

AC 2007-791: LABORATORY-SCALE STEAM POWER PLANT STUDY — RANKINE CYCLER™ COMPREHENSIVE EXPERIMENTAL ANALYSIS

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Laboratory-Scale Steam Power Plant Study – Rankine Cyclers™ Comprehensive Experimental Analysis

Abstract

The *Rankine Cyclers*™ steam turbine system, produced by Turbine Technologies, Ltd., is a table-top-sized working model of a fossil-fueled steam power plant. It is a tool for hands-on teaching of fundamentals of thermodynamics, fluid mechanics, heat transfer, and instrumentation systems in an undergraduate laboratory.

Inevitably, when a power generation plant is scaled-down and it has few efficiency-enhancing components (*e.g.*, feedwater heaters), energy losses in components will be magnified, substantially lowering the cycle efficiency from values presented in textbooks and realized in real world power plants. Therefore, faculty and students at two different universities undertook a study of the *Rankine Cyclers* to determine its effectiveness as a pedagogical tool and to characterize the device with a comprehensive experimental analysis. This analysis can be useful to faculty and students who use the equipment and can also be useful to potential customers of Turbine Technologies.

This is the authors' third and final paper about the *Rankine Cyclers*, continuing the work started in 2004-05. In the first paper two important objectives were met. First, to determine the effectiveness of the *Rankine Cyclers* as a learning tool, an indirect assessment was performed (*i.e.*, a measure of student opinion). The results were positive. Second, a parametric study of the effects of component losses on *Rankine Cyclers* thermal efficiency was performed. The results showed that the range of component losses used in the parametric study accurately reflect experimental thermal efficiencies for the device, and pointed to future experimental work.

In the second paper, two further objectives were met. First, assessment of the RC's effectiveness as a learning tool was continued. The indirect assessment was extended through more student surveys, and a more direct assessment was performed based on graded student reports and exams. Assessment results were positive and pointed toward how the equipment can be used in the best possible manner in the undergraduate curriculum. Second, experimental work performed to characterize the *Rankine Cyclers* was reported. Multiple steady state runs were performed to seek the optimum operating point and methods for measuring steam flow more accurately were proposed.

For this paper, significant experimental work was concluded to further characterize the *Rankine Cyclers*. First, more steady state runs were performed at higher voltages than previous tests to determine an optimum operating point. Second, a method for accurate steam flow measurement was developed. Third, the fuel (LP) flow calibration was verified. Fourth, the turbine and generator were studied to discover discrepancies in power output. Finally, boiler efficiency is discussed along with some recommendations.

1. Introduction

At colleges around the world, mechanical engineering students are required to learn something about the Rankine cycle. Plants using this cycle with steam as the working fluid produce the majority of the world's electricity. For many students, this is simply a pencil-and-paper exercise, and only ideal or theoretical Rankine cycles are analyzed. Therefore, many mechanical engineering graduates have only a vague understanding of this nearly ubiquitous method of power generation.

One of the best ways to enhance student learning about the steam power cycle is to visit an actual power plant and perhaps analyze some data from the plant. Many students do not have the option of visiting a power plant (because of location, time, or class size constraints), so the next best option is to operate a working model of a steam power plant in a laboratory. There are various arrangements of educational steam power generating laboratory models available, but all but a few of these operate with a reciprocating piston engine instead of a turbine. The educational units employing a piston are very costly and often too large for many university laboratories. Most of the educational units employing a turbine are also too large. The "Rankine Cyclor™", produced by Turbine Technologies Ltd. of Chetek, Wisconsin (hereinafter called the "RC"), is a tabletop steam-electric power plant that looks and behaves similarly to a real steam turbine power plant (see Figure 1). It also has the advantage of relatively low cost. About the size of an office desk, the plant contains three of the four major components of a modern, full-scale, fossil fuel fired electric generating station: boiler, turbine, and condenser. Note that the RC does not operate in a true cycle (there is no pump); it is a once-through unit (see Figure 2). Nonetheless, many of the key issues regarding steam power generation are illustrated by the device.

The RC is outfitted with sensors to measure key properties. The data are displayed in real time on a computer so that students can instantaneously observe the behavior of the plant under differing scenarios. The unit burns propane to convert liquid water into high pressure, high temperature steam (over 450°F and 120 psia) in a constant volume fire-tube boiler (see Figure 3). The steam flows into a turbine causing it to spin (see Figure 4). The turbine is attached to a generator which produces electricity. The generator can produce up to approximately 4 Watts. The steam flows from the turbine through a relatively long exhaust tube to a condenser which operates at atmospheric pressure and condenses about 1/6th of the steam into liquid water. The remaining steam is vented to the atmosphere. Because of the small scale of the unit, as well as the high back-pressure, overall efficiency is inherently very low (on the order of a few hundredths of one percent).

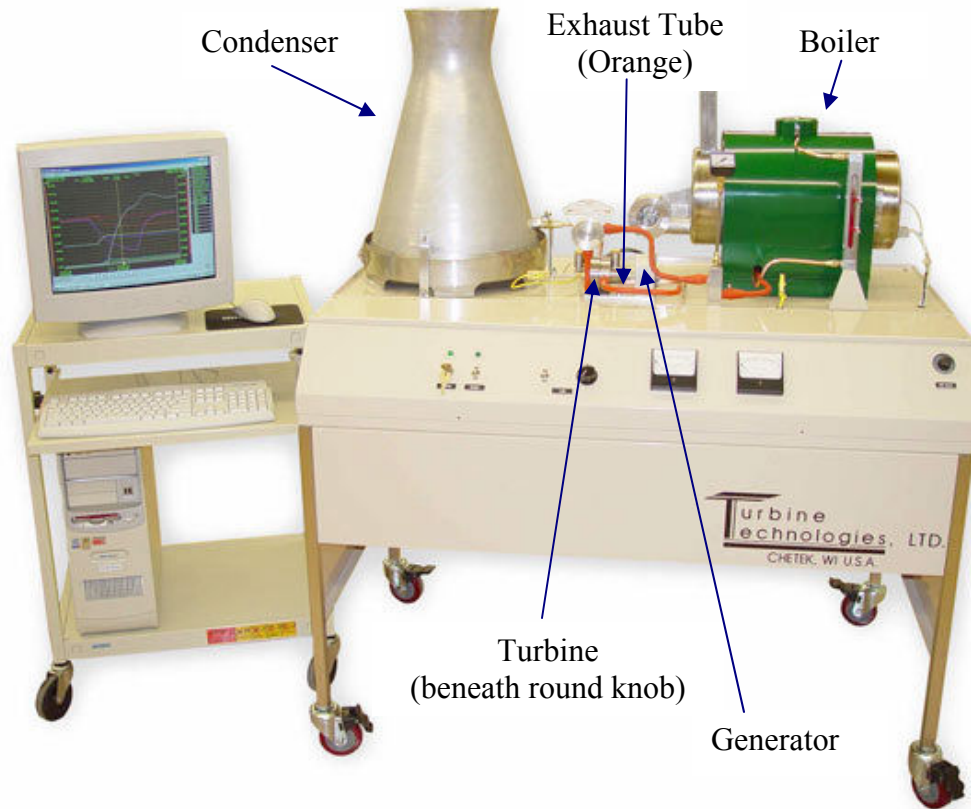


Figure 1. The Rankine Cycler. Note that newer models include a USB port data acquisition system and laptop computer that is mounted to the tabletop¹.

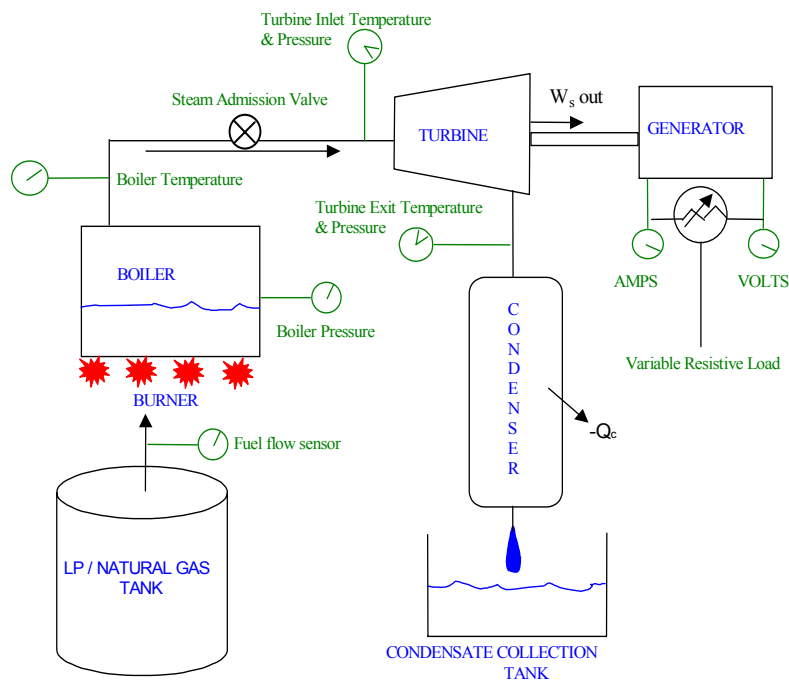


Figure 2: Schematic of the Rankine Cycler¹.

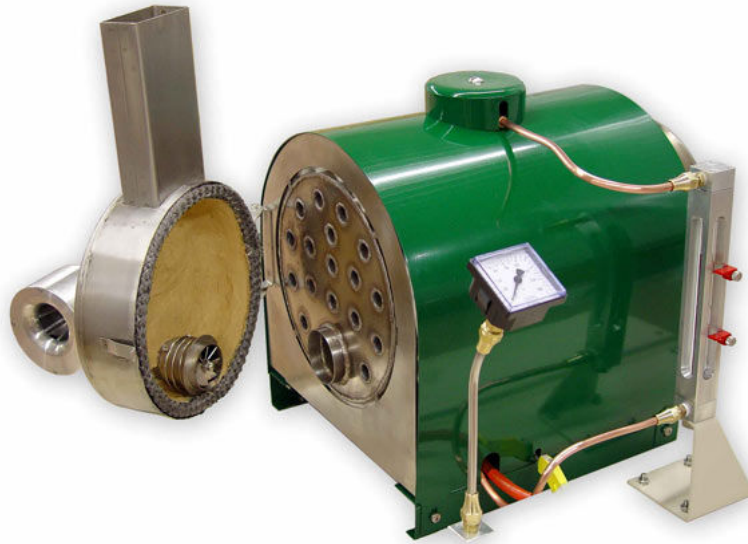


Figure 3. The dual pass, flame-through tube type (constant volume) boiler, with super heat dome¹. See Appendix A for more details.

