AC 2011-404: JUST-IN-TIME APPROACH TO INTEGRATE A DESIGN PROJECT INTO MECHANICS OF MATERIALS

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Abstract

This paper presents a just-in-time approach developed and used by the authors to integrate a design project into an introductory undergraduate mechanics of materials course. The design project discussed in-depth is a statically determinate hoist frame structure. The hoist is used to lift an object of weight. It is assembled with smooth pins and is symmetric about the two-dimensional plane. Three other design projects discussed briefly include a brace structure, beam hanger, and simple hoist structure. Lecture examples, homework problems, and design project problems are solved with all equations formulated symbolically. One major advantage is that symbolic equations can be solved for any variable value. Furthermore, the design process generally requires solving problems over a range of variable values to obtain a satisfactory design. The design project involves all or almost all topics, covered in an introductory undergraduate mechanics of materials course. The project is divided into seven phases. The background required to complete each phase is based on the material covered up to that point in the course. After a topic is covered in lecture, reinforced through homework and classroom quizzes, the project phase related to the topic area is assigned.

Introduction

Engineering design defined in Criterion 5 by ABET\textsuperscript{1} is “the process of devising a system, component, or process to meet desired needs. It is a decision making process (often iterative), in which the basic sciences, mathematics, and the engineering sciences are applied to convert resources optimally to meet these stated needs.” An educational goal of our course is to introduce design through homework problems and short, simple and well-defined projects. As the student progresses to more advanced courses, i.e., machine design, structural design, etc., projects become lengthier, open-ended and difficult, leading to the major design experience.

In accordance to ABET\textsuperscript{1}, an engineering program must demonstrate that the graduates of a program have satisfied Criterion 3(c) “an ability to design a system, component, or process to meet the desired needs within realistic constraints…” The approach proposed in this paper can be used to demonstrate Criterion 3(c) applied to individual structural components. Furthermore, if the approach is used in other courses, i.e., statics, machine design, structural design, etc., then this can be used to demonstrate ABET EC2000\textsuperscript{1} Criterion 5 as follows: “Students must be prepared for engineering practice through the curriculum culminating in a major design experience based on the knowledge and skills acquired in earlier course work…”.

Some mechanics of materials textbooks that introduce design include Beer & Johnston\textsuperscript{2,3}, Craig\textsuperscript{4}, Pytel & Kiusalaas\textsuperscript{5}, Shames\textsuperscript{6}, Shames & Pitarresi\textsuperscript{7}, Ugural\textsuperscript{8} and Yeigh.\textsuperscript{9} In general, the presentations involve homework problems or special problems identified under the category of computer application. The problems tend not to have a structured format and request a single solution for a single set of specific requirements. In other words, the solutions are not developed in general symbolic form. This certainly limits the opportunity for solution verification testing and extension to iterative design studies.
The proposed approach in this paper is based on implementation of symbolic equations and therefore allows easy extension to design as proposed by Rencis and Grandin.\textsuperscript{10} The authors are not aware of any past efforts that use a general symbolic approach to formulate all mechanics of materials problem types. With equations written in symbolic form, they are entered into a modern engineering tool (equation solver) and validated through thorough testing.\textsuperscript{11} The equations then may be used not only for repetitive analysis of a structure, but also for design of a similar structure, where the dimensions and materials must be selected for a given loading. Incorporating a computer equation solver with the ‘raw’ fundamental symbolic equations, as proposed in our approach, not only leads to easy design applications, but also has the added benefits of reduced opportunity for algebraic errors and increased engineering productivity.

This paper will first present the structured problem solving process that is used in the author’s mechanics of materials course for the lecture examples, homework problems, quizzes, and design project (focus of this paper) in our mechanics of materials course. We then discuss the verification step that is part of the structured problem solving process. An in-depth discussion of a statically determinate hoist structure design project is then considered. The next section discusses the seven phases associated with the hoist structure design project. Three additional design projects are then discussed. Finally, the conclusions of the paper are presented.

