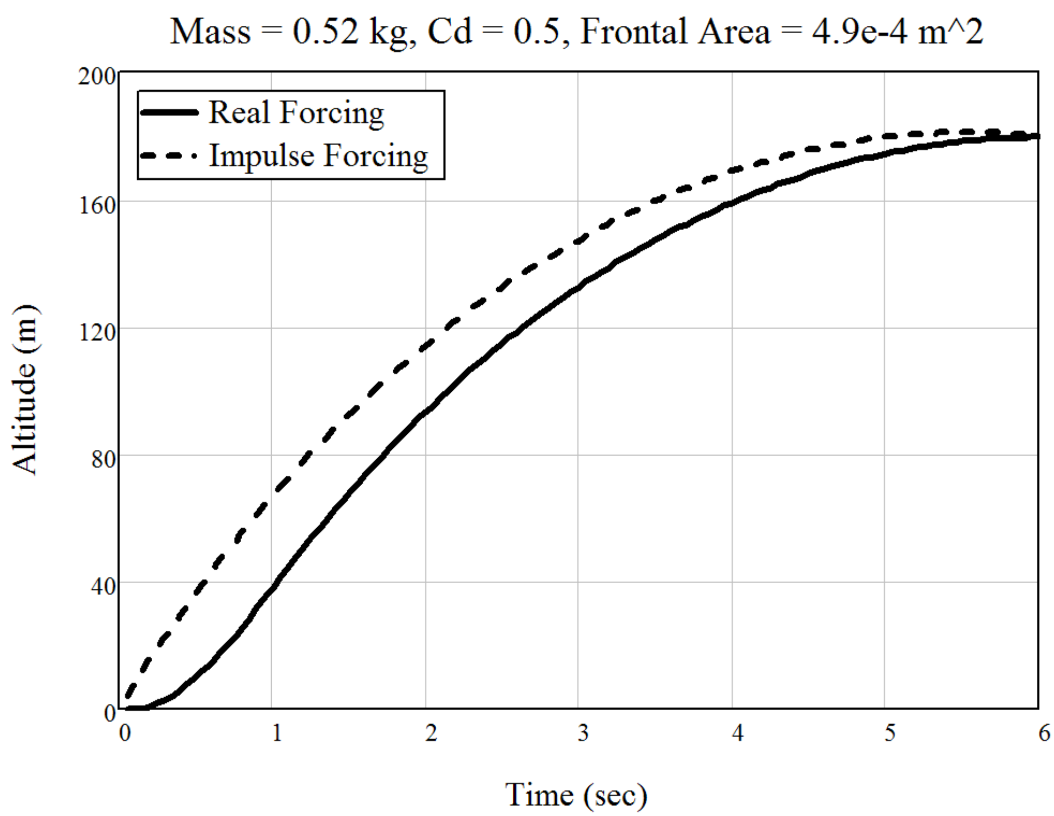

Figure 6 – Flight altitude comparison between using the measured forcing directly in the simulation and assuming impulse loading.

Examination of both Figure 5 and 6 reveals several learning opportunities for the students.

1) Though the rocket thrust profile does not satisfy the requirements of a true mathematical impulse function, the impulse loading assumption adequately models the global displacement of the rocket.

2) While the global displacement is well modeled, the altitude predictions exhibit significant inaccuracies near the time of the impulse application. This should be expected as the assumed step change in velocity does not physically occur.

3) The two models predict slightly different times for key events; the point at which apogee is reached for example. The model with impulse loading predicts apogee will be reached nearly 7% earlier than the model that uses the average engine thrust profile directly. This can again be linked to the instantaneous velocity change generated by the impulse model.

4) The velocity and acceleration results are significantly different between the two cases. This dramatically shows students that one must carefully consider the intended use of analysis results prior to selecting a mathematical model. For instance, if the intent of the study were to estimate the stress state in the rocket's engine mount during flight, the impulse loading assumption would be disastrous as the transient acceleration results are radically changed.

Project Assessment

At the time of this writing, this project has been used only twice. Given relatively small class sizes (10 to 14 students) and this limited number of offerings, quantitative outcome assessment results are currently not available. Qualitative feedback from students, however, has been very positive. The perceived strengths of the project fall into three categories.

1) The direct comparison of the two analysis methods for a single dynamic system appears extremely effective. The results of the two methods clearly expose characteristic differences between the techniques that rarely are demonstrated by homework problems. In fact, students in the class have expressed surprise when the results of the two methods do not match. At some level the students believe that all solution methods are interchangeable. Clearly demonstrating the fallacy of this belief arguably justifies the project.

2) The experimental aspects of the project greatly increase student interest and effort. This finding is not unexpected. Many publications could be referenced that praise the merits of hands-on projects. Two recent publications by Self, Borchert, and Redfield are particularly interesting, however[14,15]. Their published results indicate that while student motivation is enhanced by hands-on projects, mastery of the material is not necessarily increased (as reported by students). The work by Self, Borchert, and Redfield focused on an introductory course in dynamics in which the teaching of theory was a key course objective. By contrast, the presented project is required of more mature students already having completed classical dynamics and focuses on strengthening the students' understanding of prerequisite theory.

3) The integrated nature of the project serves to link the content of several prerequisite courses into a real-world experience. The project draws together theory from solid mechanics, dynamics, mechanical instrumentation, differential equations, and fluid mechanics. Student feedback qualitatively indicates this integration of topics helps disprove the commonly held belief by students that each course is distinct from the rest of the curriculum.

Conclusions

A project is presented that addresses model rocket flight dynamics as a tool to demonstrate both the strengths and the potential weaknesses of adopting impulse loading when modeling real-world systems. Though not strictly required, the use of experimentally measured rocket engine data adds a level of reality to the project and greatly enhances student engagement. The project results clearly identify critical limitations that must be considered when selecting an appropriate model for design work. By challenging the students to carefully evaluate and critique their findings; past offerings have shown this project to provide considerable student insight into impulse loading beyond that gained in theory courses. Sufficient detail is presented to allow implementation within a typical mechanical engineering program.

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