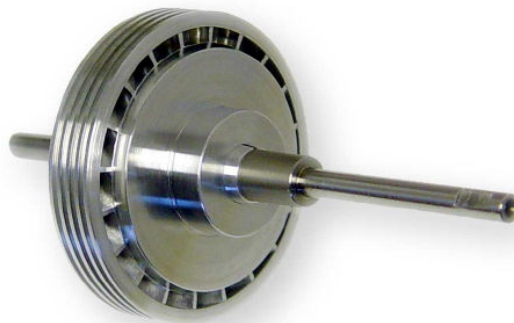


Figure 4. Single stage axial flow impulse steam turbine outside of its casing¹. See Appendix A for more details.

Previous Study and Current Objectives Lawrence Technological University (LTU) and the University of Evansville (UE) use the *Rankine Cycler* in an upper-level laboratory course, and have completed a comprehensive study of the effectiveness of the RC. This is the third and final paper, continuing the work started in 2004-05. In the first paper², two important objectives were met. First, to determine the effectiveness of the RC as a learning tool, an indirect assessment was performed; students were surveyed to assess the RC as a learning tool. Preliminary results showed that the RC and the associated calculations and reports performed quite well as a learning tool, according to the students. They reported that their knowledge of the Rankine cycle (and its associated thermodynamic concepts) increased. They indicated that discussing and operating the RC are more valuable than performing calculations with the data. The level of the material was appropriately challenging for upper-level engineering students. A few keys to successful use of the RC were also given in the paper.

Second, a parametric study of the effects of component losses on RC thermal efficiency was performed. The results showed that the range of component losses used in the parametric study accurately reflects experimental thermal efficiencies, and the results pointed to future experimental work that can be accomplished with the RC. The overall conclusion of the paper was that the benefits of the RC seem to outweigh the idiosyncrasies of the device. For its relatively low cost, the RC is useful in a mechanical engineering curriculum.

In the second paper³, two more objectives were met which extend and support the conclusions and recommendations from the first paper. First, assessment of the RC's effectiveness as a learning tool was continued. The indirect assessment of the first paper was extended through more student surveys (from both universities). With the larger sample of surveys, the results from the first paper were verified. The indirect assessment was also used to compare the RC as a learning tool between two universities with different geographical locations and RC learning objectives. The results were favorable and very similar between each university. In addition to the indirect assessment, a more direct assessment was performed based on graded exam questions and graded student reports. The assessment of exam questions indicated that the students understand the small-scale idiosyncrasies of the device, but do not understand how it differs/compares to a full-scale plant. The graded student reports indicate that the concepts of thermodynamics are reinforced through use of the RC, and that the students are relatively adept at performing the associated thermodynamic analysis.

The second objective of the second paper was to extend the comprehensive technical analysis of the RC. Multiple steady state runs were performed to seek the optimum operating point (*i.e.*, the load at which RC system or component efficiency is optimum). It was determined that a relative maximum efficiency is not attainable with current component limitations. The RC should simply be run at the highest possible load (*i.e.*, wattage, voltage, etc.). Also, a method for measuring steam flow rate easily and more accurately was suggested, and preliminary calculations with it were favorable.

For the current paper, four final objectives are met through the completion of significant experimental work which concludes characterization of the RC. First, several more steady state runs were performed at higher voltages than previous tests to seek an optimum operating point. Second, a method for accurate steam flow measurement was tested and is outlined. Third, the fuel (LP) flow calibration was verified. Fourth, the turbine and generator were studied to discover discrepancies in power output. In addition, boiler efficiency is discussed along with some recommendations.

2. Optimum Operating Point

Typical ranges of data gathered from the RC are shown in Table 1. Because the RC has several significant differences from and is much smaller than a real-world plant, the efficiencies calculated from the data are inherently low. It would therefore be most beneficial for the students to use the RC at its highest overall efficiency or optimum operating point.

	Boiler	Turbine Inlet	Turbine Outlet
Pressure (psia)	60 – 120	20 – 27	17 – 19
Temperature (°F)	350 – 600	300 – 450	275 – 400
Steam mass flow (lb/sec)	0.006 – 0.03		
Fuel flow rate (lb/sec)	0.00038		
Generator power (W)	2 - 5		

Table 1. Typical ranges of RC experimental data.

Multiple steady state runs were carefully performed to determine the optimum operating point (*i.e.*, turbine/generator performance versus load). There are multiple experimental parameters to investigate to determine optimum operating point. Figures 5 through 7 display three variations. Figure 5 shows overall efficiency (generator power divided by fuel energy input) plotted against generator power output. The relationship appears linear, so a straight line has been fit to the data. The linear equation is shown on the figure along with the Pearson product moment correlation coefficient (also known as the “r-squared value” which gives an indication of the quality of the line fit with 1.0 being a perfect fit). The figure implies that there is not an optimum point at which to operate the RC; it should simply be run at the highest possible load. However, at some power output, the efficiency should begin to drop. According to the generator manufacturer’s specifications, the maximum operating load may be the point at which the generator RPM is nearly 4500. The data collected with a maximum of 4200 RPM have nearly covered the operational range. It should be noted that it is difficult to maintain a generator RPM over 4000 with long-term steady state conditions (*i.e.*, > 60 seconds). Nonetheless, this data would indicate that the RC should be operated at the highest achievable steady power output.

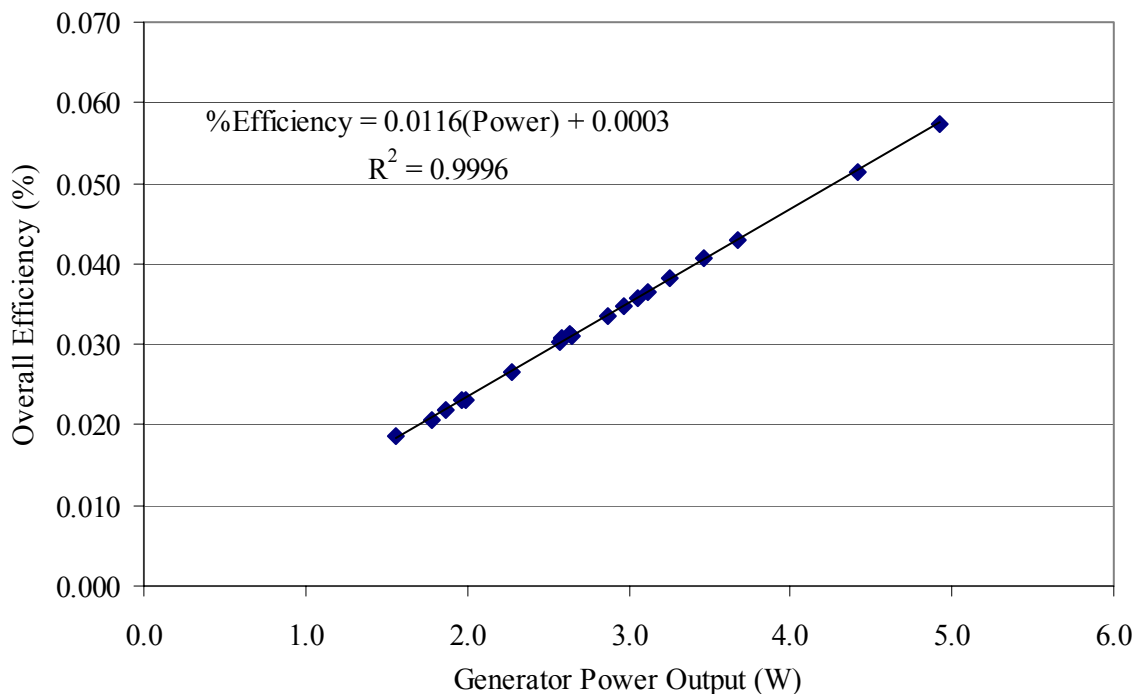


Figure 5. Overall Efficiency vs. Power Output

Figure 6 shows overall efficiency plotted against generator voltage. Generator RPM is implicitly indicated since it is related to DC output voltage by $\text{RPM} = 366.7 \text{ V}$ (information supplied by Turbine Technologies, LTD.). The efficiency vs. voltage trend is similar to the one in Figure 5 but not as distinct. This reinforces the idea that the RC should be run at maximum loading (maximum speed) for optimum conditions.

Figure 7 shows efficiency plotted with isentropic enthalpy drop from turbine inlet to atmospheric outlet. The increasing trend is again apparent.

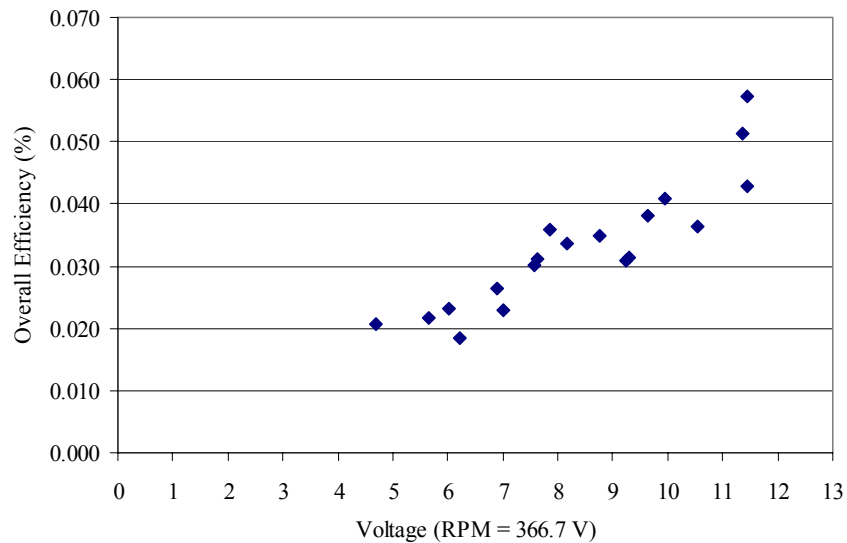


Figure 6. Overall Efficiency vs. Voltage/RPM

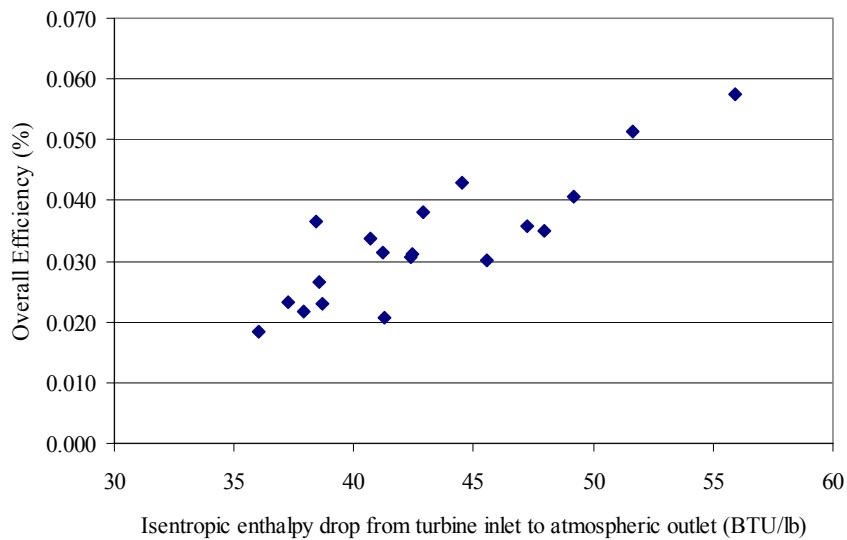


Figure 7. Overall Efficiency vs. Isentropic Enthalpy Drop from turbine inlet to atmospheric outlet

3. Using Turbine Exhaust Tube Pressure Drop to Measure Steam Flow

Possibly the least satisfying aspect of running an experiment with the Rankine Cycle is determining the steam flow rate. This requires marking the boiler water level (using a sight-glass) at the beginning and end of the data collection period, waiting a few hours for the system to cool, then draining and refilling the boiler, noting the volume of water added to move the level between the two points on the sight-glass. The volume flow rate for the test is then determined by dividing the make-up water volume by the elapsed time for the run^{*}; mass flow would then be obtained by multiplying by liquid water density. Steam mass flow rate determined by this method is highly inaccurate because of the uncertainty involved in marking water level, in draining and refilling the boiler, and in the differences in density between hot and cold water. The uncertainty in steam flow rate is likely no better than 10% - probably higher when inexperienced students are making the measurements. Clearly, a more direct and real-time steam mass flow measurement is highly desirable.