**Structured Problem Solving Process Used for Design Project**

This section provides an overview of the structured problem solving process used for the design project in our mechanics of materials course.\textsuperscript{10,11} The authors have used this structured problem solving process for lecture examples and also require the students to use this process for homework problems and quizzes.

A primary goal in an introductory mechanics of materials course is to show the student that force and elastic deformation analysis of single or multiple connected bodies is based on the application of only three fundamental sets of equations:

- rigid body equilibrium equations,
- material load-deformation equations derived from Hooke’s Law, and
- equations defining the known or assumed geometry of deformation.

The commonality of a general approach to all problems is emphasized, an approach that is identical for determinate and indeterminate structures containing axial, torsional and/or bending loads. This general approach is formulated to emphasize:

- identification of applicable fundamental independent equation set(s) being written,
- formulation of the necessary governing equations in symbolic form, with no algebraic manipulation to isolate unknowns,
- matching the number of unknowns with the number of independent equations and
- entering the known numerical data and solving for the unknown variables.
For the general problem involving deformation, we use a non-traditional structured problem solving format that contains eight analysis steps. The students are required to follow the steps listed below for every in-class problem, homework problem, and quiz problem they solve, including the proposed design project.

1. **Model.** The success of any analysis is highly dependent on the validity and appropriateness of the model used to predict and analyze its behavior in a real system, whether centric axial loading, torsion, bending or a combination of the above. Assumptions and limitations need also be stated. This step is not explicitly emphasized in any mechanics of materials textbook.

2. **Free Body Diagrams.** This step is where all the free body diagrams initially thought to be required for the solution are drawn. The free body diagrams include the complete structure and/or parts of the structure. Very importantly, all dimensions and loads, even those which are known, are defined symbolically.

3. **Equilibrium Equations.** The equilibrium equations for each free body diagram required for a solution are written. All equations are formulated symbolically. There is no attempt made at this point to isolate the unknown variables. However, every term in each equation must be examined for dimensional homogeneity.

4. **Deformation Formulas.** In problems that are statically indeterminate or require the structural stiffness we apply the deformation formulas. The deformation formulas are written for each part of a structure based on the Model in Step 1. All equations are formulated symbolically and there is no algebraic manipulation. Every term in each equation must be examined for dimensional homogeneity.

5. **Compatibility and Boundary Conditions.** One or more compatibility equations are written in symbolic form to relate the displacements. A compatibility diagram is used when appropriate to assist in developing the compatibility equations. All equations are formulated symbolically and there is no algebraic manipulation. Every term in each equation must be examined for dimensional homogeneity. Although compatibility equations are commonly written for indeterminate problems, the authors emphasize their use for determinate problems just as is done in the textbooks by Craig, Crandall et al., Shames, and Shames & Pitarresi.

6. **Complementary and Supporting Formulas.** Steps 1 through 5 are sufficient to solve for the (primary) variables force and displacement in a structures problem. Step 6 includes complementary formulas for other (secondary) variables such as stress and strain, variables which may govern the maximum allowable in service values of force and displacement, but which do not affect the governing equilibrium or deformation equations. Supporting formulas are those which might be required to supply variable values in the material law equations and complementary formulas; formulas such as area, moment of inertia, centroid location of a cross-section, volume, etc.

The complementary and supporting formulas are written symbolically and are necessary to develop a complete analysis. The complementary formulas might involve solution
governing variables such as stress, strain and stiffness. Supporting formulas may be necessary to completely define variables in Steps 3 through 5 and in the complementary formulas. These formulas might include cross-sectional area, polar moment of inertia, centroid location, moment of inertia, section modulus, effective length, radius of gyration, etc.

7. **Solve.** The independent equations developed in Steps 3 through 6 solve the problem. The students compare the number of independent equations and the number of unknowns. The authors emphasize that the student should not proceed until the number of unknowns equals the number of independent equations.

