LaPREP: Louisiana's Successful Enrichment Program and Its Engineering and Discrete Math/Probability and Statistics Components

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Abstract

LaPREP (Louisiana Preparatory Program) is a highly successful enrichment program in engineering and the mathematical sciences for high-ability middle school students. It has been honored by the Mathematical Association of America, the National Science Foundation and the Jacqueline Kennedy Onassis Foundation. It has been featured in several newspaper articles and has been given wide television coverage. Recently, it was featured on CSPAN, and a PBS video tape describing the program was aired nationally.

LaPREP identifies, encourages, an instructs competent middle school students, particularly women and minorities. Its long-term goal is for them to successfully complete a college program, preferably in engineering, math or science. Participants attend seven weeks of intellectually demanding classes and must maintain a 75+ average to remain in the program. Activities include course work in engineering and the mathematical sciences. Lab experience, including "hands-on" research activities, introduces engineering, math, and science as active and participatory processes. Other activities include ACT preparation, study skills and college awareness seminars, and field trips to local businesses and industries. Professionals in the engineering and technological fields, including many minorities, discuss career opportunities.

The program has been very successful in identifying and educating high ability middle school students. Evaluations by the participants, their parents, and by local and state officials who have visited LaPREP have been excellent. No current or former participant has dropped out of high school and 84% of exiting participants have indicated that LaPREP has increased their desire to study math and science. Moreover, all 55 former participants who have graduated from high school have enrolled in college and more than 90% of those responding to a survey indicated they were majoring in math or science. This is considerable improvement when compared with a similar survey taken of entering LaPREP students in which only 54% expressed an interest in attending college and majoring in math or science.

This paper will describe the program and its academic components, paying particular attention to the engineering and the discrete math/probability and statistics components.

I. Background

The battle for education and technical competence is not won at the college level, nor even the level of senior high school. By the time a student has finished middle school, he or she will have strong feelings concerning a direction in life and a career. LaPREP has been developed at LSU in Shreveport, Louisiana in an attempt to educate and influence students at this early stage in their lives.

LaPREP is an enrichment program that identifies those students who are potential engineers or scientists, and gives them the necessary reinforcement needed so that they may successfully pursue such studies in college. It is based on traditional values such as hard work, commitment, and discipline. Academic excellence, rather than remediation, is stressed.

Traditionally, the principal academic obstacles for such students include the problem-solving component of mathematics. A major objective of the program is to strengthen these skills. This is done through seven weeks of intellectually demanding classes and laboratories. For example, in the problem-solving component of the program, students study together in small groups and attack rigorous mathematical problems using techniques employed by Dr. Philip UriTreisman at the University of California at Berkeley. In his program, the failure rate for high-ability minority students in freshman calculus went from sixty percent to only four percent[1].

Over a period of two summers, LaPREP students study topics that are not substitutes for the usual courses in the middle or high school curricula. For example, they study course work in Engineering, Logic, Algebraic Structures, Probability and Statistics, Computer Science, Discrete Mathematics, Technical Writing, Problem Solving, Medical Careers Preparation and ACT Preparation.

II. Need

Statistics on poverty, violence, and the high school dropout rate paint a bleaker picture for children in Louisiana than in any other state, according to the recently released report by the Annie E. Casey Foundation of Baltimore [2]. Increasingly, students graduating from public high schools are required to take remedial courses upon entering state colleges and universities[3]. A recent report by the Federal Office of Minority Concerns states that the decline of black college enrollment and graduation has reached “alarming proportions” and is creating “a crisis of substantial dimensions for the American society” [4]. Because Louisiana has the highest percentage of children living in poverty and the lowest high school graduation rate in the nation (56%), the situation is extremely serious in this state[5]. Clearly, a large number of students are finding the path to a college degree to be a very difficult one.

According to a 1989 report from The American Council on Education, there is a heavy concentration of black and Hispanic students in the fields of education and social sciences, but there are “still insufficient numbers of black and Hispanic students majoring in science and mathematics” [6]. Moreover, “the extent of the underrepresentation (in each field) is in direct proportion to the amount of mathematics employed in the field. For lack of a proper foundation in mathematics, Blacks, Hispanics, and Native Americans are shut out of many scientific and business careers"[7]. This is particularly true in Louisiana where only 22% of its black eighth-grade students are achieving at or above the basic level of proficiency in mathematics [8].