Possible approaches to obtaining such measurements would involve the installation of a small-scale flow meter such as an orifice or turbine meter. This would be problematic because it would require purchasing another instrument and it would introduce yet another pressure drop into an already highly inefficient system. Such devices would also require calibration, especially a turbine meter which would be required to operate at elevated temperatures in the steam environment.

An alternate approach that shows considerable promise is to use the turbine-to-condenser exhaust tube pressure drop to indicate the steam flow rate. The exhaust tube is a surprising 34.5 inch (88 centimeters) long and contains several bends (See Figure 1 and Figure 8), making the effective length even longer. The steam pressure drop through the tube is the order of 3 lb/in² (21 kPa). In a typical experimental run, the steam at the turbine exit is superheated, and the pressure drop along the tube ensures that it remains superheated – facilitating modeling the steam as a gas. In addition to giving a potentially reliable measurement of steam flow, the evaluation of the flow rate requires students to apply methods from fluid mechanics (compressible or incompressible), providing one more link between the laboratory and the classroom.

^{*} An alternate method of determining water volume used for a test run: 1) Note the amount of water initially placed in the boiler. 2) After the test run, let the unit cool and drain the remaining water into a graduated cylinder. 3) Subtract initial volume from final volume. This method does not account for water used in the process of warming up the unit.

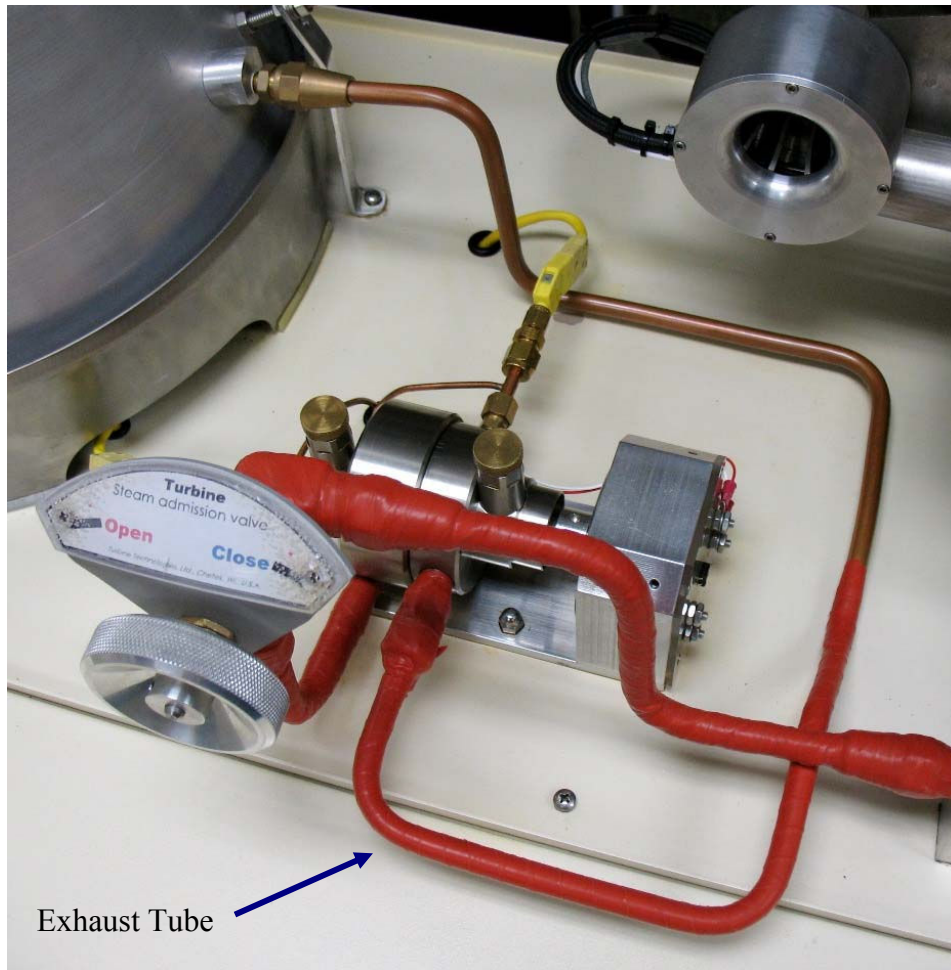


Figure 8. The turbine exhaust tube as installed on the Rankine Cycler.

With measurements of the turbine exit pressure and temperature (essentially stagnation values) available and knowing that the steam discharges to atmospheric pressure at the condenser, at least four different flow models can be used to calculate the steam flow rate. In increasing order of sophistication they are:

- a. Model as incompressible flow in a pipe, using steam density determined from turbine exhaust conditions
- b. Model as incompressible pipe flow, but account for compressibility by using steam density averaged between tube inlet (turbine exhaust) and tube exhaust conditions. This requires using the energy equation to determine the steam exhaust temperature, use of the steam tables, and a few cycles of iteration.
- c. Model as compressible, adiabatic, frictional constant area flow (Fanno flow); treat steam as an ideal gas
- d. Model as compressible, adiabatic, frictional flow (Fanno flow); treat steam as a real gas, using the adiabatic exponent from steam tables and a compressibility factor ($pv = ZRT$)

Essentially, these four models fall into two categories: “constant” density and Fanno flow. Model a can be easily derived from Model b and Model c from Model d (by setting $Z = 1$) so

only Model b and Model d will be developed here. In the subsection after model development, the calibration of the exhaust tube as a flow measurement device is described, and a resulting parameter for use in the models is determined. The final subsection compares the various steam flow measurement methods.

Model b: Constant (average) density The core of this model is the expression for pressure drop in constant density flow through a horizontal pipe

$$\Delta p = \left(f \frac{L}{D} + \sum K \right) \times \frac{1}{2} \bar{\rho} V^2 \quad (1)$$

where f is the Darcy friction factor, L and D the pipe length and diameter, $\sum K$ the sum of local loss coefficients, in this case representing tube bends, entrance loss, and exit loss, and $\bar{\rho}$ is an appropriate density. In this case, $\bar{\rho}$ is estimated as the arithmetic mean of the steam inlet (*i.e.*, turbine discharge stagnation) density and the outlet (exhaust tube discharge static) density:

$$\bar{\rho} = \frac{\rho_1 + \rho_2}{2} . \quad (2)$$

It is convenient to express the losses due to the tube bends using the “equivalent length” concept[†] so that Equation 1 becomes

$$\Delta p = \left(f \frac{L_{eq}}{D} + K_{entrance} + K_{exit} \right) \times \frac{1}{2} \bar{\rho} V^2 .$$

Solving for velocity

$$V = \frac{\sqrt{2\Delta p}}{\sqrt{\bar{\rho} \left(f \frac{L_{eq}}{D} + K_{entrance} + K_{exit} \right)}} . \quad (3)$$

Using

$$\dot{m} = \bar{\rho} V \frac{\pi}{4} D^2$$

gives

$$\dot{m} = \frac{\pi}{4} D^2 \times \frac{\sqrt{2\bar{\rho}\Delta p}}{\sqrt{f \frac{L_{eq}}{D} + K_{entrance} + K_{exit}}} . \quad (4)$$

If all parameters on the right-hand side of Equation 4 are available, the mass flow can be determined. The exhaust tube is made from 1/4 -inch nominal type K copper tubing. The standard inside diameter is $D = 0.305$ in. If more accuracy is desired, the tube can be removed and D measured with a micrometer; the tube we tested had $D = 0.302$ in. The pressure drop Δp is simply the measured gage pressure at the turbine outlet (available in standard RC data collection). The loss coefficients can be taken as $K_{entrance} = 0.5$ (square-edge entrance) and $K_{exit} = 1.0$.

[†] A calibration for the parameter $f \frac{L_{eq}}{D}$ is presented later in this paper.

Determining values for $\bar{\rho}$ and $f \frac{L_{eq}}{D}$ is somewhat more involved. For preliminary calculations, f can be evaluated in the usual way, via the Moody Chart or Colebrook-White formula from the Reynolds number ($Re = \frac{\bar{\rho}VD}{\mu} = \frac{4\dot{m}}{\pi\mu D}$) and relative roughness[‡]. The equivalent length of the pipe can be estimated from measurement of the actual length (we measured 34.5 inches) plus standard handbook values for the equivalent length for the 5-90° and 2-45° bends.

More accurate calculations of steam mass flow rate can be made by calibrating the exhaust tube to obtain a more representative value for $f \frac{L_{eq}}{D}$. Of course the presence of V (and $\bar{\rho}$) in the Reynolds number make the calculations iterative.

The final item needed for Model b is the average density (See Equation 2). The inlet density ρ_1 can be easily determined because the RC data include exhaust temperature and pressure and the steam is invariably superheated, so a steam table look-up[§] gives the specific volume, $v_1(p_1, T_1)$, and $\rho_1 = 1/v_1$. The tube outlet density is obtained by noting that the exhaust pressure, p_2 is atmospheric (zero gage). A second property for determining the exhaust density is obtained by using the energy equation^{**} and assuming that the flow process is adiabatic, giving

$$h_1 = h_2 + \frac{V_2^2}{2}$$

where h is enthalpy, and velocity at 1 is zero.