The solution may be obtained by hand, and this generally requires algebraic manipulation. Alternatively, the solution of any number of equations, linear or non-linear, can be obtained with a modern engineering tool. With intelligent application of verification (Step 8), the computer program is a much more reliable calculation device than a calculator. (ABET\textsuperscript{1} criterion 3(k) states that engineering programs must demonstrate that their students have the “ability to use the techniques, skills, and modern engineering tools necessary for engineering practice”.) The students are allowed to select the modern engineering tool of their choice, and this might include Mathcad\textsuperscript{13}, Matlab\textsuperscript{14} and TKSolver.\textsuperscript{15} The authors have not seen this solution procedure in any mechanics of materials textbook.

8. **Verification.** This important step is a critique of the answer, and is discussed in-depth in the next section. This step is considered only in the mechanics of materials textbook by Craig.\textsuperscript{7} This step has been included to educate our students on how to question and test solutions to verify their ‘answers’. An in-depth discussion on verification can be found in Rencis and Grandin.\textsuperscript{11}

Problems in statics require only Steps 1, 2, 3, 6, 7, and 8. These six steps have not been employed in the treatment of statics problems in any statics or mechanics of materials textbook. Furthermore, Steps 1 through 8 have not been suggested in any mechanics of materials textbook. A simple example problem using this eight step structured problem solving format can be found in Rencis and Grandin.\textsuperscript{10}

Pedagogically the step-by-step problem solving process allows a student to build a structure in their minds of how to efficiently approach a problem and solve it. The authors have found through observation that this step-by-step procedure helps students build logic, promote analytical thinking, provide a true physical understanding of the subject and, hopefully, extend the same disciplined process to other courses. Furthermore, both authors and other instructors have found that students who take the follow up undergraduate machine design course are well prepared to start a design project on day one and the project considered is more comprehensive than past design projects.
Verification

One of our educational goals is to convince students of the wisdom to question and test solutions to verify their ‘answers’. We do this by integrating verification as part of the structured problem solving format discussed in the previous section. There are very few textbooks that have addressed verification. It has been considered in statics by Sandor\textsuperscript{16} and by Sheppard & Tongue\textsuperscript{17}, and in mechanics of materials by Craig.\textsuperscript{4} Verification is new to almost all undergraduates, but it is critical and really must be formally integrated into the solution process! Once our students graduate and become professionals, they must be prepared to stand behind their ‘answers’.

In our approach, verification Step 8 is carried out after solution Step 7 is performed once. The power of our proposed use of the modern engineering tool rests in the ability to quickly and easily run many cases to verify the problem solution. How does one test the problem solution? Listed below are some suggested questions that students may apply for the purpose of verification of their ‘answers’.

- *A hand calculation?* A longhand analysis for the complete solution, a partial solution and supporting calculations, e.g., geometric properties. The pitfall here is that a longhand solution of incorrect equations might check the computer solution (of the same incorrect equations) leaving a false impression of verification of the ‘answer’.

- *Comparison with a known problem solution?* A known problem solution may be found in references, e.g., handbooks, appendices, textbooks, etc.

- *Examination of limiting cases with known solutions?* Limiting cases are constructed which establish a problem with a known solution. For example, removing the static indeterminacy by reducing the stiffness (lowering of the elastic modulus) of structural components yields an example that may be tested with a hand calculation or compared to other known solutions. Altering the placement of load(s) is another example. Known problem solutions may be found in handbooks, appendices, textbooks, etc.

- *Examination of obvious known solutions?* These are problems that are simple and which yield quick, very apparent known solutions. For example, zero applied loads must yield no response. Other examples, a concentrated applied load positioned at a rigid support would result in zero response, a load reversal would yield the same magnitudes but opposite signs.