Across the country, students need to be encouraged to major in the engineering and technological fields. In the state of Louisiana, where education reform has become a top governmental priority, students in general, and minorities in particular, need extra encouragement to pursue such studies. In addition, Louisiana has the third highest percentage of black residents, the highest percentage of children living in poverty, and the lowest high school graduation rate in the nation. The need for creative intervention into the academic progress of these students is necessary.

LaPREP has been developed and established as an intervention program in Louisiana that will succeed in preparing and encouraging students, including minorities, to pursue and successfully complete college studies in engineering, mathematics, and other related fields. It is not a "one-shot" program, but a two-year program that influences and encourages students through their middleschool and/or early high school years.

III. Program

LaPREP is an enrichment program in which the purpose is to identify, encourage, and instruct competent middle school students (including women and minority students) who, as a result of the program, will pursue college studies, preferably in mathematics, science, or engineering. Because the measure of under representation of minorities in a particular field is in direct proportion to the amount of mathematics contained in the field (see paragraph 2 of II), LaPREP emphasizes the development of abstract reasoning, problem solving, and technical writing skills, primarily through mathematics enrichment courses and seminars. Class assignments, laboratory projects, and scheduled examinations are included in the program.

Other aspects include field trips to local industries, lecturers speaking on science and engineering opportunities, drug and violence prevention activities, well-known minority speakers, and ACT preparation.

Program faculty includes college, high school and middle school teachers. Minority college science majors serve as program assistants and are excellent role models.

IV. Local and National Recognition

LaPREP has received local, regional, and national attention. A resolution applauding LaPREP "for making a positive impact on the lives of young people and for contributing to the future prosperity of the community and the nation" was unanimously passed and presented to LaPREP by the Shreveport City Council. It is one of fewer than 150 intervention programs recognized by the National Science Foundation and the Mathematics Association of America in the *Directory of Mathematics-based Intervention Projects (SUMMA 1996).*

The Mathematical Association of America gave LaPREP an award in 1996 recognizing its contribution to mathematics in Louisiana. Moreover, the LaPREP director has been invited to speak about LaPREP and share information relevant to intervention programs by the National

Science Foundation together with the Mathematical Association of America (SUMMA), the Department of Energy, the Minority Science Improvement Program (under the Department of Education), the American Society of Engineering Education, the Louisiana Association of Teachers of Mathematics, and the Louisiana Governor's Safe and Drug-Free Schools and Communities Conference.

In 1998, LaPREP received national attention as its director was named recipient to the prestigious Jefferson Award for his contribution in the area of public service. The honor resulted from his work with LaPREP. An even greater honor was his selection as one of five recipients, selected from over 14,000 nominees, for the Jacqueline Kennedy Onassis Award, referred to as the Nobel Prize for public service. The Jefferson Award ceremony was featured on national television with special coverage of the Jacqueline Kennedy Onassis Award winners on CSPAN. A nationally circulated PBS video tape describing the program and interviewing instructors and participants in the program was aired nationally.

Also, in 1997 and 1998, its director won the Carnegie Foundation's award as Louisiana Professor of the Year, the Mathematical Association of America's award for Distinguished College Teaching of Mathematics in Louisiana and Mississippi, and the Governor's Award of Excellence, all due in part to his work with LaPREP.

V. Engineering Component

The seventh grade participants are taught the principles of buoyancy by dividing them into teams and having the teams attempt to construct submarines of neutral buoyancy. The project introduces sound engineering practices and allows the participants to develop team approaches to solving problems.

In the first part of the course, calculation skills are developed by having the participants compute both measurements of mass and the amount of mass needed to create neutral buoyancy in different geometric shapes. The groups are then given pieces of 4 inch PVC schedule 40 plastic pipe along with end caps and 4 inch plastic hemispheres. The pipe pieces range in random lengths form about 40 centimeters to 1 meter. Weights of the different parts are determined, the caps are glued onto the pipes, and the hemispheres are taped onto the caps. The purpose of the hemispheres is to streamline the flow of water around the blunt PVC end caps. Students then calculate the amount of iron necessary to create neutral buoyancy of their subs. The weights of iron are toe-wrapped below the center of buoyancy of each sub to provide balanced weights and stability.