Solving for h_2

$$h_2 = h_1 - \frac{V_2^2}{2} = h_1(p_1, T_1) - \frac{8\dot{m}^2}{\rho_2 \pi^2 D^4} \quad (5)$$

Finally,

$$\rho_2 = \frac{1}{v_2(h_2, p_2)}$$

In practice, the calculations are iterative (we did them on a spreadsheet). The steps are:

1. Evaluate $\rho_1(v_1)$, h_1 , μ from available p_1, T_1 data using steam property information
2. Assume a value for \dot{m} (The value from the “drain-and-fill” method is a good starting point)

[‡] The exhaust tube is essentially smooth so $\frac{\epsilon}{D} \approx 0$

[§] Actually, we used steam property software

^{**} Left on their own, students will often assume an isentropic process for the tube flow. Although the entire Model b is approximate, modeling a process whose central feature is fluid friction as isentropic is inappropriate.

3. Calculate Re and f
4. Calculate h_2 from Equation 5 (assume $v_2 = v_1$ for the first cycle)
5. Determine v_2, ρ_2 from h_2, p_2 using steam property information
6. Calculate $\bar{\rho}$
7. Calculate a new value for \dot{m} from Equation 4 [recall $\Delta p = p_1(\text{gage})$]
8. Return to step 3 and repeat until convergence

If mass flow evaluation using Model a is desired, the average steam density, $\bar{\rho}$, is replaced by the turbine exhaust/tube inlet density ρ_1 and steps 4-6 of the iteration are omitted.

Model d: Adiabatic Compressible Flow of a Real Gas Any number of textbooks on Fluid Mechanics and/or Gas Dynamics/Compressible Flow develop the theoretical model for adiabatic, frictional flow of an ideal gas in a constant area duct; a process widely known as Fanno flow (See, for example references 4,5,6,7,8). The NASA *Compressed Gas Handbook*⁹ extends the model to real gasses by using the “compressibility factor” equation of state, $pv = ZRT$. The essence of the model is to replace “ R ” everywhere in ideal-gas relationships by “ ZR .” In addition, rather than using the ratio of specific heats, k , the NASA *Compressed Gas Handbook* suggests the use of a generalized “isentropic exponent for real gasses,” k_s (or γ), so that, for example, the speed of sound is

$$c = \sqrt{k_s ZRT}$$

Note that values for Z can be obtained from a compressibility factor chart, and k_s can be determined from extensive steam tables (such as those published by ASME). The real gas Fanno flow model can be adapted to determining the steam flow rate in the RC in a fairly straightforward manner. Using the development in the *NASA Compressed Gas Handbook* as the basis, the following equations and parameters are involved:

- Stagnation Properties: $T_{01} (= T_{1RC}), T_{02}, p_{01} (= p_{1RC}), p_{02}$
- Mach Number: $M \equiv \frac{V}{c} = \frac{V}{\sqrt{k_s ZRT}}$; M_1, M_2
- Adiabatic flow, no work: $T_{01} = T_{02} = T_0$
- Mass flow rate (evaluated at tube inlet, $T_{01} = T_1; p_{01} = p_1$) :

$$\dot{m} = \frac{p_{01} \pi D^2}{4 \sqrt{ZRT_0}} \sqrt{k_s} M_1 \left(1 + \frac{k_s - 1}{2} M_1^2 \right)^{\frac{k_s - 1}{2(k_s + 1)}} \quad (6)$$

- Choking length parameter: $f \frac{L^*}{D} = \frac{1 - M^2}{kM^2} + \frac{k + 1}{2k} \ln \left[\frac{(k + 1)M^2}{2 + (k - 1)M^2} \right]$ (7)

where L^* is the length of duct (downstream from the point represented by M) that would result in flow choking ($M = 1$)

- Stagnation pressure ratio function: $\frac{p_0}{p_0^*} = \frac{1}{M} \left[\frac{2 + (k_s - 1)M^2}{k_s + 1} \right]^{\frac{k_s + 1}{2(k_s - 1)}}$ (8)

where p_0^* is the stagnation pressure that would exist at the choking point if the flow were to become choked (*i.e.*, if $L = L^*$)

- Static-to-stagnation pressure ratio (valid for any flow process):

$$\frac{p}{p_0} = \left(1 + \frac{k_s - 1}{2} M^2 \right)^{\frac{k_s - 1}{k_s}} \quad (9)$$

In flow through a pipe of length L , the Mach number changes from M_1 to M_2 in accordance with the relationship

$$\frac{fL^*}{D}(M_2) = \frac{fL^*}{D}(M_1) - f \frac{L}{D} \quad (10)$$

where the first two terms are calculated by Equation 7 and the latter term from the actual pipe values of f , L , and D . Additionally, the stagnation pressure change that occurs is calculated by:

$$\frac{p_{02}}{p_{01}} = \left[\frac{\frac{p_0^*}{p_0}(M_2)}{\frac{p_0^*}{p_0}(M_1)} \right] \quad (11)$$

where the two pressures on the left are the actual values and the pressure ratios on the right are evaluated from M_1 and M_2 using Equation 8.

In theory, the Fanno flow model is complicated by the possibility of choking and the change of the essential behavior of the flow that occurs at $M = 1$. Happily for the Rankine Cycler application, the flow rates are low enough and the tube is short enough that the Mach numbers remain well below 1.

For application to calculating the flow rate for the RC exhaust tube, we use the measured turbine outlet/tube inlet pressure, p_1 , as p_{01} , the measured turbine outlet/tube inlet temperature, T_1 , as T_0 , the atmospheric (absolute) pressure as p_2 , and $f \frac{L_{eq}}{D}$ as $f \frac{L}{D}$. Issues involving f , Re , D and L_{eq} are identical to those for Model b, discussed above. The calculations now involve a nested iteration. The object is to determine the inlet Mach number and mass flow rate that cause the exit static pressure to match the atmospheric pressure. Calculations proceed as follows:

1. Determine values of R , Z , k , and μ appropriate for the steam temperature and pressure.
2. Assume a value for M_1 (0.2 ~ 0.25 is appropriate) to calculate $\frac{fL^*}{D}(M_1)$ and

$$\frac{p_0^*}{p_0}(M_1)$$

3. Calculate mass flow from Equation 6
4. Calculate Reynolds number and friction factor

5. Calculate $f \frac{L_{eq}}{D}$ and then $\frac{fL^*}{D}(M_2) \left[= \frac{fL^*}{D}(M_1) - f \frac{L_{eq}}{D} \right]$
6. Calculate M_2 from Equation 7 by iteration
7. Calculate p_2 as $p_2 = p_1 \times \frac{P}{P_0}(M_2) \times \frac{\frac{P_0}{P^*}(M_2)}{\frac{P_0}{P^*}(M_1)}$
8. Compare calculated p_2 to absolute atmospheric pressure; if they are not equal adjust M_1 and repeat steps 3-8.

For our calculations, we use an Excel[®] spreadsheet. Step 6 is done by fixed point iteration (actually, we iterate on M_2^2); about 5 steps is sufficient, starting from an initial guess $M_2 = 1.1M_1$. The overall iteration for M_1 is done using the Excel Solver Add-In.

If a mass flow calculation using Model c (ideal gas)^{††} is desired, simply put $Z=1$ and use the ratio of specific heats for k .

Calibration of Exhaust Tube as a Flow Measurement Device While the models outlined above are interesting, accuracy of flow measurement depends on the information supplied to the model. In practice, most flow measurement devices such as orifice plates, flow nozzles, or turbine meters must be calibrated in order to achieve the best accuracy. Calibration data for these devices is typically expressed in terms of a correction coefficient; for an orifice plate of diameter

d installed in a pipe of diameter D , the working equation would be $\dot{m} = C_d \times \frac{\pi}{4} d^2 \sqrt{\frac{\rho \Delta p}{1 - (d/D)^4}}$

with the *discharge coefficient*, C_d determined by a set of calibration experiments.

While it would certainly be possible to use the correction coefficient approach to express a calibration for the RC exhaust tube, it seemed more rational and straightforward to use

experimentally determined values for the friction-length parameter, $f \frac{L_{eq}}{D}$. Of course, parts of this parameter are easily determined, namely the actual tube length and diameter. Essentially, calibration is required because of the indefinite nature of the friction factor, f , and, especially, the equivalent length of the bends.

The friction-length parameter is expected to depend on the Reynolds number and, possibly, the Mach number. Because the Mach number of the steam flow in the exhaust tube is reasonably low ($M \sim 0.25$), it was assumed that Mach number effects on the friction-length parameter are negligible. The calibration experiments were done using cold water. Figure 9 shows the calibration apparatus.

^{††} For the actual conditions in a Rankine Cycler run, departure from ideal gas behavior is insignificant ($Z \approx 1$) and there is no discernable difference between Model c and Model d



Figure 9. Two views of the exhaust tube flow calibration apparatus

A spare exhaust tube was provided to the authors by Turbine Technologies. A plenum chamber was fabricated from steel and fittings were provided to attach an ordinary garden hose, a pressure gage, and the exhaust tube. For a typical calibration run, the hose was turned on and the system allowed to come to steady state. After steady state was established, the discharge from the tube was directed to a collection vessel and both water volume and elapsed time were measured, as was the pressure in the plenum chamber. Since the water was discharging to atmospheric pressure, the gage pressure in the chamber was also the pressure drop. Volume flow rate is obtained by dividing the volume collected by the elapsed time. This process was repeated many times at different flow rates (Flow rate was varied by opening the faucet to which the hose was attached.). Figure 10 shows the raw data.

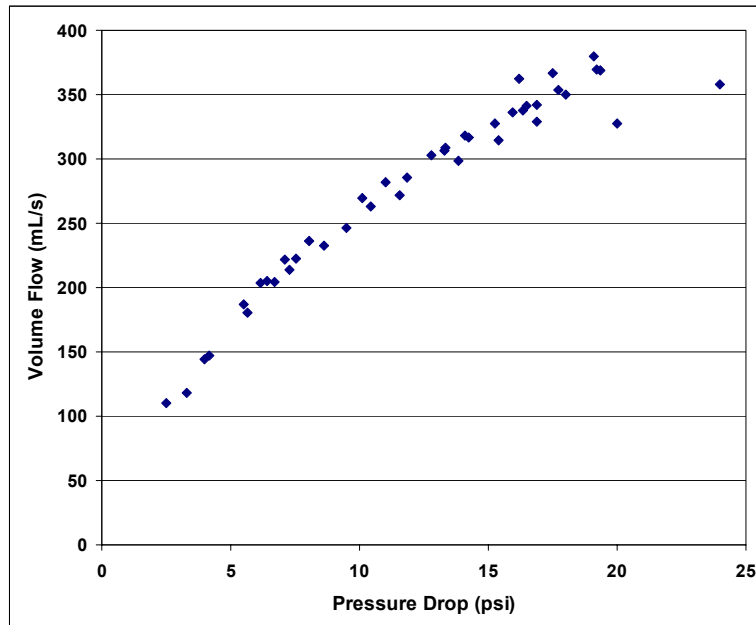


Figure 10. Raw Data for Flow Rate and Pressure

The data show a very clear trend, with the exception of the very highest flow rates, where it appears that between two and six points are outliers. The discrepancy is likely attributed to difficulty coordinating timing at high flow rates; the three most egregious outliers have been dropped from the following analysis.