- *Your best judgment?* This is where an examination of the answer points to obvious quantitative and/or qualitative errors. In a quantitative sense, are answers of the correct order of magnitude? From a qualitative perspective, do the applied loadings produce reactions and displacements in directions obvious from a physical understanding of the problem? Are the signs correct?
• **Comparison with experimentation?** Experimentation gives substance to theoretical concepts and provides a means of augmenting insights gained from analytical studies. Furthermore, it can also be used to verify results. Due to time limitations in our course, experimentation is not considered.

As indicated above, attempts at solution verification may take many forms, and, although in some cases it may not yield absolute proof, it does improve the level of confidence. The authors have observed that the verification Step 8 helps students build logic, promote analytical thinking and provide a better physical understanding of the subject. Example problems that use the verification step can be found in Rencis and Grandin.\textsuperscript{10,11}

**Overview of the Hoist Structure Design Project**

Our mechanics of mechanics course requires one large design project and small homework design problems assigned throughout the term. The large design project is a statically determinate hoist structure shown in Figure 1. The structure is assembled with smooth pins and is symmetric about the XY plane. This hoist is to be made by the manufacturer in several models, each with a different maximum lifting capacity and overall dimension, but all with the same basic configuration. The manufacturer also offers the option of custom size and capacity of the basic design. For every model and customer option, you, as an engineer (student) in the company, are given the responsibility of specifying:

- The material for the pins.
- Dimensions of three pins A, B, and C.
- The material for the structure Members 1 and 2.
- The dimensions of each structural member, Member 1 and Member 2.

Developing the mathematical model in symbolic form, and solving the equations with an equation solver program will create a tool for rapid and accurate sizing of all components for any proposed hoist model of this configuration. Customer inquiries about specific design capacity changes may be answered very quickly; a huge benefit for sales potential.
Assigned Phases for the Hoist Structure Design Project

The analysis and design of the hoist components will be ongoing as the students proceed through the course, each new topic contributing the necessary knowledge to continue the process. The hoist structure design project in Figure 1 included the following seven phases:

1. **Phase 1 – Member and Pin Force Analysis.** This phase is assigned after we expose the students to the force analysis of a frame structure, i.e., equilibrium and free-body diagrams of frame members and pins. The authors cover pin free-body diagrams since this topic is not commonly covered in statics textbooks. The overall goal of Phase 1 is, given the general symmetric configuration of the hoist design, determine the force exerted on and by each component of the hoist structure when lifting a load, W. The students are required to develop a model that defines the maximum shear forces in each pin and the forces at each pin joint of Members 1 and 2 in terms of the applied loading and dimensions. Because there will be several models, and possibly an endless number of options, with different dimensions and capacities, all equations are expressed symbolically. These equations are to be entered into the engineering tool selected by the students. The students are required to draw free-body diagrams and label each diagram in symbolic form. The students are also required to do thorough verification using the engineering tool and carry out one numerical hand solution. Once the model has been verified, the students are required to determine the force exerted on and by each
component of the hoist structure when lifting a load given numerical values of variables lifting load \( W \) and structure geometric parameters \( X_B, X_C, Y_A, R_1, \) and \( R_2 \) shown in Figure 1. The overall challenge of this phase is that the students must formulate all equations symbolically, identify all required equations, and draw the corresponding free-body diagrams that show symbolic variables.

2. **Phase 2 - Pin Connection and Member Stress Analysis.** The students are first exposed to the concepts of stress, that includes normal and shear stresses for members and pins and stress concentration factor, and then assigned this phase. The overall goal of Phase 2 is, given the general symmetric configuration of the hoist design, determine the shear stress in each pin and normal stress in Member 2 when lifting a load, \( W \). The students must first develop the symbolic equations for the maximum shear stress in each pin and normal stresses in Members 2 and draw all appropriate figures with labels in symbolic form. The stress analysis is quite limited in this phase, but considers the following: 1. Shear stress in pins \( A, B \) and \( C \); 2. normal stress on the cross-section of Member 2; 3. compressive bearing stress on the pin surfaces and on the pin hole surfaces of Members 1 and 2, and; 4. shear tearout stress in Members 1 and 2 at the pin hole. Finally, the solution will be subjected to thorough verification. The students are required to always carry out verification for the current phase, but also revisit the verification process carried out for the previous phase.