A trip to the LSU in Shreveport swimming pool follows where each sub is placed in the pool at a depth that is one half the total depth of the pool. The subs are released and the time is measured for the sub to sink to the bottom or float to the top. Students are then given a chance to readjust the total weight of the subs and reposition the weight as needed. A final trial is then performed. Subs are also given a shove underwater to observe how they travel. With a fairly high mass and streamlined structure, most travel considerable distances.

VI. Probability and Statistics / Discrete Mathematics

This academic component of LaPREP is divided into two parts. The first covers counting principles and some basics of probability and statistics. The second unit covers several discrete mathematics concepts with an emphasis on matrix applications.

A. Unit I: Counting Principles/Basic Probability and Statistics

1. Misuse and Abuse of Statistics in the Media
Unless informed otherwise, students seem to believe that if something appears in print, it is true. To dispel this notion and to encourage them to be more critical of what they read, numerous examples of misuse of statistics and terminology, misinterpretation of results, and meaningless graphs and tables as found in newspapers and advertising are presented. Some of these abuses are minor and are covered quickly, but others, far more significant because they could have an impact on public policy, are discussed at length.
2. Counting Principles
The Fundamental Principal of Counting, combinations and permutations, the formulas, and practical examples of each are covered.
3. Estimation/Data Collection
Students participate in several estimation/data collection activities, as described below.

a. Estimation of Fraction of Expired Safety Inspection Stickers: After a short discussion of vehicle safety inspection stickers, students give their individual estimates of the proportion which are expired. The class than adjourns to the university parking lot where each student notes whether his/her 20 randomly selected vehicles has an expired sticker. Back in the classroom, an overall fraction of expired stickers is calculated. Also noted is the variation in the fraction expired in the individual samples and how closely the overall observed fraction approximated their original guesses. Further discussion centers on the appropriateness of this sample to estimate the statewide fraction expired, and the changes necessary to get a more representative sample.

b. Estimation of Frequency of Letters in English Alphabet Texts: In this activity, students are asked to guess the most frequently occurring letter and their guesses are recorded. Each student is given a randomly selected page from a novel such as *The Three Musketeers* and uses a random number table to pick an initial starting letter from their page, and does a frequency distribution for the next 100 letters. About half of the student's results are transferred to a transparency and displayed on an overhead projector. A discussion follows concerning how close their results were to their initial guesses, the variability of the individual letter frequencies, and the combined results.

c. Estimation of Percent Defective in a Lot of New Fuses: To demonstrate that results may differ depending on the novel chosen, students are then given randomly selected pages from *Gadsby* by Ernest Vincent Wright, a 50,000 word novel written without using the letter "e". Students are admonished to remain silent, no matter how different their results seem to be from their previous experiment. After each has completed their frequency distribution, some of the individual results, as well as the combined results are displayed on the overhead. Finally, the importance of letter-frequency analysis in cryptology is discussed.

Typically, sample fraction defectives ranges from 0/30 to 7/30, with very few " estimates" being the exact population defective—10%. Students are truly amazed that in looking at a population which obviously contains a mixture of colored beads—blue and orange—that several samples of n=30 would contain no orange beads. They are able to see first hand, that for small samples, the sample estimate may be considerably different from the population proportion. The idea of limit is introduced by re-calculating the estimate after each new sample is taken, giving a new "overall" estimate of the population fraction defective by including previous sample data, illustrating that the average of the sample fractions approaches 10% as more samples are taken.

d. Estimate of Survival Time Following By-Pass Surgery: In this activity, twenty checkers, each bearing a distinct integer from one through twenty, are used to represent survival times in years for twenty heart by-pass patients. The students are given the arithmetic mean (10.5 years) for these 20 patients, and then each draws a random sample of n=3 from the population of N=20, with the results recorded and the sample means determined for each sample. The discussion which follows includes the total number of samples possible (combinations), the impossibility of obtaining a sample mean identical to the population mean, the smallest and largest possible sample means (2 and 19 respectively), and the variability of the sample means.