The data of most interest is friction-length parameter versus Reynolds number. The friction-length parameter was calculated by

$$f \frac{L_{eq}}{D} = \frac{\Delta p \pi^2 D^4}{8 \rho Q^2} - 1.5$$

where ρ is water density, Q is measured volume flow rate, and the “1.5” is the sum of loss coefficients for the entrance and the exit. Reynolds number is calculated by $Re = \frac{4Q}{\pi D \nu}$ with ν the kinematic viscosity of the water. Figure 11 shows the friction-length parameter data.

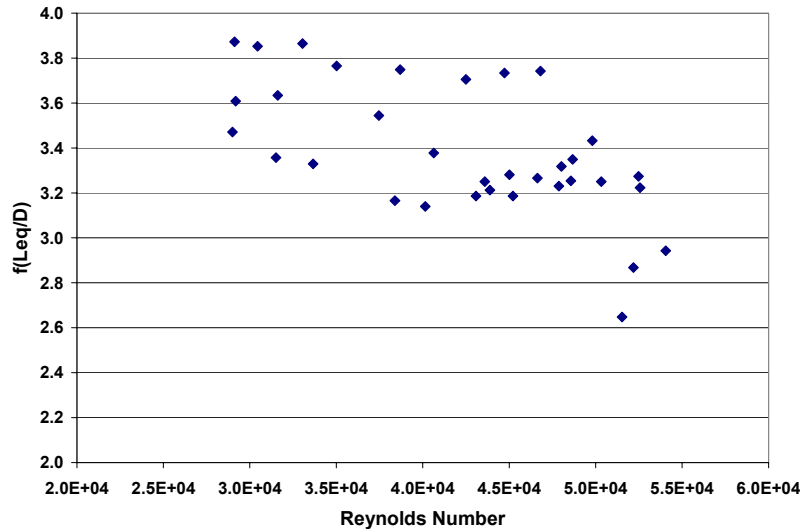


Figure 11. Friction-Length Parameter as Function of Reynolds Number

It is possible to fit the data to a correlation equation; this would suggest $f \frac{L_{eq}}{D} \sim Re^{-0.27}$. This relationship is suspect for two reasons. First, there is considerable scatter in the data, indicating significant experimental error and little reason for confidence in the fit. Second, the data cover only a small range of Reynolds number.^{‡‡} In order to bring more order to the data it was decided to imbed the Reynolds number dependence in a standard equation for f and then use the calibration data to determine a value for $\frac{L_{eq}}{D}$. We chose the Petukov equation for f , mainly because it is reported to be applicable over a wide range of Reynolds number¹⁰:

^{‡‡} The actual flow in the RC exhaust tube has $Re \sim 2.5 \times 10^4$, which is at the low end of the calibration data.

$$f = (0.790 \ln(Re) - 1.64)^{-2} \quad (12)$$

By dividing each value of $f \frac{L_{eq}}{D}$ by f calculated with Equation 12, the results shown in Figure 12 were obtained.

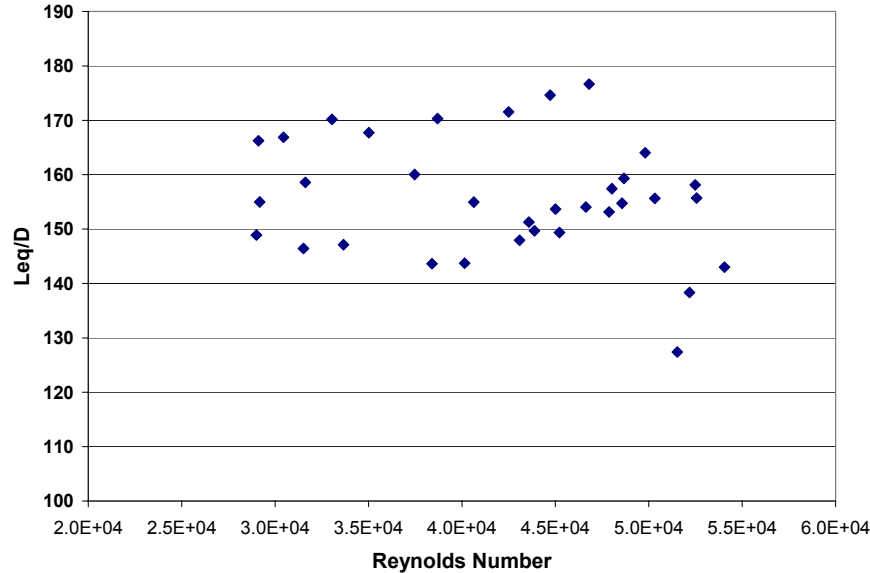


Figure 12. Equivalent Length vs. Reynolds Number from Calibration

Clearly $\frac{L_{eq}}{D}$ shows little systematic variation with Reynolds number ($\sim Re^{-0.08}$ as-is; if the remaining three “outlier” values are removed, then $\sim Re^{0.001}$). The final recommendation is to use a constant value for $\frac{L_{eq}}{D}$, calculated by averaging the data

$$\frac{L_{eq}}{D} = 154.36 \pm 3.65 \quad (12)$$

Combining Equations 11 and 12, the calibration experiments imply that the following expression be used when calculating Rankine Cyler steam flow from exhaust tube pressure drop data

$$f \frac{L_{eq}}{D} = \frac{154.36}{(0.790 \ln(Re) - 1.64)^2} \quad (13)$$

This equation may be used with any of the flow models.

Comparison of Results for Flow Measurement Steam flow was determined for 8 sets of data taken from Rankine Cyler runs using Method b, Method d, and the “drain and fill” process. (The data were taken from the runs previously reported³ and are included in the previous section on optimum operating point). Figure 13 shows a cross-plot of the flow data.

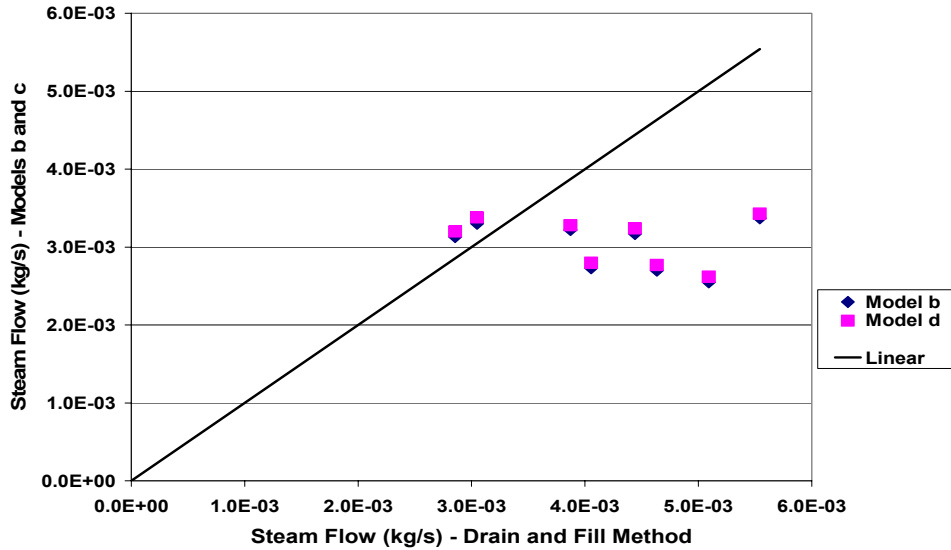


Figure 13. Comparison of Three Methods for Measuring Steam Flow

If all three methods were in perfect agreement, the data points would fall on the straight line. With experimental error, some scatter about the line would be expected. It is clear that Method b and Method d agree quite well, but neither is in agreement with the “drain and fill” method.

Figures 14 and 15 show the flow measurements plotted against the parameter $\frac{p(absolute)}{\sqrt{T(absolute)}}$.

Figure 14 is for the turbine inlet and Figure 15 is for the turbine outlet.

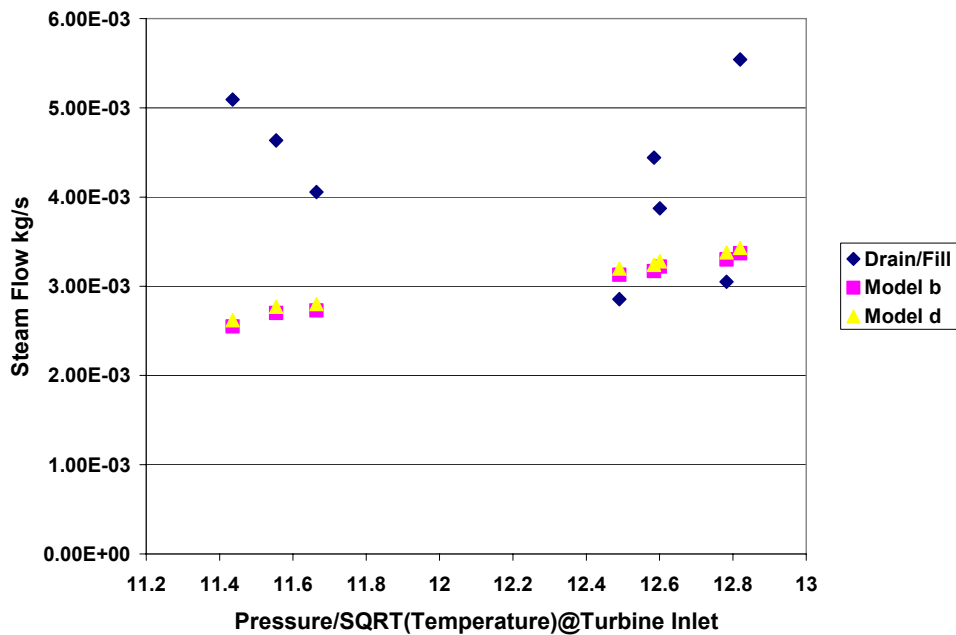


Figure 14. Measured Flow vs Turbine Inlet Compressible Flow Parameter

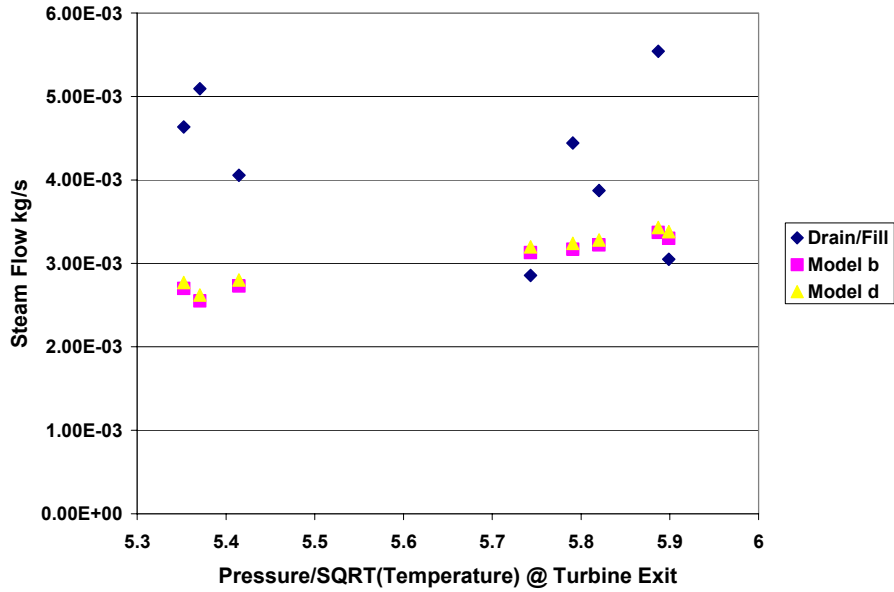


Figure 15. Measured Flow vs Turbine Exit/Exhaust Tube Inlet Compressible Flow Parameter

For any compressible flow process or device, the mass flow should correlate with p/\sqrt{T} , at least to a good approximation. It is clear that steam flow determined by Methods b and d correlate reasonably well with this parameter; it is equally clear that steam flow determined by “drain and fill” does not correlate.