3. **Phase 3 - Initial Design of the Pins and Members 1 and 2.** This is considered the simplest phase of the design project and where students are first introduced to a small aspect of design through sizing the pins and members. The students are exposed to the concepts of material properties in the classroom and then assigned this phase. The overall goal of Phase 3 is to compare the stresses in the pins and Member 2 with the strengths of the materials and, using a specified factor of safety, make a preliminary determination for the diameter of each pin and cross-section dimensions of Member 2. Where appropriate, standard sizes will be selected. The solution will once be subjected to thorough verification and the students must revisit the verification process carried out for all previous phases.

4. **Phase 4 - Initial Design of Structure Stiffness.** The students are first exposed to statically determinate axial members in lecture and then assigned this phase. The overall goal of Phase 4 is to determine the hoist structure stiffness based on the elasticity of the axial loaded Member 2 with Member 1 assumed rigid. The hoist structure stiffness is defined as \( k = \frac{W}{v_D} \), where \( W \) is the vertical load at point D and \( v_D \) is the vertical deflection at point D in Figure 1. Symbolic equations are written which define the deformation of centric axially loaded members and the corresponding displacement of points in the structure caused by loads applied to the structure. If the stiffness is not within specification, the cross-section dimensions of Member 2 will be changed. The solution will be subjected to thorough verification and the students must revisit the verification process carried out for all previous phases.

5. **Phase 5 - Beam Stress Design and Refined Structure Stiffness.** After the students are exposed to concepts of beam bending, i.e., equations defining beam flexure stress and
transverse displacement of beam and stress concentration factors, they are then assigned this phase. The overall goal of Phase 5 is to determine the hoist structure stiffness and the superposition of uniform axial and bending stresses in Member 1. Equations for Member 1 will be entered into the program and tested. The Member 1 dimensions will be modified to yield a stress within a specified factor of safety. The stiffness of the structure will be recalculated to include the axial and transverse deformation of Member 1, and its dimensions adjusted to meet the required stiffness specification. The solution will be subjected to thorough verification and the students must revisit the verification process carried out for all previous phases.

6. **Phase 6 - Column Analysis and Design.** After the students have been exposed to centrically loaded long columns, they are assigned this phase. The overall goal of Phase 6 is to test the possibility of column failure of any member supporting a compressive load and modify cross section dimensions as necessary. The solution will be subjected to thorough verification and the students must revisit the verification process carried out for all previous phases.

7. **Phase 7 – Final Hoist Structure Design.** The students now have the technical background and mathematical model in symbolic form to design a hoist structure using an engineering tool. The overall goal of Phase 7 is to design a hoist structure based on customer specifications as discussed in the previous section. The customer has specified the maximum lifting load W and structure geometric parameters X_B, X_C, Y_A, R_1, and R_2 shown in Figure 1. The students are required to carry out the following: 1. Select the material for the pins.; 2. determine the dimensions of pins A, B, and C.; 3. select the material for the structure Members 1 and 2, and; 4. determine the dimensions of each structural member, Member 1 and Member 2. The students are required to use the same material throughout the entire structure and select pins of the same size. The solution will be subjected to thorough verification and the students must revisit the verification process carried out for all previous phases.