Students are then introduced to hypothesis testing at the most elementary level -- they are asked to look at the different sample means and see which ones are so far away from the mean of 10.5 that had they gotten such a sample mean, they would conclude that the population mean was not equal to 10.5. Understandably, most select only those sample means in the 8 to 13 range as being not too far away from the population mean, and incorrectly conclude that the remaining more extreme sample means would give one reason to doubt that the population mean is actually 10.5.

4.LaPREP Lotto: The current Louisiana lottery in which 6 numbers are chosen from 40 (6/40) and the connection with the combinations counting principle is discussed, after which a smaller version, the LaPREP Lotto in which 6 numbers are chosen from 13 (6/13) is introduced. A clear glass fish bowl with 1715 blue beads and one orange bead is brought out and the students are told that each of these 1716 beads represents one of the combinations of six from thirteen and, one of the 1716 possible outcomes of the lottery, only one of which is a winner. Using a paddle, each student randomly draws one bead or "purchases one ticket." Incidentally, although the probability of getting the "winning ticket" is 1/1716, this activity has never produced a winner. The very productive discussion which follows centers not only on the difficulty of obtaining the one orange bead among the 1716 in the bowl, but the considerably more difficult task of being a winner in the current Louisiana lottery in which there are 3,838,380 possible outcomes--more than 2220 times that of the in-class activity.

To illustrate this idea, the students are told that if the 1716 beads in the fish bowl were laid out on the floor, they would occupy an area about three square feet one layer deep -- roughly equivalent to three tiles on the floor. By comparison, the Louisiana lottery would occupy a column with a base of approximately 3 square feet and a height of more than 2220 beads, or 92.5 feet since each bead is about 1/2 inch in diameter. This would be equivalent to a column over seven stories tall if one allows 12 feet per story in which there is only one lucky bead. Not surprisingly, after this discussion, students express no interest in purchasing lottery tickets.

B. Unit II: Matrices and Discrete Math

Students are introduced to the basics of matrices including addition, multiplication, determinants and inverses as these are needed in the applications which follow. Multiplication of matrices is presented in the traditional way, then taught as a repeated application of dot products. Finding determinants of square matrices by hand is discussed only for 2 x 2 matrices. Finally, the procedure for finding the inverse for 2 x 2 matrices whose determinant equals one is shown.

As a homework assignment, students are asked to fabricate their own scenarios for each of the following applications. The best ones are presented in class.

1. Encryption Using Square Matrices: First, a monoalphabetic substitution -- an alphanumeric cipher -- is introduced (a = 1, b = 2, c = 3, d = 4, ... , z= 26) and the connection between the total number of such ciphers and the product rule counting principle is covered. Then a simple message such as HELP is transformed into 8, 5, 12, 16. This is the numerical version of the plaintext or cleartext. Next, any 2 x 2 matrix (the encryption matrix) with a determinant of one is chosen and its easy-to-find inverse (the decryption matrix) is determined.

At this point, the cleartext (from left to right) is broken up into two 1 x 2 matrices and each is multiplied by the encryption matrix. The result is two 1 x 2 matrices which contains the cyphertext.

To decode the cyphertext, matrix multiplication is again required. This time the cyphertext is broken up into two 1 x 2 matrices and each is multiplied by the decryption matrix which results in two 1 x 2 matrices containing the numerical cleartext. After referring to the alphanumeric cipher, the original message is recovered.

Cleartext containing an odd number of characters (nine for example) can be handled by using a 3x3 encryption matrix. Since the procedure for finding the inverse for a 3x3 matrix was not demonstrated, a graphics calculator such as the TI83 can be used to find the inverse, and at the same time illustrate some of the calculator's matrix features. Typically, five students in a class of twenty will have access to a graphics calculators. If an additional five calculators can furnished by the instructor, student enthusiasm remains very good. Instructions on using the calculator to enter matrices, find sums and products, determinants and inverses are shown using the overhead projector. At this point students are able to send and receive coded messages of any reasonable length.