Finally, consider correlation with the measured generator power output. Figure 16 shows steam flow determined by Method b and by “drain and fill” plotted against power. One would expect at least a rough correlation between flow and power if the turbine inlet properties are held relatively constant.

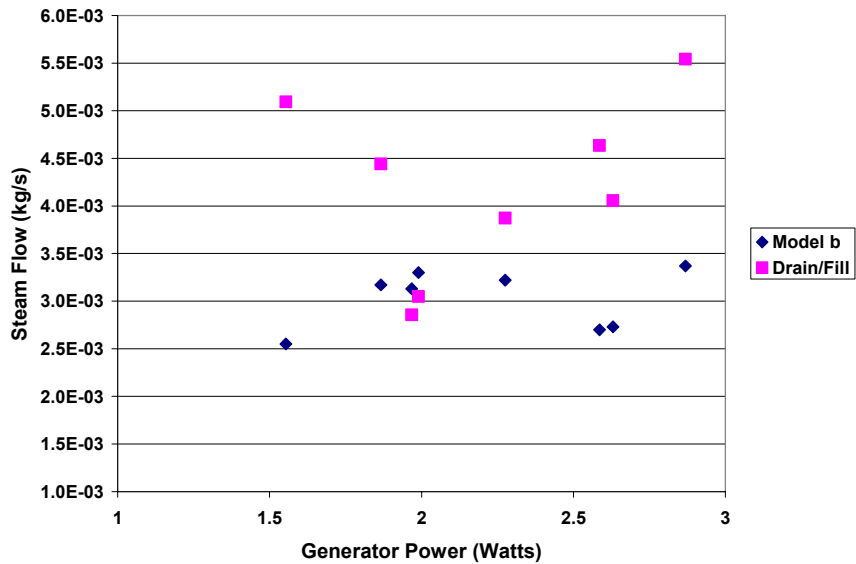


Figure 16. Steam Flow vs Generator Power

While the situation is perhaps not as clear as the previous three figures, it is at least possible to imagine a correlation between power and Method b flow, but again, there seems to be no correlation between power and “drain and fill” flow.

There are two questions to be answered: First, which of the two exhaust tube based methods is best; second, what is the best method for actually determining the steam flow? As shown in Figures 12-14, there is little difference between values of steam flow calculated from the same data by Methods b and d. Values calculated by Method b are always slightly lower than those calculated by Method d; this is probably because b explicitly accounts for local losses at the tube inlet while d ignores them. Because the Mach number is inherently low, the “average density” model (Model b) is an excellent approximation to the actual compressible flow.

The second question is the more important; which type of method gives the better answer (or, more significantly, which is closer to the correct answer). Unfortunately, the question of absolute accuracy must remain open, because no “true” values for steam flow are available. It is clear, however, that the “exhaust tube” methods are consistent with the basic physics of the system and produce results with high precision, while the “drain and fill” method, at least as executed by typical students, produces wildly varying, and hence unreliable, results. For this reason alone, the exhaust tube methods are to be preferred.

Aside from the question of accuracy, there are three other reasons to prefer exhaust-tube-based methods. First, the method can provide steam flow data in real time. It would certainly be possible to incorporate coding in the data acquisition program to indicate steam flow continuously in the same way that pressure, temperature, and fuel flow are. Second, all results are in hand when the laboratory session is complete; there is no need to wait for the system to cool and depressurize and then return to do the drain and fill. The third advantage is pedagogical. The exhaust tube methods introduce the disciplines of fluid mechanics and compressible flow into an RC experiment, thus enriching the learning experience.

For all of these reasons, the authors strongly recommend that users of the Rankine Cyclers abandon the “drain and fill” procedure and instead use the exhaust tube method for steam flow determination.

4. Fuel (LP) Flow Calibration

A second flow rate is needed for a Rankine Cyclers experiment: the fuel gas (propane or natural gas) flow^{§§}. Ultimately, the mass flow rate of propane is needed so that the energy input to the unit can be determined. In addition to the fuel flow rate, the heating value of the fuel is required. Generally likelihood of an error can be avoided by using a mass-based heating value with mass flow rate as opposed to volume flow rate and volume-based heating value. This is because of the inevitable confusion about what temperature and pressure to which the volume-based quantities are referred.

^{§§} The RC units owned by LTU and UE both operate on liquid propane (LP) gas

The RC comes from the manufacturer equipped with an in-line turbine flow meter for fuel gas volume flow measurement. The data are displayed on the computer in units of liters/minute. There are two issues that might arise concerning this measurement. First, in some early units, a volume flow correction factor was needed. Apparently, the turbine meters are calibrated with air flow and in these early units, it was necessary to correct the volume flow by multiplying by a factor corresponding to $\sqrt{\frac{\text{Air Density}}{\text{Propane Density}}}$; the manufacturer gave this factor as $\sqrt{1/1.53}$. Later units have this factor incorporated into the data acquisition software. The issue is knowing which type of unit one has.

The second issue involves the seeming ubiquitous question of determining appropriate conditions of pressure and temperature to associate with the volume flow. This is important because users must either evaluate density to use with volume flow to obtain mass flow or, alternately, obtain fuel volumetric heating values that match the volume flow reference. The perennial question is: “Do I use the pressure and temperature values corresponding to those that existed when the turbine meter was calibrated, or do I use values that exist in the laboratory when the experiment was run, or do I use some ‘standard’ value?”

These two issues can be avoided by measuring mass flow instead of volume flow, or at least a few mass flow measurements can be made simultaneously with the volume flow measurements so that questions of pressure, temperature, and density can be sorted out. If the RC is fueled from a “standard” propane bottle such as is used on a gas grill, obtaining a reference LP flow is relatively easy. To do this, we set the propane bottle on a digital scale capable of high precision measurement. Students simply note the indicated weight of the gas bottle at the beginning of a run and at the end of a run. The difference in weight can be converted to mass and divided by the elapsed time to determine the mass flow average. A few such runs can be used to cross-check the volume flow-based data to determine if the “correction factor” is needed and to determine appropriate values for reference pressure and density. Alternatively, the scales can be used for every run to give an independent reading for fuel flow, enhancing accuracy.

The tank-weighing method was investigated at the University of Evansville, leading to a very helpful outcome. When UE’s Rankine Cyclor was acquired, the information available indicated that it was an “old-type” unit which required that the indicated volume flow rate be corrected from an “air” value to a “propane” value. The operational assumption was that the fuel flow meter had been calibrated using air at 740 mm.Hg pressure and 23°C, so that the indicated volume flow would be multiplied by $\sqrt{1/1.53}$ to produce the correct propane flow. Mass flow was then obtained by multiplying by the density of propane at 740 mm.Hg pressure and 23°C (0.001764 kg/L). Table 2 shows the results of two runs made by students in Spring 2006.

Indicated Volume Flow(L/min)	"Corrected" Volume Flow (L/min)	Propane Used (kg)	Time (min)	"Corrected" Mass Flow (kg/min)	Uncorrected Mass Flow (kg/min)	Mass Flow $\Delta m/\Delta t$ (kg/min)
6.17	4.99	0.073	5.842	0.00880	0.01088	0.01250
6.19	5.00	0.055	5.068	0.00883	0.01092	0.01085

Table 2. Comparison of Fuel Flow Results

These data clearly indicated that the correction factor should not be applied in UE's unit; although it is inconclusive as to whether to use the propane density at the calibration conditions of 740 mm.Hg and 23°C or at the actual laboratory conditions, which were fortuitously nearly the same as the calibration conditions.

5. The Turbine Enigma

In attempting to model and evaluate the components of the Rankine Cyclor, the authors have uncovered some serious issues; collectively we call these the "Turbine Enigma". In general, there are discrepancies among the actual generator output, the calculated turbine output, and the maximum theoretical output. The essential points are outlined as follows.

The following are either given (by the manufacturer) or assumed/deduced information about the steam turbine, the generator, and the steam property data acquisition system:

- The turbine is a single-stage impulse type.
- The turbine is directly coupled to the generator so that
TURBINE RPM = GENERATOR RPM .
- GENERATOR RPM is related to DC output voltage by $RPM = 366.7 V$ (this information supplied by Turbine Technologies and verified in a 2006 e-mail correspondence).
- Sensors for turbine inlet steam properties (p, T) and outlet steam properties (p, T) are located in (relatively) large volume "plenum" regions indicating that the properties measured are stagnation values.
- Steam turbine wheel diameter is about 2.0 inches (Information supplied by Turbine Technologies).

Using as a typical set of experimental data "Run 11/23/04-1" from LTU (reported to be "excellent data" – data from this run were included in the authors' previous paper²) the following have been measured and calculated:

- Steam flow: 0.00305 kg/s (roughly verified by "turbine exhaust tube pressure drop" calculation)
- Voltage: 6.03 V
- Generator Output Power: 1.99 W
- Enthalpy drop in turbine (from p, T measurements and steam tables): 10.65 Btu/lb

Using these values, various specific energy terms can be calculated:

- Enthalpy drop ($10.65 \text{ Btu} / \text{lb} \times 2.2 \text{ lb} / \text{kg} \times 1054 \text{ J} / \text{Btu} =$) 24,700 J / kg
- Specific work from enthalpy drop ($q \approx 0; \Delta PE = 0; \Delta KE \approx 0$ so $w \approx \Delta h =$) 24,700 J/kg
- Maximum possible work extraction by a single-stage impulse turbine (See Appendix A)

$$w_{\max} = 2U^2 = 2(r\omega)^2 = 2\left(\frac{D}{2} \frac{2\pi}{60} RPM\right)^2 = 2 \times \left(\frac{2.0 \text{ in}}{2} \times 0.0254 \frac{\text{m}}{\text{in}} \times \frac{2\pi}{60} \times 366.7 \frac{RPM}{V} \times 6.03V\right)^2$$

$$w_{\max} \approx 69.2 \text{ J} / \text{kg}$$

- Specific work indicated by generator output $w = \frac{P_{gen}}{\dot{m}} = \frac{1.99 W}{0.00305 kg/s} = 653 J/kg$

The first point to note is that instead of $w_{gen} \approx w_{\Delta h} \approx w_{max}$ as expected, we have $w_{gen} \approx \frac{1}{40} w_{\Delta h}$, which implies a very inefficient generator and/or a very low system mechanical efficiency.