After a topic is covered in class, reinforced through homework and classroom quizzes, the phase related to the topic area is assigned. Some homework problems assigned for a topic are design oriented. For each phase the students are required to use the eight steps of the structured problem solving process discussed in the section entitled Structured Problem Solving Process Used for Design Project. As each phase is completed, the 8th step verification, of the process educates the students on how to question and test solutions to verify their ‘answers.’ After the second, fourth and sixth phases of the project are completed, students are given the entire solution up to that phase. The solution includes the engineering tool documentation and supporting figures labeled with variables. The design project is carried out in a two student team. The students are required to use a modern engineering tool that can select Mathcad, Matlab, or TKSolver. Eighty percent of the students use TKSolver and the remainder used Mathcad. The students selected TKSolver since the follow up course in machine design uses the textbook by Norton that employs TKSolver and Mathcad. The students are graded on each phase of the project. The seven phases are worth 20% of the final course grade.
Overview of Additional Design Projects

The authors use a different design project each time they teach the mechanics of materials course. The projects are varied in difficulty from the complex hoist structure discussed in this paper to simpler frame structures. We will now discuss three simpler frame structures.

The first example of a simpler design project involves the brace structure shown in Figure 1. The brace consists of a pin connected frame structure shown in Figure 2 that is made of two links ABC and CE connected together and to the foundation with smooth pins. The structure is symmetric about the XY plane. The weight of the links is negligible in comparison to the applied loads, $P_B$ and $P_C$. The students are required to carry out this project in the same manner as the hoist structure just discussed.

![Figure 2. Brace structure design project.](image)

The second simpler design project involves a beam hanger assembly that is fabricated by assembling two identical outer plates and an inner plate with a bolt at B and a pin at C, as shown in Figure 3. The beam is supported by two smooth pin supports and supports a load $P_C$ by means of the yoke attached to pin C. The pin C has a slight interference fit in both inner and outer plates. The bolt hole is machined to provide clearance between the bolt and inner plate in the beam longitudinal X direction. The structure is symmetric about the XY plane. Neglect the weight of the plates and the friction between the plates at the connection. The student teams are once again required to complete this project in the same manner as the hoist structure.
A third design project involves the simple hoist structure shown in Figure 4. The hoist consists of Members 1 and 2. The statically determinate hoist structure in Figure 4a has smooth pin connection at A, B, and C. The statically indeterminate hoist structure in Figure 4b has smooth pin connection at B and C, and is fixed at A. Both structures are symmetric about the XY plane. The weight of Members 1 and 2 is negligible in comparison to the applied load, P. The students begin the design project with the statically determinate structure in Figure 4a and then add the statically indeterminate structure in Figure 4b as they obtain the necessary background to solve the problem. Since the problem was formulated using symbolic equations, the only difference between Figures 4a and 4b is the couple and rotation at A. In Figure 4a the moment of the couple is zero (known) and the rotation is unknown, whereas, in Figure 4b the moment is unknown and the rotation is zero (known). The students are required to carry out this project in the same manner as the complex hoist structure just discussed.
Conclusion

This paper presents a just-in-time approach developed and used by the authors to integrate a design project into an introductory undergraduate mechanics of materials course. Frame structure design projects that have been integrated into the course include hoist structure, brace structure, beam hanger, and simple hoist structure. The approach to formulate a problem requires a student to model a general physical problem with the fundamental equations written in symbolic form, with no variable values specified. This helps the student to more fully concentrate on the fundamental principles taught in the course. Introducing modern engineering tool to solve the equations removes the necessary manipulation of the equations to isolate the dependent variables. Training the student to examine and test the answer becomes an important goal in our course. The proposed approach has been used for small design homework problems and the design projects discussed in this paper. The proposed approach can also be used in follow up design and non-design courses that includes advanced mechanics of materials, machine design, structural analysis, structural design, etc. The first author and other instructors have observed that students who have used the proposed approach are more prepared to solve more complex design problems than previously considered in the follow up machine design course than students who were not exposed to the proposed approach. Future research will assess how successful the proposed approach compares to the previous approach in terms of student learning. Furthermore, we plan to assess how the students do in the follow-on machine design course compared to the previous approach used.
Bibliography