2. Determination of Dominance in a Group of Persons Using Matrices: This application is introduced by an example in which the identification of the leader of a group of five gang members is desired. To do this, the members are interviewed pairwise and the dominant member of the pair determined. [Here the number of pairwise interviews-combinations of five persons taken two at a time—is already known from the counting principles section of LaPREP.]

Once the ten pairwise dominances are determined, a digraph is constructed with the group members serving as the vertices and the directed edges representing the dominance of one vertex over another. From the digraph, a 5x5 "0x1" matrix is constructed by entering a "1" in the ijth position if row (person) *i* dominates column (person) j, otherwise a zero is entered. Diagonal entries are zero since a person cannot dominate himself.

The row totals give the number of one-stage dominances for that row (person) and the row (person) with the largest total is declared the winner. In the event of a tie, the square of the matrix, whose entries give the number of two-stage dominances of a given row over a given column, may help break the tie. In the squared matrix, the row totals give the number of two stage dominances for that row (person). By adding the original "0-1" matrix and its square, one gets a matrix whose row totals gives the number of one and two stage dominances for that row (person). The time required to carry out these matrix operations is reduced considerably by using the TI-83.

This same dominance structure is used as a model for other realistic examples including the spread of a rumor, the spread of an infection, and true rankings in an athletic conference.

3. Transition Matrices: A short discussion of transition matrices (entries are probabilities and rows sum to one) is given. The initial application example is one regarding three nightly network news shows. Probabilities of viewers staying put for the next news show or switching to an alternate network are provided. These probabilities make up the entries for the 3x3 transition matrix. The current viewer distribution is also given—the fraction of viewers who currently watch each of the nightly news programs. These entries are put in a 1x3 matrix (the initial state matrix).

To find the viewer distribution at the end of the first evening, the product of the initial state matrix and the transition matrix is taken. This product matrix is multiplied by the transition matrix to find the distribution of viewers at the end of the second evening. The viewer distribution on any particular night can be found by repeating this process.

4. Critical Choices Using Matrices: Consider three persons who are skilled in clandestine operations, (but not necessarily the same operations) which would prove useful in helping to de-stabilize an unfriendly government. Suppose that person 1 is skilled in forgery, bomb making, and foreign languages; Person 2 is skilled in counterfeiting, computers, and bomb making; Person 3 is skilled in foreign languages, computers, and forgery. This information can be put in a 3x5 "0-1" person (row) x skills (column) matrix, in which the ijth entry is a "1" if person *i* is skilled in activity "j", and "0" otherwise.

Furthermore, consider four target foreign countries in which one or more of these skills in needed. For example, in country A, operatives with skills in counterfeiting, forgery, computers, and foreign languages are needed, and countries B, C, and D require operatives with possibly different skills. This information is entered into a second matrix, a 5x4 "0-1" skills (row) by country (column) matrix. The product matrix is a person by country matrix whose row totals suggest to which country each operative should be assigned for maximum efficiency.

5.Using Matrices to Analyze Terrorist Activities: Another “0-1” matrix example, but somewhat different from the previous ones, is one in which there are 6 persons suspected of participation in one or more of four activities, such as bombings, hi-jacking, etc. A 6x4 “0-1” suspect (row) by activity (column) matrix whose ijth entry is a “1” if the ith person is a suspect in the jth activity, and “0” otherwise. Column sums give the number of persons suspected of taking part in each activity, and row sums give the number of activities each suspect took part in. The connections among the suspects can be shown in a carefully constructed by "association" ”symmetric matrix whose entry in the ijth position is a “1” if person *i* worked on at least one job with person j, and “0” otherwise. All diagonal entries are set at zero. Using the “association” matrix, the “association” graph (not digraph in this case, since the matrix is symmetric) can be constructed.

More information can be obtained by finding the square of the “association” matrix—easily done using the TI-83. Students are asked to interpret the entries (off-diagonal entries give the number of groups of three in which *i* and j work together and diagonal entries give the number of groups of three of which *i* is a member).

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