Even more surprising is that $w_{gen} \approx 10 w_{max}$, which implies that the system violates both the First and Second laws of thermodynamics, as well as Newton's Second Law (in the form of the Moment of Momentum equation)!

If we redo the calculations on a power basis instead of a specific work basis:

$$P_{gen} = 1.99 W ; P_{enthalpy\ drop} = 75 W ; P_{max\ possible} = 0.21 W$$

again one unlikely and one "impossible" situation.

The differences between enthalpy drop work/power and generator values could possibly be explained by heat loss to the relatively massive turbine housing and the surroundings, together with mechanical and generator inefficiency (although significant heat loss would be one more serious discrepancy between the RC and a real world power plant). Preliminary estimates seem to imply that heat loss alone is not sufficient to account for the observed difference.

The discrepancy between maximum power and measured (electrical) power is more serious, as it implies the violation of all three of Mechanical Engineering's most sacred laws. The authors have investigated several possibilities. Of course, an obvious culprit would be the comparison of "theory" with the "real world"; however, the theory predicts the maximum work/power under ideal conditions. The real work/power would certainly be expected to be lower than the ideal; but a real power ten times the ideal is predicted.

The next suspect is the data. Independent measurements of generator voltage and current have verified the RC-measured values – verifying the electrical power output. The steam flow is certainly a questionable measurement, but the "standard" (drain-and-fill) method has been verified by the exhaust tube pressure drop method. (Recall, we are searching for a 1000% effect – not a 10% effect)

The final item to consider is the rotational speed. As noted above, the RPM/Voltage relationship has been verified by the manufacturer by mounting a typical RC generator on a precision milling machine spindle and measuring the voltage and speed. Separate stroboscopic measurements of generator speed by the authors has essentially confirmed the speed at about 2200 RPM. No one has confirmed the turbine rotor speed; however drawings and photographs of the disassembled turbine indicate a direct drive between turbine wheel and generator.

It is instructive to consider what would be an appropriate speed for the turbine wheel, given the actual size of the wheel, the existing nozzle and blade geometry and the actual operating pressures and temperatures of the RC. First consider the steam velocity leaving the nozzle. Using an ideal gas model for the superheated steam, the velocity can be calculated by

$$C_1 = \left[2 \times \eta_N \times c_p T_{TurbineInlet} \times \left(1 - \left(\frac{P_{TurbineExhaust}}{P_{TurbineInlet}} \right)^{\frac{k-1}{k}} \right) \right]^{\frac{1}{2}}$$

Using values typical for a Rankine Cycler run

$$c_p \approx 0.48 \frac{Btu}{lbm \cdot R^o}; k \approx 1.3; T \approx 300F = 760R; p_{in} \approx 10 \text{ psig} = 24.7 \text{ psia}; p_{out} \approx 3 \text{ psig} = 17.7 \text{ psia}$$

and assuming for the “slot” nozzles $\eta_N \approx 75\%$ gives

$$C_1 \approx \left[2 \times 0.75 \times 0.48 \frac{Btu}{lbm \cdot R} \times 760R \times \left(1 - \left(\frac{17.7}{24.7} \right)^{0.231} \right) \times 32.174 \frac{ft \cdot lbm}{lbf \cdot sec^2} \times 778.16 \frac{Btu}{ft \cdot lbf} \right]^{\frac{1}{2}} = 1007 \frac{ft}{sec}$$

For the “optimum” impulse turbine, the blade speed should be about one-half the inlet absolute velocity ($\frac{U}{C_1} \approx \frac{1}{2}$), so the blade speed should be $U \approx 500 \frac{ft}{sec}$ and the rotational speed for a 2-inch wheel should be

$$\omega = \frac{U}{r} \approx \frac{500 \frac{ft}{sec}}{0.0833 \text{ ft}} \approx 6000 \frac{rad}{sec} \approx 57,000 \text{ RPM} .$$

The maximum work would then be

$$w_{max} = 2U^2 \approx 2 \times 500^2 / (32.174 \times 778.16) \approx 20 \text{ Btu} / \text{lb}$$

which is much more in line with the observed enthalpy drop of about 10 Btu/lb.

Having exhausted all of the possibilities (electrical data, steam flow data, speed/voltage correlation, turbine speed/generator speed relationship) the authors can find no rational explanation for the generator work/power being (significantly) larger than the “maximum” values. Any resolution of this dilemma would be very welcome.

6. Boiler Efficiency

If students are asked to develop a theoretical model of the entire unit, then information on boiler efficiency is needed. In a typical power plant, boiler efficiency is defined by:

$$\eta_{Boiler} \equiv \frac{\text{Heat added to Steam}}{\text{Energy in fuel}} = \frac{\dot{m} \Delta h)_{steam}}{\dot{m} \text{ HHV})_{fuel}} . \quad (14)$$

For simple boilers (e.g., gas-fired units), efficiency may be determined from measurements of the parameters in this equation. For actual power plant boilers, especially those fired by coal,

boiler efficiency is determined by using the so-called *energy balance method* (also called the *loss method*), in which the energy losses per unit fuel energy input are subtracted from 100%. This method has the advantage of higher accuracy but requires chemical analysis of the fuel and measurements of exhaust temperature and chemical composition. The ASME *Performance Test Code for Fired Steam Generators* can be consulted for details¹¹.

The parameters required for Equation 14 can be determined relatively easily, if not accurately. Steam and fuel flow rates are determined as part of a typical RC experimental run, either by the traditional methods or as discussed in this paper. Fuel *HHV* (Higher Heating Value) is also determined in a typical run. The only questionable quantity is the enthalpy rise of the steam. Boiler outlet enthalpy is easily determined from measured boiler pressure and temperature; it is the boiler “inlet” enthalpy that is the problem. Of course, since the RC does not operate in a steady flow mode, there is no “input” enthalpy during a run. That leaves three seemingly logical choices for evaluating input enthalpy:

1. Assume $h_{steam\ in} = 0$ (Essentially, specifying the input as liquid at 32°F).
2. Evaluate the enthalpy of liquid water at the temperature of the make-up water used to fill the boiler.
3. Use the enthalpy of saturated liquid water evaluated at the boiler discharge pressure (or essentially, the enthalpy of the water that is being evaporated to steam during a run).

In reality, none of these choices makes much sense; in fact using some of them can result in a computed boiler efficiency greater than 100%! Choices 1 and 2 include the energy used to pre-heat the water and heat up the boiler metal parts to the operating temperature as useful (steady state) output, while choice 3 excludes any energy used to heat the water to saturation temperature and is influenced by “free” heat from pre-heated boiler parts and by any steam that is generated from dropping boiler pressure during the transient process. *In short, because the boiler undergoes a transient process, conventional concepts of boiler efficiency are meaningless, at least in terms of measurement.* If a value for boiler efficiency is desired for a comprehensive steady-flow analytical model of the RC, it is safe to assume that the RC boiler efficiency is around 85% which is within the typical range of 75% to 95% for full-scale power plant boilers^{12***}.

It is perhaps meaningful to inquire about the boiler’s combustion efficiency, defined as the energy generated by burning the fuel divided by the fuel heating value. Combustion efficiency is less than 100% if some portion of the fuel is unburned. Determination of this efficiency would require a chemical analysis of the boiler exhaust gas stream to determine the presence of unburned carbon or hydrogen. Such an analysis is probably beyond the scope of an investigation of the RC as a steam-cycle device. In any event, the combustion efficiency is probably very near 100% as the fuel is gas and the air supply and air/fuel mixing of the RC are well-designed.

*** For a gas-fired boiler such as that on the RC, the dominant losses would be “dry gas loss” (*i.e.*, energy in the hot, dry products of combustion in the exhaust stack) and “water from Hydrogen in fuel loss” (*i.e.*, enthalpy of vaporization of any H₂O formed from combustion of Hydrogen).

7. Recommendations

Besides the general RC characterization given in this paper and the previous two papers^{2,3}, some further work is recommended to characterize any given individual RC. The following three studies may enhance the usefulness of the RC to determine parameters such as output and efficiency.

1. **Boiler Performance:** If an RC user (*e.g.*, student or instructor) deems boiler combustion efficiency determination a useful exercise, it should be investigated for various operating conditions. As a minimum, boiler combustion efficiency determination will require exhaust gas unburned carbon and, possibly, hydrogen measurements.
2. **Generator Performance:** An investigation of generator efficiency, separate from the turbine, should be made.
3. **Second Law Analysis:** An exercise of considerable educational value would be to conduct a Second Law analysis of the unit. Because of its small scale, and high losses together with the rather complete set of thermodynamic data available, the RC is an excellent device for performing a second law analysis. Not only would the students benefit from performing a second law analysis (a topic that receives little or no coverage in the required Thermodynamics courses at LTU and UE), it would also give a better understanding of scaling drawbacks and help identify the major sources of losses.

8. Conclusions

Pedagogical and experimental characterization of the Rankine Cyler has been carried out in three papers, of which this is the last. Pedagogically, the RC has been shown to be a useful device, as shown by both subjective and objective assessment^{2,3}. Several experimental investigations have been carried out; they have two general objectives:

- First, to suggest improvements to data collection methods and to investigate optimum operating conditions
- Second, to experimentally characterize RC components, with the aim of allowing users (faculty or students) to construct an analytical model of the unit.

We consider that the first objective has been successfully met. Major contributions are the development of a more accurate, real-time method for measuring steam/water flow by using the pressure drop along the exhaust tube, the investigation of a direct-weighing method for checking fuel flow meter calibrations, and determining the optimum operating point for the unit.

The component characterizations are of less value. Our studies showed that the two major components, the boiler and turbine, cannot be reasonably characterized from experiments. The boiler efficiency cannot be determined because it is an ill-defined concept in this transient flow system. Probably the most significant uncharacterized component is the turbine; the issue identified by us as the “Turbine Enigma” is, at least to us, a barrier to any further attempts to

characterize the turbine performance. We were successful, however, in characterizing the turbine exhaust tube!

Based on our extensive studies, we conclude that the Rankine Cyclor is of most value 1) when used to give the students an opportunity to observe and operate a real power plant, 2) when used to generate real-world data for calculations, and 3) for illustrating real-time, computer-based data acquisition in the thermal sciences. On the other hand, because of the idiosyncrasies of the transient operation, the extremely small scale of the system, and the difficulties in characterizing the principal components, the Rankine Cyclor is not a good basis for an analytical model of a power generating system. In conclusion, if used for the proper objectives, the educational benefits of the RC can outweigh the idiosyncrasies of the device. For its relatively low cost, the RC is useful in the mechanical engineering curriculum.

Acknowledgements

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Appendices

A. Experimental Apparatus Descriptions¹

The experimental hardware (Rankine Cyclor™) consists of multiple components that make up the necessary components for electrical power generation (utilizing water as the working fluid). These components include:

1. Boiler

A stainless steel constructed, dual pass, flame-through tube type boiler, with super heat dome, that includes front and rear doors. Both doors are insulated and open easily to reveal the gas fired burner, flame tubes, hot surface igniter and general boiler construction. The boiler walls are insulated to minimize heat loss. A side mounted sight glass indicates water level.

2. Combustion Burner / Blower

The custom manufactured burner is designed to operate on either LP or natural gas. A solid-state controller automatically regulates boiler pressure via the initiation and termination of burner operation. This U.L. approved system controls electronic ignition, gas flow control and flame sensing.

3. Turbine

The axial flow steam turbine is mounted on a precision-machined stainless steel shaft, which is supported by custom manufactured bronze bearings. Two oiler ports supply lubrication to the bearings. The turbine includes a taper lock for precise mounting and is driven by steam that is directed by an axial flow, bladed nozzle ring. The turbine output shaft is coupled to an AC/DC generator.

4. Electric Generator

An electric generator, driven by the axial flow steam turbine, is of the brushless type. It is a custom wound, 4-pole type and exhibits a safe/low voltage and amperage output. Both AC and DC output poles are readily available for analysis (rpm output, waveform study, relationship between amperage, voltage and power). A variable resistor load is operator adjustable and allows for power output adjustments.

5. Condenser Tower

The seamless, metal-spun condenser tower features 4 stainless steel baffles and facilitates the collection of water vapor. The condensed steam (water) is collected in the bottom of the tower and can be easily drained for measurement/flow rate calculations.

6. Data Acquisition (Note: Newer RC models have an updated system that will operate through the USB port of any newer PC.)

The experimental apparatus is also equipped with an integral computer data acquisition station, which utilizes National Instruments™ data acquisition software (modified 2004 models).

The fully integrated data acquisition system includes 9 sensors. The sensor outputs are conditioned and displayed in “real time”- on screen. Data can be stored and replayed. Run data

can be copied off to floppy for follow-on, individual student analysis. Data can be viewed in Notepad, Excel and MSWord (all included).

The system is test run at the factory prior to delivery and the “factory test run” is stored on the hard drive under the “My documents” folder. This file should be reviewed prior to operation, as it gives the participant an overview of typical operating parameters and acquisition capability.

7. Sensors

Nine (9) sensors are installed at key system locations. Each sensor output lead is routed to a centrally located terminal board. A shielded 64-pin cable routes all data to the installed data acquisition card. This card is responsible for signal conditioning and analog to digital conversion. Software and sensor calibration is accomplished at the factory prior to shipment.

Installed sensor list includes:

- Boiler pressure
- Boiler temperature
- Turbine inlet pressure
- Turbine inlet temperature
- Turbine exit pressure
- Turbine exit temperature
- Fuel flow
- Generator voltage output
- Generator amperage output

8. Overall System Dimensions

Length:	48.0 inches (122 cm)
Width:	30.0 inches (77 cm)
Height:	58.0 inches (148 cm)

B. Analysis of Axial Flow Impulse Turbine with Application to Rankine Cyclor

Figure B.1 shows one blade of an axial flow impulse turbine. The linear speed of the blade is U and is the product of blade radius and angular velocity (rotational speed). The blade speed is the same on either side of the wheel. Station (1) is the inlet and station (2) is the discharge. In the velocity diagrams, W is the velocity relative to the blade and C is the absolute velocity (relative to the nozzle and casing).

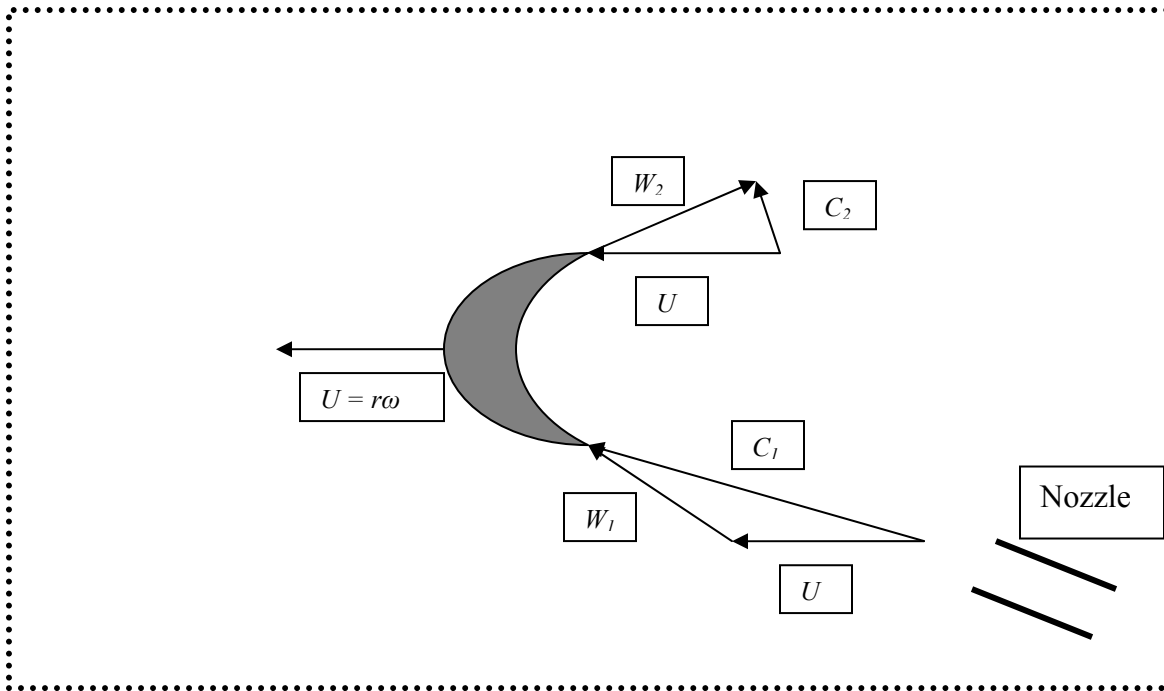


Figure B.1 Axial Flow Impulse Turbine Blade with Velocity Diagrams

In an impulse turbine, all of the pressure drop occurs in the nozzle. This produces a relatively large steam velocity (C_1). Because the flow through the rotor occurs at constant pressure, an impulse turbine can use *partial admission*, with a finite number of jets of steam placed around the wheel. The Rankine Cycle turbine uses partial admission, with several slot nozzles directing steam onto the turbine blades. Because there is no pressure drop in the wheel (blades), all of the work is done by turning the fluid and there is a large change in C ($C_2 \ll C_1$).

An ideal impulse turbine stage has the following characteristics:

- The fluid enters and leaves the blade perfectly (W_1 & W_2 tangent to blade surface)
- There is no heat transfer
- There is no friction between fluid jet and blade

Combining the last two implies that the ideal flow process is isentropic.

Now consider the relationship between the inlet and outlet velocities. Since the pressure is constant, the flow *relative to the blade* is neither accelerated nor decelerated, therefore

$$W_2 = W_1.$$

Conservation of mass for the (steady) flow through the stage demands

$$\rho_1 C_{a1} (s \times h) = \rho_2 C_{a2} (s \times h)$$

where s is the blade-to-blade spacing, h is the blade height, and C_a is the axial velocity component (perpendicular to U). Because pressure, entropy, and enthalpy are all unchanged across the blade row, density is also unchanged and

$$C_{a1} = C_{a2}$$

also, from the velocity diagram

$$W_{a1} = C_{a1} = C_{a2} = W_{a2}$$

This means that the altitude of both velocity triangles is the same.

Now consider the maximum possible specific work extraction from the steam. This will be examined two different ways: from an energy perspective and from a momentum perspective.

Energy

Obviously, the maximum work extraction requires that the bulleted conditions above be satisfied (no heat loss, no friction). Because there is no heat loss and no enthalpy change, the energy balance becomes

$$w = \frac{C_1^2}{2} - \frac{C_2^2}{2}$$

(*i.e.*, all of the work comes from changing the fluid kinetic energy). The inlet kinetic energy is set by the nozzle process so C_1 cannot be affected by the blade. The maximum work will be realized when C_2 is smallest. Because C_{a2} is fixed by mass flow rate, C_2 will be smallest when it is perpendicular to U (*i.e.*, when $C_2 = C_{a2}$ and $C_{u2} = 0$). Thus

$$w_{\max} = \frac{C_1^2}{2} - \frac{C_{a2}^2}{2} = \frac{C_{U1}^2 + C_{a1}^2 - C_{a2}^2}{2} = \frac{C_{U1}^2}{2} \quad \text{because } C_{a1} = C_{a2}$$

From the velocity diagrams

$$C_{U1} = U + W_{U1}$$

But

$$W_2 = W_1 \quad \text{so} \quad W_1^2 = W_2^2$$

and thus

$$W_{U1}^2 + W_{a1}^2 = W_{U2}^2 + W_{a2}^2.$$

However

$$W_{a1} = W_{a2} \quad \text{so} \quad W_{U1}^2 = W_{U2}^2$$

and

$$W_{U1} = W_{U2}.$$

For the maximum work condition, the exit diagram would be a right triangle, with $W_{U2} = U$

Substituting into the work equation

$$w_{\max} = \frac{C_{U1}^2}{2} = \frac{(U + W_{U1})^2}{2} = \frac{(U + U)^2}{2} = 2U^2.$$

Momentum

Euler's turbine equation (from the Moment of Momentum equation) is

$$w = U_1 C_{U1} - U_2 C_{U2}$$

Obviously, the work will be maximum when $U_2 C_{U2} = 0$ (*i.e.*, when the outlet velocity is perpendicular to U [once again, a right triangle]). The maximum work is

$$w_{\max} = U_1 C_{U1}$$

From the analysis above

$$w_{\max} = U C_{U1} = U(U + W_{U1}) = U(U + W_{U2}) = U(U + U) = 2U^2$$

So by either energy or momentum, the maximum possible specific work output for an impulse turbine is

$$w_{\max} = 2U^2$$